Isaac Physics Skills

Developing mastery of essential pre-university physics

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Marking of answers and compilation of results is free on Isaac Physics.
Register as a student or as a teacher to gain full functionality and support.

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Notes for the Student and the Teacher

These sheets are of two kinds - skill sheets and fact sheets.

The skill sheets provide practice for the student in applying a single principle of physics to a range of reasonably straightforward situations - often starting out with the substitution of values into an equation. After the first few questions have enabled the student to gain confidence in using the equation, subsequent questions may require the use of more than one equation or principle, or insight to solve a novel problem.

The fact sheets test knowledge on the parts of sixth form physics courses which require reading. While eminently suitable for revision, they may also be used to verify that the student has performed prior reading on the operation of an MRI scanner, for example, before it is discussed in class.

The sheets are equally valuable in the early stages of learning and in revision.

While the manner of use is up to the teacher and the student, we recommend that until a pass mark (we suggest 75%, as indicated in the square by each skill sheet) is obtained, the student studies further, then repeats a selection of questions. This process is repeated until the student passes. The teacher's mark book records how many attempts the student has taken rather than the mark obtained on a sole attempt. Students, likewise, have a list of the skills, and tick them off when the required level of proficiency has been obtained. A grid is provided at the front of this book for this very purpose. In this way, all students achieve mastery of the skills, and do not move on until this has been achieved. We have found that all students who have the capacity to pass an A-level course have the capacity to attain mastery of all of these skills. However, we find that if such mastery has been obtained, the student goes on to gain a greater understanding of physics and a higher grade at A-level than would be expected otherwise.

We also recommend that an answer is not considered correct unless it is numerically accurate, is given to an appropriate number of significant figures, and incorporates a suitable unit: a student cannot be satisfied with their comprehension while they are still getting the final result incorrect. After all, this would be intolerable in any practical situation in which physics principles were applied.

The Isaac Physics on-line version of this book, and its associated marking facility, follow these significant figure and unit requirements.

ACM & JJC
Westcliff-on-Sea & High Wycombe, 2015

Acknowledgements

These sheets, and the approach presented here for using them, were first devised for use in the Physics Department at the Royal Grammar School in High Wycombe, where we served as colleagues from 2010 to 2014. We are grateful to the students who have put them to such productive use, and have given valuable feedback. We are also grateful to colleagues in other schools who have also applied these methods and given us encouragement – particularly Keith Dalby at Westcliff High School for Boys.

However, our greatest thanks are to our colleagues in High Wycombe who tirelessly used these methods with their students to bring about better learning - to Peter Glendining, to Matthew Hale (who also devised an excellent spreadsheet for the monitoring of student progress on these sheets), and to Paula Dove who suggested the need for fact sheets.

We are also grateful to Prof. Mark Warner and his team at the Cavendish Laboratory for their assistance in turning a set of A4 problem sheets into a professional publication. Much credit is due to James Sharkey and Aleksandr Bowkis for the typesetting and editing. We are also grateful to Robin Hughes and Ally Davies for their thoughtful comments on the draft of this text.

Anton has three personal ‘thank yous’ to record: firstly to the Civil Aviation Authority. Its insistence that all student pilots gain mastery in all ground school topics inspired the development of a similar approach to A-level Physics teaching. Secondly, to Jennifer for her support and feedback in the implementation of the project, not to mention the enthusiasm with which she has championed its adoption, and her extensive written contributions. Most importantly, he wishes to acknowledge Helen Machacek. For not only has she remained the most supportive, encouraging and loving wife imaginable, but she bore much more than her fair share of the responsibility of looking after their children during the holiday periods when her husband was writing many of the sheets.

Jennifer wishes to thank Anton, for being such an inspiration, both as a physicist and as a friend. He has opened her eyes to so many ideas and possibilities in the teaching of Physics and in interactions with students, and this skills sheet approach is just one example of that. He has been unendingly supportive and his wisdom, advice and example have contributed hugely to the teacher and to the person she is today.

And finally, we thank you for being willing to try them out too. We wish you well with your studies, and hope that these sheets help you and your students attain understanding and success.

Soli Deo Gloria,

ACM & JJC
Westcliff-on-Sea & High Wycombe, 2015

Using Isaac Physics with this book

Isaac Physics offers on-line versions of each sheet at `isaacphysics.org/physics_skills_14` where a student can enter answers. This on-line tool will mark answers, giving immediate feedback to a student who, if registered on isaacphysics.org, can have their progress stored and even retrieved for their CV! Teachers can set a sheet for class homework as the appropriate theme is being taught, and again for pre-exam revision. Isaac Physics can return the fully assembled and analysed marks to the teacher, if registered for this free service. Isaac Physics zealously follows the significant figures (sf) rules below and warns if your answer has a sf problem.

Uncertainty and Significant Figures

In physics, numbers represent values that have uncertainty and this is indicated by the number of significant figures in an answer.

Significant figures

When there is a decimal point (dp), all digits are significant, except leading (leftmost) zeros: 2.00 (3 sf); 0.020 (2 sf); 200.1 (4 sf); 200.010 (6sf)

Numbers without a dp can have an *absolute accuracy*: 4 people; 3 electrons.

Some numbers can be ambiguous: 200 could be 1, 2 or 3 sf (see below). Assume such numbers have the same number of s.f. as other numbers in the question.

Combining quantities

Multiplying or dividing numbers gives a result with a number of sf equal to that of the number with the smallest number of sf:

$x = 2.31$, $y = 4.921$ gives $xy = 11.4$ (3 sf, the same as x).

An absolutely accurate number multiplied in does not influence the above.

Standard form

On-line, and sometimes in texts, one uses a letter 'x' in place of a times sign and ^ denotes "to the power of":

1800000 could be 1.80x10^6 (3 sf) and 0.0000155 is 1.55x10^-5

(standardly, 1.80×10^6 and 1.55×10^{-5})

The letter 'e' can denote "times 10 to the power of": 1.80e6 and 1.55e-5.

Significant figures in standard form

Standard form eliminates ambiguity: In $n.nnn \times 10^n$, the numbers before and after the decimal point are significant:

$191 = 1.91 \times 10^2$ (3 sf); 191 is $190 = 1.9 \times 10^2$ (2 sf); 191 is $200 = 2 \times 10^2$ (1 sf).

Answers to questions

In this book and on-line, give the appropriate number of sf: For example, when the least accurate data in a question is given to 3 significant figures, then the answer should be given to three significant figures; see above. Too many sf are meaningless; giving too few discards information. Exam boards also require consistency in sf.

Physical Quantities

Quantity	Magnitude	Unit
Seconds per year	3.156×10^{7}	-
Permittivity of free space (ϵ_0)	8.85×10^{-12}	$s^4\ A^2\ kg^{-1}\ m^{-3}$
Electrostatic force constant ($1/4\pi\epsilon_0$)	8.99×10^{9}	$m^3\ kg\ A^{-2}\ s^{-4}$
Universal gravitational constant (G)	6.67×10^{-11}	$N\ m^2\ kg^{-2}$
Mass of neutron (m_n)	1.67493×10^{-27}	kg
Mass of neutron (m_n)	1.00867	u
Mass of proton (m_p)	1.67262×10^{-27}	kg
Mass of proton (m_p)	1.00728	u
Mass of electron (m_e)	9.10938×10^{-31}	kg
Mass of electron (m_e)	5.48580×10^{-4}	u
Gas constant (R)	8.31	$J\ mol^{-1}\ K$
Electron volt (eV)	1.60×10^{-19}	J
Avogadro's number (N_A)	6.02×10^{23}	-
Boltzmann's constant	1.38×10^{-23}	$J\,K^{-1}$
Speed of light (*in vacuo*)	3.00×10^{8}	$m\,s^{-1}$
Atomic mass unit (u)	1.66054×10^{-27}	kg
Planck's constant (h)	6.63×10^{-34}	J s
Charge on electron	1.60×10^{-19}	C
0° C	273.15	K
1 parsec (pc)	3.086×10^{16}	m
Acceleration due to gravity	9.81	$m\,s^{-2}$
Light year	9.46×10^{15}	m
Specific heat capacity of water	4180	$J\,kg^{-1}\,K^{-1}$

Contents

Checklists

Unit	Skill	
A1	Choose an appropriate equation from a list, re-arrange it, substitute numbers for variables, and calculate the unknown quantity.	
A2	Express units in terms of SI base units.	
A3	Express a measurement in standard form (2.43×10^{-8}) or using a prefix (24.3 ns) to a given number of significant figures.	
A4	Convert measurements from one unit to another.	
A5	Calculate the gradient or y-intercept of a straight line on a graph and give its unit.	
A7	Estimate the area under the line on a graph and give its unit.	
A8	Estimate areas under lines using units with prefixes.	
B1	Determine the horizontal and vertical components of a vector (displacement, velocity, force or the electric field in Malus' Law).	
B2	Determine the sum of two vectors by scale drawing or trigonometry where the triangle of vectors always has a 90° angle.	
B3	Solve problems of uniform accelerated motion in 1-dimension (SUVAT problems).	
B4	Solve trajectory problems using the independence of horizontal and vertical motion.	
B5	Find the missing force in a question using the principle of moments.	
B6	Stress, strain and Young's modulus.	
B7	Force and energy calculations using springs separately and in combinations.	
B8	Mechanical calculations of energy and power.	
B9	Energy and force in springs and stretched materials.	
C1	Resistors & resistivity.	
C2	Understands the relationship between charge carrier motion and electric current.	
C4	Solve circuit problems using Kirchhoff's Laws.	
C5	Find the voltage across components in a potential divider circuit.	
C6	Work out the terminal p.d. of a battery given the current supplied, e.m.f. and internal resistance.	
D1	Calculate amplitudes and intensities from power.	
D2	Intensities of light after passing a polarizer.	
D3	Determine whether a wave, traveling by two different routes to a detector will interfere constructively or destructively.	

Unit	Skill	
D4	Select the right equation to use for two-source interference problems, re-arrange it, and solve it to obtain the correct answer.	
D5	Understand standing waves.	
D6	Select an equation to use in solving photoelectric effect problems, re-arrange it, and solve it to obtain the correct answer.	
D7	Perform calculations relevant to quantum physics.	
D8	Calculate refractive indices, angles of refraction, and critical angles.	
E1	Estimate absolute uncertainties.	
E2	Calculate relative uncertainties.	
E3	Estimate the relative uncertainty in a calculated result from the uncertainties of the original measurements.	
E4	Assess whether measurements are accurate or reliable.	
F1	Calculate the force needed to change an object's momentum in a given time.	
F2	Solve a 1-d problem in conservation of momentum.	
F3	Convert angles from degrees to radians, can convert ordinary speeds into angular speeds, can convert between frequency, time period & angular frequency.	
F4	Work out the size and direction of the force needed to keep an object in uniform circular motion.	
F5	Work out the gravitational force on an object using Newton's law of gravity, either directly (knowing M) or by comparison with another object where the force is known.	
F6	Work out the time period of a circular orbit from its radius (or vice versa) without looking up Kepler's 3rd Law.	
F7	Perform calculations related to oscillators.	
G1	Convert Celsius into kelvin, and know when K must be used.	
G2	Use the right form of the gas law (pV=nRT or $pV = N$kT or pV/T=const) to solve a problem involving gases.	
G3	Calculate energies required to cause temperature changes, and to calculate the final temperature of mixtures.	
G4	Calculate energies required to cause changes of state.	
H1	Work out the force (direction & magnitude) on an electron or alpha particle between two charged plates.	
H2	Calculate the electric field E near one or two point charges.	
H3	Find the speed of electrons accelerated from rest in an electric field.	

Unit	Skill	
H4	Work out the force (direction & magnitude) on a wire carrying a current in a magnetic field.	
H5	Work out the force (direction & magnitude) on a moving charged particle in a magnetic field.	
H6	Work out the radius of the circular path followed by a charged particle in a magnetic field.	
H7	Work out the e.m.f. (magnitude & direction) induced in a coil of wire when it is moved in a magnetic field.	
H8	Calculate the voltage on the secondary of a transformer.	
I1	Work out the charge & energy stored on a capacitor from the charging voltage.	
I2	Work out the capacitance of a network of capacitors.	
I3	Sketch the current / voltage / charge on a capacitor as a function of time as it discharges through a resistor, labelling key points.	
J1	Complete a nuclear equation (including beta decay and nuetrinos.)	
J2	Calculate the half life of a radioactive sample from a knowledge of number of nuclei & activity.	
J3	Sketch the no. of nuclei remaining in / activity of a radioactive source as a function of time as it decays, labelling key points.	
J4	Calculate the energy released in a nuclear reaction from the masses of reactants & products OR a graph of binding energy per nucleon.	
K1	Work out galaxy velocity from spectral shift and thus its distance using Hubble's law.	
K2	Perform a variety of exponential calculations.	

Explanation Checklist

Unit	Skill	
L1	How a mass spectrometer works.	
L2	The main categories of fundamental particles.	
L3	The construction of a nuclear (fission) reactor.	
L4	What happens when X-rays hit tissue, and how X-ray images can be improved.	
L5	How an ultrasound image is taken, the significance of acoustic impedance, and the difference between A and B scans.	
L6	How MRI works, and the advantages & disadvantages of MRI in comparison with other techniques (X-ray, PET).	
L7	The life cycle of a star.	
L8	The history of the Universe according to the Big Bang model.	

Chapter A

General Questions

A1 Using and Rearranging Equations

9/12

Use the following equations:

$$s = ut \qquad a = \frac{(v-u)}{t} \qquad F = ma \qquad v = f\lambda$$

$$V = IR \qquad P = IV \qquad E = Pt \qquad Q = It$$

where the letters have the following meanings:

s = distance	u, v = velocity	t = time	m = mass
V = voltage	I = current	F = force	a = acceleration
Q = charge	E = energy	P = power	f = frequency
λ = wavelength	R = resistance		

A1.1 $F = 3.0$ N, $m = 2.0$ kg, what is a?

A1.2 $I = 0.20$ A, $t = 200$ s, what is Q?

A1.3 Calculate the resistance needed if you want 0.030 A to flow through a component when a 9.0 V battery is connected to it.

A1.4 Calculate the distance travelled by a car going at 30 m s^{-1} in 2.0 minutes.

A1.5 Calculate the wavelength of a wave that travels at 3.0×10^8 m s^{-1} if its frequency is 2.0 GHz (2.0×10^9 Hz).

A1.6 Calculate the power of a 0.25 A, 240 V light bulb.

A1.7 A Corsa accelerates from 15 m s^{-1} to 25 m s^{-1} in 8.0 s. Calculate the acceleration.

A1.8 If a jet has a maximum acceleration of 20 m s^{-2}, what is the time it would take to get from 0 m s^{-1} to 100 m s^{-1}?

A1.9 Calculate the power if 5.0 A flows through a 2.0 Ω resistor.

A1.10 My kettle needs to be able to give 672, 000 J of heat energy to water in 240 s. Assuming that it is connected to the 240 V mains, what current is needed?

A1.11 Calculate the force needed if my 750 kg car needs to accelerate from rest to 13 m s^{-1} in 5.0 s.

A1.12 Calculate the electrical energy used by a 240 V light bulb with a resistance of 60 Ω in 600 s.

A2 Derived and Base SI Units

30/37

Express the following derived units in terms of the SI base units. The first one has been done for you:

Derived Unit	in Base Units	Power of each base unit			
		m	s	kg	A
m s^{-2}	m s^{-2}	1	-2	0	0
J		A2.1	A2.2	A2.3	A2.4
N		A2.5	A2.6	A2.7	A2.8
C		A2.9	A2.10	A2.11	A2.12
V		A2.13	A2.14	A2.15	A2.16
Ω		A2.17	A2.18	A2.19	A2.20
Pa		A2.21	A2.22	A2.23	A2.24
N C^{-1}		A2.25	A2.26	A2.27	A2.28
V m^{-1}		A2.29	A2.30	A2.31	A2.32

Express the following derived units in terms of the unit specified and base units. The first one has been done for you.

A2.33 a) Express the ohm in terms of the volt and base units: $\Omega = \mathrm{V\,A^{-1}}$

b) Express the joule in terms of the newton and base unit(s).

c) Express the pascal in terms of the joule and base unit(s).

d) The answer to A2.33c means that pressure in effect measures an amount of energy per unit ________________

e) Express the $\mathrm{V\,m^{-1}}$ in terms of the joule and base unit(s).

f) Express the unit of density in newtons and base unit(s).

$^{9}/_{12}$

A3 Standard Form and Prefixes

You will be penalized if you give the wrong number of significant figures where the question specifies the required number of significant figures. [NOTE: standard form means that there is always one non-zero digit before the decimal point.]

Write the following as 'normal' numbers:

A3.1 3×10^4

A3.2 4.89×10^6

A3.3 3.21×10^{-3}

A3.4 2×10^0

Write the following in standard form to three significant figures:

A3.5 2000000

A3.6 34580

A3.7 23.914

A3.8 0.000005638

Write the following as 'normal' numbers with the unit (but without the prefix):

A3.9 3 kJ

A3.10 20 mA

Write the following using the most appropriate prefixes:

A3.11 5×10^7 m

A3.12 6×10^{-10} s

A4 Converting Units

$^{9}/_{11}$

Convert between units as specified. Express your answer in standard form if the power of ten is ≥ 3, or ≤ -3. Your answer must include units, as indeed it must in *all* questions with units in this book.

Convert:

A4.1 34.5 mm to nm

A4.2 34.5 mm to pm

A4.3 2.4 ps to ms

A4.4 465 μA to mA

A4.5 465 μA to kA

A4.6 43×10^{-7} GW to μW

A4.7 34 m^2 to cm^2

A4.8 58 N m to N cm

A4.9 9600 μm^2 to cm^2

A4.10 0.035 N cm^{-2} to Pa

A4.11 450 kg m^{-3} to kg mm^{-3}

Chapter F

Mechanics

8/10

F1 Force and Momentum

In these questions ignore the effects of friction & drag.

F1.1 What is the momentum of a 750 kg car going at 31 m s^{-1}?

F1.2 What is the momentum of an electron (mass = 9.1×10^{-31} kg) travelling at 3.0×10^{7} m s^{-1}?

F1.3 If a 20000 kg bus accelerates from 10 m s^{-1} to 25 m s^{-1}, what is the change in momentum?

F1.4 A 50 g ball is travelling at 2.0 m s^{-1} when it hits a wall and rebounds at 1.5 m s^{-1}. Calculate the change in momentum.

F1.5 A 750 kg car takes 15.3 s to accelerate from 5.0 m s^{-1} to 31 m s^{-1}. Calculate the force needed to do this.

F1.6 A 70 kg person jumps in the air and is travelling downwards at 2.0 m s^{-1} when their feet touch the ground. If it takes the person 0.30 s to stop, calculate the resultant force on them.

F1.7 I am trying to push start a car which has stopped. If the biggest force with which I can push the car is 420 N, and the car has a mass of 1025 kg, how fast will it be going after 8.0 s of pushing?

F1.8 Calculate the force needed to accelerate a 50000 kg spacecraft from rest to 7000 m s^{-1} in four minutes.

F1.9 An alpha particle (mass = 6.7×10^{-27} kg) is fired at the nucleus in a gold atom with a speed of 3.5×10^{6} m s^{-1}. It bounces off at the same speed in the opposite direction. If the collision takes 10^{-19} s, what is the average force?

F1.10 How long would it take a 637 N force to accelerate a 65 kg physics teacher from rest up to a speed of 100 m s^{-1}? (NB this is over 200 mph)

F2 Conservation of Momentum

$^{8}/_{10}$

Two masses, called Alfie and Beth, collide and stick together under four different circumstances, as shown in the four rows of the table below. Calculate the missing measurements:

Before collision				After collision
Alfie's mass /kg	Alfie's velocity /m s^{-1}	Beth's mass /kg	Beth's velocity /m s^{-1}	Velocity /m s^{-1}
30	+2.0	40	+1.5	F2.1
60	-1.4	30	+2.8	F2.2
120	+1.5	80	F2.3	0.0
120	+3.0	F2.4	-31	+2.0

F2.5 Charlie is driving her 20000 kg bus. She stops at a roundabout. Percy is driving his 750 kg Corsa at 15 m s^{-1} behind her. He fails to stop and rams into the back of the bus, sticking to it. The impact releases the brakes on the bus. How fast will the smashed up wreck be travelling immediately after the collision?

F2.6 A neutron (mass = 1 u) is moving at 300 m s^{-1} when it smacks into a stationary ^{235}U nucleus (mass = 235 u), and sticks to it. What will the velocity of the combined particle be?

F2.7 A 7.90 g bullet is travelling at 200 m s^{-1}. It hits a 3.00 kg sack of sand which is hanging by a rope from the ceiling. The bullet goes into the sack, and is stopped inside it by friction with the sand. How fast is the sack going immediately after the bullet has "stopped" inside it[1]? NB you must give your answer to 3 significant figures to be awarded the mark.

F2.8 A rocket (containing a space probe) is travelling at 7000 m s^{-1} in outer space. The 2000 kg probe is ejected from the front of the rocket

[1] "stopped" means stopped relative to the sand, not stopped relative to a stationary observer.

(forwards) using a big spring. If the speed of the probe afterwards is 7200 m s^{-1}, and the rest of the rocket has a mass of 6000 kg, what is the speed of the rest of the rocket?

F2.9 In a strange form of billiards, the cue ball is one third the mass of the other balls, which are stripey. There is no spin, and I hit a stripey ball centrally with the cue ball (travelling at 1.4 m s^{-1}) such that the cue ball rebounds in the opposite direction with half of its initial speed. What is the speed of the stripey ball?

F2.10 I am stranded, stationary, in space, but near to my spacecraft. I detach my 30 kg oxygen cylinder, and fling it away from the spacecraft with a speed of 3.0 m s^{-1}. If my mass (without the cylinder) is 80 kg, how fast will I travel in the other direction towards my spacecraft?

F3 Units of Rotary Motion

$^{18}/_{20}$

F3.1 How big is 3 rad, when expressed in degrees?

F3.2 How many radians are there in 90°?

Complete the questions in the table by converting the units:

Time period /s	Frequency /Hz	Angular Velocity /rad s^{-1}	Angular Velocity /rpm
0.50	F3.3	F3.4	F3.5
F3.6	F3.7	3.0	F3.8
F3.9	F3.10	F3.11	3800
F3.12	50	F3.13	F3.14
2700	F3.15	F3.16	F3.17

F3.18 A car travels 10 km. One of its wheels has a radius of 30 cm. Calculate the angle the wheel turns as the car travels this distance (answer in radians).

F3.19 An astronaut's training centrifuge has a radius of 4.0 m. If it goes round once every 2.5 s, calculate the velocity of the end of the centrifuge arm (4.0 m from the pivot).

F3.20 My washing machine has a spin speed of 1200 rpm, and a drum radius of 20 cm. Calculate how fast clothes go, when up against the side of the drum.

Isaac Physics

Developing problem solving skills.

L. Jardine-Wright
Co-director, Isaac Physics Project

About the author

Lisa Jardine-Wright is an astrophysicist, and a tutor and director of studies in physics at Churchill College, Cambridge. She lectures first year maths and physics within the Cambridge Natural Sciences degree and, for the last 10 years, has also directed and developed the educational outreach programmes of the Cavendish Laboratory.

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LJW, 2015

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Also available from Periphyseos Press

A Cavendish Quantum Mechanics Primer, M. Warner and A. C. H. Cheung.

The Primer accompanies readers from school to university. It briefly sketches the formal basis of the subject, and then the reader quickly starts solving problems as diverse as quantising motion in potentials, quantum mechanical tunnelling, muon catalysed fusion, pair production, motion in nanostructures, and the interference of particles as waves. Mathematics from the penultimate school year suffices, until complex numbers and partial derivatives are introduced for travelling waves and quantum mechanics in higher dimensions, Chapters 4 and 5.

Chapters 1–3 guide the transition from school to university, and are a practical aid for the fluency and understanding that are tested in admissions to higher physics, maths and engineering. Chapter 1 is available online at `www.cavendish-quantum.org.uk`. It revises maths and physics skills.

Contents

Physics Problem Solving

Isaac Physics - You work it out

isaacphysics.org

The key concepts and numerical manipulations that you practise in Chapters A — L are an excellent step on the road to studying physics, engineering or any STEM course at university. To really be able to think like physicists, we also need to develop our ability to combine these key concepts and manipulations, in order to solve longer and more involved problems. These synoptic or problem solving type questions can be challenging for experienced physicists, but completion of such problems gives a real sense of achievement and success.

The main mission of Isaac Physics is to provide problem solving questions for students who have completed their GCSEs (or equivalent) through to those who have finished their A-levels and are preparing for university. We have over 500 questions graded from Level 1 (post-GCSE) through to Level 6 (pre-university and NOT for the faint-hearted).

In the following chapters you will find:

- advice on how to problem-solve,
- sample solutions to two problems from our site (Level 3 Dynamics: Pop-up Toy, and Level 5 Statics: Prism)
- a sample question from each of the topics, and each of the levels on isaacphysics.org
- a url link to a webpage containing this and the other questions from the same level and topic.

Take up the challenge and put your new found skills to the test — but before you do, make sure you pick up a large **pad of paper** and **a pencil** so that **you** can work it out.

A Guide to Solving Physics Problems

isaacphysics.org/solving_problems

Physicists develop highly desirable skills through their extensive experience of problem solving – logic, determination, resilience and mathematical ability to name just a few. You will become an ace problem solver by answering ***lots*** of questions – these 5 key steps will help you develop a logical, structured method and universal approach.

5 key steps to problem solving

- Step 1: Keywords
- Step 2: Diagram
- Step 3: Concepts
- Step 4: Symbols
- Step 5: Dimensions & Numbers

When faced with a new question, we employ a strategy to break the problem down into a series of 5 steps to digest and analyse it. Each step helps us to understand the information given in the question and establish what it is that we are being asked to calculate or discover. Using these steps for each new question we attempt can, with practice, make solving physics problems extremely satisfying and rewarding.

- **Step 1: Keywords**
  Are there words in the question that contain additional information about the problem? Frictionless; light; uniform... Highlight these words so that they stand out – they will help to simplify the solution and allow us to neglect concepts that we don't need to consider.

- **Step 2: Draw a diagram**
  The action of drawing a diagram helps us to process and summarise the information in a question. Drawing a diagram will save time and effort later and is the key to finding a solution. When drawing the diagram, label the quantities that are given with symbols (e.g. u for a velocity, d for a displacement); staying in symbols rather than using numbers is vital – see Step 4.

- **Step 3: Key concepts & mathematics**
  Fluency with mathematical rearrangements is essential but we need to

be sure that we are logical with our approach and consider the physical concepts that we need before throwing algebra at the problem. Identify which concepts are relevant to the problem. Write down the relevant physical principles and equations that might be useful – particularly if they connect the quantities that are given in the question.

- **Step 4: Stay in symbols**
  Even if the question gives numerical values, represent each of them with a symbol. This may appear to overcomplicate the problem, but it really helps when checking the solution or trying to find a mistake.

  For example, imagine that as part of the calculation we want to find the magnitude of the displacement of a cyclist who has travelled 13 km West and 5.0 km North.

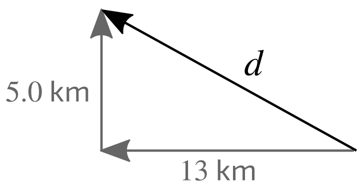

Figure 1: The displacement of a cyclist who has travelled 13 km West and 5.0 km North.

  You identify that you need to use Pythagoras' theorem

$$\text{displacement, } d = \sqrt{144} = 12\,\text{km} \qquad \text{WRONG} \qquad (1)$$

  The numbers hide information about the calculation that letters would not. Let $w = 13$ km and $n = 5$ km

$$\text{displacement, } d = \sqrt{w^2 - n^2} = 12\,\text{km} \qquad \text{WRONG} \qquad (2)$$

  You can see straight away that the square of the numbers have accidentally been subtracted rather than added.

- **Step 5: Check dimensions, then put in numbers**
  Before putting numbers into an algebraic expression, experienced physicists check whether their answer has the correct ***dimensions*** (you may have seen this before as checking your units). For example, if we are trying to find a quantity of time, our expression must have dimensions of ***only*** time – no length, or mass or charge!

  Imagine that we are trying to calculate how fast the Earth travels around the Sun and we know that the radius of the Earth's orbit is $r = 1.50 \times 10^{11}$ m and that it takes $t = 365.25$ days. We make a mistake and write down that the distance travelled by the Earth is $2\pi r^2$. If we check our dimensions on both sides of the equation, we can see that this is incorrect.

$$\text{speed} = \frac{2\pi r^2}{t} \quad (3)$$

$$\text{Dimensions:} \quad \frac{[L]}{[T]} = \frac{[L]^2}{[T]} \quad (4)$$

The length dimension, $[L]$, that we have on the top of the left hand side of expression (4) does not match the length squared, $[L]^2$ on the top of the right hand side of expression, so we determine that there is a mistake with our r^2 part of the expression.

Now we have the steps, we apply them to two example problems from Levels 3 and 5 of isaacphysics.org.

Example Solution - Level 3 Dynamics: Pop-up Toy

`isaacphysics.org/s/i8OQ9P`

Q: A pop-up toy consists of a head and sucker of combined mass $m = 1.5$ kg stuck to the top of a light spring of natural length $l_0 = 0.30$ m and spring constant $k = 250$ N m^{-1}. The centre of mass of the system can be taken to be at the top of the spring. The spring is compressed to length $l_1 = 0.10$ m when the pop-up toy is stuck to the ground.

What height above the ground does the bottom of the unstretched spring jump to when it is smoothly released?

Step 1: Keywords

Highlight the key words in the text above.

What are the key words in the question that will help you to draw your diagram and understand the physical concepts that might be useful? Note also what the question is asking for.

Solution:

- combined mass at the top – we can consider the sucker and mass as one object of mass, m, at the top of the spring.
- centre of mass – the point at which we should consider the weight of the combined mass to act.
- light – we can consider the spring to have no mass or weight, it is negligible.

- spring — is extensible.
- smoothly released — the toy is not caused to jolt in anyway so there is no energy converted to heat or sound, we conserve kinetic and potential energies.
- **Q:** Height of the bottom of the spring.

Step 2: Draw a diagram

Draw and annotate a diagram with all of the information from the question, in particular draw each stage of the toy's behaviour.

Solution:

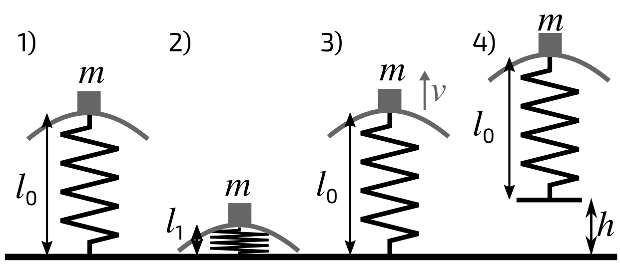

Figure 2: The 4 stages of the toy's behaviour. 1) Before compression. 2) Compressed and ready to jump. 3) Returned to its natural length, l_0, but now the mass and sucker has a velocity, v. 4) At maximum height, the velocity of the mass and sucker = 0.

Steps 3 & 4: Concepts, mathematics & symbols

Identify the concepts that you think might be useful in answering the question.

Solution: Hooke's Law, spring constant, forces, work done, gravitational potential energy (GPE), elastic potential energy (EPE), kinetic energy (KE) and conservation of energy.

Method 1: Energy

How do you know whether to analyse a problem using forces (method 2) or energy? Using conservation of energy in this problem is simpler, as the total amount of energy remains the same throughout all the stages of motion and therefore we can consider just the beginning and end situations (stage 2 and stage 4).

Write down an expression for the conservation of energy. The total energy of the toy at stage 2 is equal to its total energy at stage 4.

- **Take care to define a position of zero gravitational potential energy.**

Solution: For the whole of this solution I will choose for the ground level to be the position of zero gravitational potential energy.

$$\begin{aligned}
\text{Total energy at stage 2} &= \text{Total energy at stage 4} \\
\text{GPE}_2 + \text{EPE}_2 &= \text{GPE}_4 \\
mgl_1 + \frac{1}{2}k(l_0 - l_1)^2 &= mg(h + l_0) \\
h + l_0 &= l_1 + \frac{1}{2mg}k(l_0 - l_1)^2 \\
h &= l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2 \qquad (5)
\end{aligned}$$

Method 2: Force and Work Done

Using a force method for the pop-up toy is more challenging because the force due to the spring is not constant throughout the motion. However, analysing the problem through forces should give us the same answer.

How does the force exerted by the spring on the mass cause the toy leap off the ground?

Solution: Hooke's Law tells us that the magnitude of the force, F, needed to compress a spring by an amount x, is given by $F = kx$. To compress the spring by an additional small amount δx we need to do work **on** the spring. The amount of work done is $\delta W = F\delta x$. As δx becomes very small (tends to zero), δW tends to dW.

We can then find the total work done to compress the spring from stage 1 to stage 2 by integrating with respect to dx between the limits of the compression at stage 1 to the compression at stage 2.

$$\begin{aligned}
&\text{Stage 1, compression} = 0 \\
&\text{Stage 2, compression} = (l_1 - l_0).
\end{aligned}$$

The total work done on the spring between these two stages is then released between stages 2 and 4, and converted to gravitational potential energy.

$$\begin{aligned}
\text{Work done from 1 to 2} &= \text{change in GPE from 2 to 4} \\
\int_0^{(l_1 - l_0)} (kx)\mathrm{d}x &= \text{GPE}_2 - \text{GPE}_4 \\
\left[\tfrac{1}{2}kx^2\right]_0^{(l_1 - l_0)} &= mgl_1 - mg(h + l_0)
\end{aligned}$$

$$h + l_0 = l_1 + \frac{1}{2mg}k(l_1 - l_0)^2$$
$$h = l_1 - l_0 + \frac{1}{2mg}k(l_1 - l_0)^2$$

Our result is indeed consistent with that of method 1, equation (5).

This is not the only force method we could use, the problem can also be solved using Newton's Second Law to equate the resultant force on the toy, at a general time, to its acceleration. The acceleration must be written in terms of $\frac{dv}{dx}$ rather than $\frac{dv}{dt}$ so that we can then integrate to find an expression for v^2 which can be related to the height h.

Step 5: Dimensions & numbers

The question asks for a height, which has dimensions of length $[L]$ – each term of the right hand side of our expression (5) should therefore also have dimensions of length.

Solution: Not all of the terms have dimensions that we can just write down; for example, what are the dimensions of k and g?

$$g = \text{acceleration} = [L][T]^{-2}$$
$$k = \frac{\text{force}}{\text{length}} = \frac{[M][L][T]^{-2}}{[L]} = [M][T]^{-2}$$

$$h = l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2$$
$$= [L] + [L] + \frac{[1]}{[M][L][T]^{-2}}[M][T]^{-2}([L])^2$$
$$= [L] + [L] + [L] = \text{correct}$$

Now we substitute the values given in the question and consider carefully the number of significant figures we should give in our answer.

$m = 1.5$ kg, $l_0 = 0.30$ m, $l_1 = 0.10$ m, $k = 250$ N m^{-1} and $g = 9.81$ m s^{-2}.

Solution:

$$h = l_1 - l_0 + \frac{1}{2mg}k(l_0 - l_1)^2$$
$$h = 0.10 - 0.30 + \frac{1}{2 \times 1.5 \times 9.81} \times 250 \times (0.30 - 0.10)^2 = 0.14\,\text{m}$$

Example Solution - Level 5 Statics: Prism

`isaacphysics.org/s/i8OQ9P`

A prism has a cross section that is an isosceles triangle. It has a unique angle of 30°, and a mass of $m = 100$ g. You wish to lift it by touching the upper two faces only.

Q: If the coefficient of friction between the prism's surface and your skin is $\mu = 0.400$, what is the minimum normal force you need to apply to each face in order to support the prism?

Step 1: Keywords

Highlight the key words in the text above.
What are the key words in the question? Note exactly what the question asks.

Solution: isosceles, friction, minimum, normal, unique angle.
Q: We are asked for the minimum normal force applied to **each** face.

Step 2: Draw a diagram

Redraw and annotate a diagram of the prism.

- Label your angle θ to stay in symbols until the end of the calculation.
- Annotate the diagram with **all** the forces acting on the prism. Think carefully about the forces that act **on the prism** as opposed to those **on your fingers**.

Solution: To add the frictional force on the prism we think about the direction that the prism wants to fall – we know that friction will oppose this motion. Using Newton's third law, the frictional force on our fingers must be equal in magnitude but opposite in direction to that on the prism.

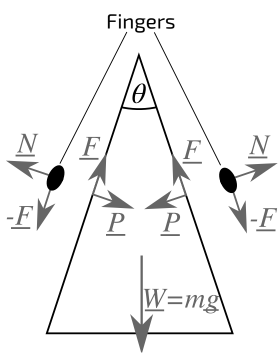

Figure 3: The forces on the isosceles prism and on your fingers as you lift the prism and hold it in static equilibrium.

Steps 3 & 4: Concepts, mathematics & symbols

What concepts may we need to solve this problem?

Solution: Newton's first law, Newton's third law and the law of friction.

Write down the magnitude of the frictional force in terms of the coefficient of static friction μ and the normal component, P of the force that you apply to each upper face of the prism.

- Note that the force that you apply to the prism is equal and opposite to the reaction force of the prism on your finger, by Newton's third law.

Solution: From the law of friction: $F \leq \mu N = \mu P$
To find the minimum value of P we use the maximum value of F: $F = \mu P$

Apply Newton's First Law vertically.

Solution: From Newton's First Law and resolving forces vertically:

$$2F\cos\left(\frac{\theta}{2}\right) - \left(W + 2P\sin\left(\frac{\theta}{2}\right)\right) = 0$$

Using $F = \mu P$:

$$2\mu P\cos\left(\frac{\theta}{2}\right) - \left(W + 2P\sin\left(\frac{\theta}{2}\right)\right) = 0$$

By rearranging the latter expression we can write down an expression for the force we apply, P, in terms of μ, the angle θ and the weight of the prism mg.

Solution:

Applied normal force, $P = \dfrac{W}{2\left[\mu\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right]}$

Step 5: Dimensions & numbers

We are asked to find a force so we should check that the dimensions of our expression are the same as that of a force, $F = [M][L][T]^{-2}$

Solution:

$$\frac{W}{2\left[\mu\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right]} = \frac{[M][L][T]^{-2}}{\text{dimensionless quantities}}$$

Our expression is indeed dimensionally correct.

Now using the values given and taking care with the number of significant figures in our answer, $m = 100$ g, $g = 9.81$ m s^{-2}, $\mu = 0.400$ and $\theta = 30.0°$.

Solution:

$$P = \frac{W}{2\left[\mu\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right]} = \frac{0.100 \times 9.81}{2\left[0.400\cos(15.0) - \sin(15.0)\right]} = 3.85\,\text{N}$$

Level 1: Mechanics

Statics

isaacphysics.org/s/Dbv5Dj

Q: Bed of Nails

A uniform rod of weight 500 N is supported by two pegs at either end of the rod.

Part A: Draw a free body diagram showing the forces acting on the rod. What is the magnitude of the forces exerted by each of the pegs on the rod?

Part B: If there are now eight pegs evenly spaced along the rod to support its weight, what force is applied by each peg on the rod?

Part C: Using the previous answers, explain how it is possible to lie on a bed of nails, but putting weight on one nail is extremely painful.

Dynamics

isaacphysics.org/s/CMjHTE

Q: A Hamburger

A hamburger has 2.2 MJ of chemical potential energy which can be released by eating it. (Take $g = 9.81\ \mathrm{m\,s^{-2}}$)

Part A: How much energy is needed to lift a 50 kg student upwards through a height of 0.40 m?

Part B: How many step-ups of 0.40 m would the student need to make in order to burn off the energy of a hamburger?

Part C: How many hamburgers would the student need to consume in order to reach the top of Mount Everest, a height of 8.8 km?

Part D: If the energy of a hamburger could be used by itself to propel itself upwards, how high could it rise? (The mass of a hamburger is 0.22 kg; assume this remains constant. Assume that g also remains constant).

Kinematics

isaacphysics.org/s/IUV78T

Q: A Strange Planet

A lost astronaut lands her spaceship on an unknown planet. She decides to work out the value of the acceleration due to gravity on this planet so that she can check her onboard computer and find out her location. She knows that on Earth (where $g = 9.81\ \mathrm{m\,s^{-2}}$ downwards) she takes 1.0 s to jump up and land again. On this planet, a jump takes 1.4 s.
What is the magnitude of the downward acceleration due to gravity on the strange planet? Assume she can jump up at the same speed on any planet.

Level 2: Mechanics

Statics

isaacphysics.org/s/B16Mzd

Q: Force on Table Legs

A uniform table consists of a circular wooden board of mass $m = 3.00$ kg resting on top of three vertical legs, each of mass $M = 0.500$ kg. The legs are equidistant from the centre of the table and form an equilateral triangle. (Take $g = 9.81$ m s^{-2}).

Part A: What is the magnitude of the reaction force from one of the legs on the tabletop?

Part B: What is the magnitude of the reaction force from the ground on one of the legs?

Dynamics

isaacphysics.org/s/4JKSmy

Q: The Lift

A lift, of mass 500 kg, is travelling downwards at a speed 5.0 m s^{-1}. It is brought to rest by a constant acceleration over a distance 6.0 m.

Part A: What is the tension, T, in the lift cable when the lift is stopping?

Part B: What is the work done by the tension whilst stopping the lift?

A lift, of mass 500 kg, is travelling upwards at a speed 5.0 m s^{-1}. It is brought to rest by a constant acceleration over a distance 6.0 m.

Part C: What is the tension, T, in the lift cable when the lift is stopping?

Part D: What is the work done by the tension whilst stopping the lift?

Kinematics

isaacphysics.org/s/L1TER3

Q: A Stoplight

A car is travelling along a road at constant speed $u = 7.00$ m s^{-1} . The driver sees a stoplight ahead change to amber and applies the brakes. Consequently, the car experiences a total resistive force equal to $\frac{1}{6}$ of its weight and stops just at the stoplight. Taking $g = 9.81$ m s^{-2}, how far does the car travel during the braking period?

Level 3: Mechanics

Statics
isaacphysics.org/s/T2B6VD

Q: Trapdoor
[UCLES]

A uniform rectangular trap door of mass $m = 4.0$ kg is hinged at one of its ends. It is held open, making an angle $\theta = 60^\circ$ to the horizontal, with a force of magnitude F at the open end acting perpendicular to the trap door.

Part A: Find the force on the trapdoor.

Part B: Find the value of $\tan\varphi$ to two significant figures, where φ is the angle made by the line of action of the reaction force at the hinge above the horizontal.

Dynamics
isaacphysics.org/s/klHQYn

Q: Energy of a Bullet
[UCLES]

A bullet of mass m, moving horizontally with speed u, meets a block of wood of mass M travelling along the same line but in the opposite direction with speed U, and remains embedded in it.

Show that the loss of kinetic energy is of the form $\frac{1}{2}kMm$, where k is in terms of u, U, m, M and find the loss in KE when the bullet of mass 20 g, travelling at 400 m s^{-1} hits a block of wood, mass 4.0 kg moving at 2.0 m s^{-1}.

Kinematics
isaacphysics.org/s/ktiMKD

Q: The Bolt Thrower

A castle wall has bolt throwers which fire a bolt horizontally at a speed v. In order to fire over the enemy's shields, the bolt must make an angle of at least $\theta = 45^\circ$ to the horizontal when it hits the ground. The bolt throwers can be mounted at different heights in the wall and set to fire at different speeds.

Part A: Find the maximum range of a bolt fired from a height $h = 10$ m.

Part B: Find the speed required to reach the maximum range calculated in Part A.

Level 4: Mechanics

Statics

isaacphysics.org/s/YUWgEx

Q: A Uniform Ladder

[UCLES]

A uniform ladder AB of mass $m = 20.0$ kg rests in equilibrium. A is in contact with a smooth vertical wall and B is in contact with a rough horizontal floor. The ladder lies in a vertical plane perpendicular to the wall and makes an acute angle $\alpha = 25.0^\circ$ to the horizontal. A horizontal force $P = 400$ N is applied to the ladder in a direction away from and perpendicular to the wall. This force acts at the point one quarter of the way up the ladder.

Part A: Find, in terms of P, m and α, the magnitude of the normal reaction force at A, N_A and calculate its value.

Part B: Find, in terms m, the magnitude of the normal reaction force at B, N_B and calculate its value.

Part C: Find the magnitude of the frictional force on the ladder at B.

If the coefficient of friction between the ladder and the floor at B is μ , and the value of P is gradually increased from zero.

Part D: Find how the equilibrium will be broken if $\mu = \cot\alpha$

- ☐ Equilibrium will be broken by the ladder toppling.
- ☐ Equilibrium will be broken by the ladder sliding.

Part E: Find how the equilibrium will be broken if $\mu = 4\cot\alpha$

- ☐ Equilibrium will be broken by the ladder toppling.
- ☐ Equilibrium will be broken by the ladder sliding.

Dynamics isaacphysics.org/s/4ox4q1

Q: Braking a Car [UCLES]

When a car of mass 1000 kg is travelling along a level road at a steady speed of 20 m s^{-1}, its engine is working at 18 kW.

Part A: Find the magnitude and direction of the resistive force due to friction, which may be taken to be constant.

Part B: The engine is suddenly disconnected and the brakes applied, and the car comes to rest in 50 m. Find the force, assumed constant, exerted by the brakes.

Part C: Find also the distance in which the car, travelling at 20 m s^{-1}, would come to rest if the engine were disconnected and the same braking force applied on an upward incline of angle θ, where $\sin\theta = \frac{1}{20}$.

Part D: By how much does this change if the car is travelling down the same hill at 20 m s^{-1}?

Kinematics isaacphysics.org/s/8BB7Rw

Q: Broken Cannon

A cannon on horizontal ground, at point C, is used to target a point T, 25 m behind a narrow wall. Unfortunately the cannon is damaged and can only fire at a 45° angle and at one speed. So, the only way to aim the cannon is by moving it towards and away from the target. The gunners aren't sure if they can actually hit the target.

Part A: If the cannonball leaves the cannon at $u = 35$ m s^{-1}; at what distance, d, must the cannon be placed in front of the wall in order to hit the target, if the wall is ignored and the target is at the same height as the cannon?

Part B: The wall is 15 m high. Does the cannonball actually go over the wall and hit the target? If so, by how much?

SHM

isaacphysics.org/s/DZYEBC

Q: Weighing at Sea

[UCLES]

A man is standing on a weighing machine on a ship which is bobbing up and down with simple harmonic motion of period $T = 15.0$ s.

Assuming that the motion is vertical calculate the amplitude of the ship's motion, if the scale reading of the machine varies between limits of 55.0 kg and 65.0 kg.

Angular Motion

isaacphysics.org/s/mcIYvI

Q: Hammer Throwing

At a hammer throwing event, the speed that an athlete is spinning is such that they complete one revolution in 0.50 s. Most of the weight of the hammer is in the ball, so it can be approximated as a point mass $m = 7.0$ kg at a distance $l = 1.2$ m away from the end of the handle.

The athlete stretches their arms out such that they are holding the end of the handle of the hammer a distance $d = 0.60$ m away from their axis of rotation. Assume that the hammer has no vertical motion.

Part A: What is the linear speed of the ball at the end of the hammer?

Part B: What is the kinetic energy of the hammer?

Circular Motion

isaacphysics.org/s/GJMwf0

Q: Car on a Roundabout

A car approaches a level roundabout of radius $R = 10$ m.

Part A: What is the maximum speed that the can travel around the roundabout without slipping, if the coefficient of friction between the road and car is $\mu = 0.80$?

A racing car approaches a corner of radius $R = 50$ m, banked at an angle $\theta = 30^\circ$ to the horizontal.

Part B: What is the maximum speed at which the car can travel if it is icy so there is no friction between the wheels and the road?

Part C: What is the maximum speed at which the car can travel around the banked corner and not slip if the road is rough, with coefficient of friction $\mu = 0.55$?

Part D: What is the minimum speed at which the car needs to travel around the banked corner when the road is rough, with coefficient of friction $\mu = 0.55$?

Level 4: Fields

Electric Fields

isaacphysics.org/s/q6IUoh

Q: Potential Difference

[UCLES]

A charge of $3.0\,\text{C}$ is moved from infinity (where the electric potential is $0\,\text{V}$) to a point X in an electric field. The work done by the electric field in this process is $15\,\text{J}$.

Part A: What is the electric potential at X?

- ☐ $5.0\,\text{V}$
- ☐ $45\,\text{V}$
- ☐ $23\,\text{V}$
- ☐ $0.20\,\text{V}$
- ☐ $15\,\text{V}$

Part B: How much work must be done by the electric field in order to return this charge back to infinity?

Part C: An electron moves from infinity to point X. The only force acting on it in this time is that due to the electric field. What kinetic energy will it gain over this distance?

Magnetic Fields

isaacphysics.org/s/RjHZfp

Q: Electron in a Magnetic Field

An electron, of charge $-1.6 \times 10^{-19}\,\text{C}$ and velocity $2.35 \times 10^{7}\,\text{m s}^{-1}$, moves perpendicularly to a magnetic field with magnetic flux density $3.4 \times 10^{-2}\,\text{T}$.

Calculate the size of the force on the electron.

Gravitational Fields

isaacphysics.org/s/b9I6Cv

Q: Escaping from the Moon

The mass of the Moon is $7.4 \times 10^{22}\,\text{kg}$ and its radius is $1.7 \times 10^{6}\,\text{m}$. Newton's gravitational constant is $G = 6.67 \times 10^{11}\,\text{N m}^2\,\text{kg}^{-2}$. Assume that the Moon is spherical.

Part A: Calculate the value of the gravitational field strength at the Moon's surface.

The mass of the Earth is 6.0×10^{24} kg and its radius is 6.4×10^{6} m. Assume that the Earth is spherical.

Part B: What is the escape velocity on the Moon as a fraction of the Earth's escape velocity?

Combined Fields

isaacphysics.org/s/kGlyBR

Q: Electric Levitation

[UCLES]

In an experiment a small, charged, plastic sphere is maintained in a stationary position by the application of a suitable vertical electric field between two large, horizontal plates in an evacuated chamber.

Part A: The mass of the sphere is 3.20×10^{-14} kg and it carries a net charge equal to that of 10 electrons. If the charge on an electron is $q = -e = -1.60 \times 10^{-19}$ C and the acceleration due to gravity is $g = 9.81$ m s^{-2}, what is the strength of the electric field needed to keep the sphere stationary?

Part B: If, when the sphere is stationary, a uniform vertical magnetic field is applied in the same direction as the electric field, what will happen to the negatively charged sphere?

- ☐ It will move in a horizontal circle.
- ☐ It will remain stationary.
- ☐ It will move upwards in a helical path.
- ☐ It will move in a vertical circle.
- ☐ It will move downwards in a helical path.

Part C: If, with no magnetic field, the sphere loses one electron and the electric field is unchanged, will the sphere accelerate...

- ☐ downwards with an acceleration, $a < g$?
- ☐ upwards with an acceleration, $a > g$?
- ☐ upwards with an acceleration, $a < g$?
- ☐ downwards with an acceleration, $a > g$?
- ☐ downwards with an acceleration, $a = g$?

Level 5: Mechanics

Statics

isaacphysics.org/s/ILSBvf

Q: Bead on a Wire

[UCLES]

A rough wire is bent into shape such that it obeys the parametric equations $x = 3p^2$ and $y = 2p^3$ for $p \geq 0$. The wire is then placed in a vertical plane such that the y-axis is vertical and the x-axis horizontal. A bead of mass m is then threaded onto the wire. The bead is pulled in the direction of increasing x by a force F of magnitude $3mg$. The coefficient of friction μ between the wire and the bead is $\frac{1}{2}$.

Find the value of the x coordinate (as an integer) at which the bead is on the point of slipping.

Dynamics

isaacphysics.org/s/NhtQS1

Q: Three Particles

[UCLES]

Three identical particles A, B and C are moving in a plane and, at time t, their position vectors, $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, with respect to an origin O are (in metres):

$$\begin{aligned}
\boldsymbol{a} &= (2t+1)\underline{\hat{\boldsymbol{i}}} + (2t+3)\underline{\hat{\boldsymbol{j}}} \\
\boldsymbol{b} &= (10-t)\underline{\hat{\boldsymbol{i}}} + (12-t)\underline{\hat{\boldsymbol{j}}} \\
\boldsymbol{c} &= (t^3-15t+4)\underline{\hat{\boldsymbol{i}}} + (-3t^2+2t+1)\underline{\hat{\boldsymbol{j}}}
\end{aligned}$$

Part A: Find the magnitude of the velocity of particle C relative to particle A when $t = 2.00$ s and give the angle which this relative velocity makes with $\underline{\hat{\boldsymbol{i}}}$ at this time.

Part B: Verify that particles A and B are both moving along the straight line with equation $y = x + 2$. If they collide, at what time will they do so?

Part C: Given that the collision between particles A and B is elastic, what are the position vectors (in metres) of A and B a time τ after the collision?

Kinematics

isaacphysics.org/s/XarSrv

Q: Crossing a River

A river has banks at $x = 0$ and $x = a = 20$ m, and flows parallel to its banks with a speed $u = kx(a - x)$, where $k = 0.010$ m^{-1} s^{-1}.

The captain of a ship wishes to moor as far upstream on the opposite bank as possible. If his initial speed is $v = 0.50$ m s^{-1}, what angle θ to the x direction should he set off at? You should assume that the ship's power output is constant, and that it always thrusts in the same direction relative to the banks.

SHM

isaacphysics.org/s/da09Es

Q: Mercury in a U-Tube

[UCLES]

Mercury of density $\rho = 13.6$ g cm^{-3} is contained in a U-tube with parallel, vertical arms. Initially the height of the mercury in the two arms is different.

Part A: Neglecting damping, what is the period of oscillation of the mercury if the total length of the tube occupied by mercury is $l = 40.0$ cm and the level of the mercury remains in the vertical part of the arms at all times?

Part B: If the area of cross-section of the tube is $S = 2.00$ cm^2, what is the total energy of the motion when the amplitude is $A = 5.00$ cm and the level of the mercury remains in the vertical part of the arms at all times?

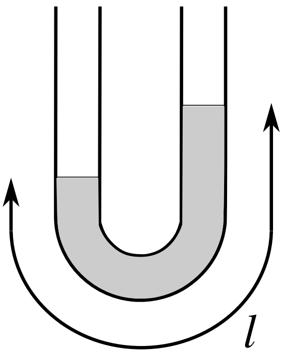

Figure 4: A length l of mercury sitting in a U-tube. Initially the height of the mercury is different in the left and right arm of the tube .

Angular Motion

isaacphysics.org/s/HyTNE6

Q: Halley's Comet

[UCLES]

Halley's comet moves in an elliptical orbit about the Sun; no external torques act on the system. At the furthest point (aphelion) the comet is a distance $r_1 = 5.3 \times 10^{12}$ m away from the Sun, and its speed at that point is $v_1 = 913$ m s^{-1}. The orbit has its closest approach to the Sun (perihelion) at $r_2 = 8.8 \times 10^{10}$ m.

What is the highest speed the comet will attain in its orbit?

Circular Motion

isaacphysics.org/s/dS3Pg1

Q: A Particle in a Bowl

A smooth hemispherical bowl of radius $a = 15$ cm is placed with its axis of symmetry vertical, and a particle of mass $m = 50$ g moves in a horizontal circular path on the inside of the bowl with a speed v. The plane of this circle is situated half way down the axis.

Part A: Find the value of v.

Part B: Find an expression for the reaction, N, of the bowl on the particle. What is the magnitude of N?

Level 5: Fields

Electric Fields

isaacphysics.org/s/G7rS6I

Q: Inkjet Printing

In some methods of inkjet printing, different droplets are given different amounts of charge as they are fired horizontally out of the nozzle. These droplets are then deflected by different amounts by a deflection voltage that is applied across two electrodes.

In one printer the deflection voltage is 480 V, the length of the deflecting electrodes is $x = 1.0$ mm and the distance between them is $d = 0.20$ mm. The electric field between these electrodes can be assumed to be constant. The droplets produced are approximately spherical with a density of $\rho = 1050$ kg m^{-3} and a radius of $a = 30$ µm. They leave the nozzle with an initial speed of $u = 5.5$ m s^{-1}.

What is the difference in the magnitude of charge between two droplets that hit the paper at the top and bottom of a letter 'I' of height $h = 2.0$ mm a distance of $L = 0.50$ cm away from the edge of the deflecting electrodes? Gravitational effects can be ignored.

Magnetic Fields

isaacphysics.org/s/Nb5XzE

Q: Electrons Orbiting the Earth

[UCLES]

Part A: Find the minimum magnetic flux density at the equator which will permit an electron of speed 10^7 m s^{-1} (to 2 significant figures) to orbit the earth close to the surface. Assume that the geographic and magnetic equators coincide. Ignore relativistic effects.
Radius of the Earth = 6.4×10^6 m
Electron mass = 9.1×10^{-31} kg
Electron charge = 1.6×10^{-19} C

Part B: If 1.0×10^{23} electrons in the ionosphere, 80 km above the equator, orbit the earth with a speed of 1.0×10^7 m s^{-1} find the current flowing in the ionosphere. Please give your answer to 3 significant figures.
Radius of the Earth = 6.37×10^6 m.

Part C: The 1.0×10^{23} electrons now travel on a circular orbit of height 50×10^3 km. They maintain the same current as before. Find the value of the magnetic flux density at the height of these electrons.
Radius of the Earth = 6.37×10^6 m.

Gravitational Fields
Q: A Binary Star System

isaacphysics.org/s/y0Df63

[UCLES]

Two stars, separated by a large distance d, rotate in circular orbits about their common centre of mass.

Part A: Given that one star has a mass m and the other a mass $2m$, find an expression for their period of rotation in terms of d, m, and G, the universal gravitational constant. If $d = 7.8 \times 10^{13} m$ and $m = 5.0 \times 10^{32}$ kg, what is the period of rotation of the two stars?

Part B: Show that the total kinetic energy of the system is proportional to its gravitational potential energy and find the constant of proportionality.

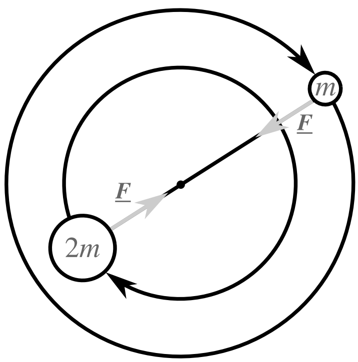

Figure 5: Two stars in a binary star system orbiting about their centre of mass with the force due to gravity acting on each of them.

Combined Fields
Q: Thunder Cloud

isaacphysics.org/s/rRP40E

[UCLES]

A simple pendulum is made from a plastic sphere of mass 1.0 g suspended by a light nylon thread. The sphere is given a positive electrostatic charge of 1.0×10^{-8} C. A thunder cloud carrying negative charge in its base passes overhead, giving rise to a uniform potential gradient of 3.0×10^{3} V m^{-1}.

Part A: Calculate the magnitude of the electrostatic force per unit mass on the sphere.

Part B: Calculate the new tension in the thread due to the combined effect of gravity and the electrostatic force on the sphere.

Part C: Determine the ratio of the time period of the uncharged pendulum to the time period while under the cloud (you may take $\sqrt{(1+x)} \approx \left(1+\frac{x}{2}\right)$, when x is small, if you wish). Give your answer to 3 significant figures.

Level 6: Mechanics

Statics

isaacphysics.org/s/i2mLQr

Q: Hanging a Non-uniform Bar

A non-uniform bar of mass m is hung horizontally between two walls, using two light ropes attached to the ends. One of the ropes makes an angle $\theta = 36.9^\circ$ to the vertical, and the other makes an angle $\phi = 53.1^\circ$ to the vertical.

If the bar is $l = 1.00$ m long, how far away from the closest edge is the centre of mass?

Dynamics

isaacphysics.org/s/aA2xdL

Q: Space Justice

A police spaceship, of mass $m = 10000$ kg, travelling at a speed $u = 2.00$ km s^{-1} needs to arrest another ship travelling ahead of it at 2.50 km s^{-1}. The police spaceship is capable of splitting itself into two equal parts and supplying them with kinetic energy from a single reserve of 2.00 GJ.

Part A: Calculate the speed of the part of the police spaceship that remains pursuing the other ship. Was the energy reserve sufficient?

Part B: If the reserve is sufficient; how long would it take for the police spaceship to catch the other craft, given that it was initially 3000 km behind?

Kinematics

isaacphysics.org/s/WgAvf5

Q: The Bouncing Ball

A ball is dropped from rest at a height h_0 and bounces from a surface such that the height of the n^{th} bounce, h_n, is given by $h_n = \alpha h_{n-1}$, where h_{n-1} is the height of the previous, $(n-1)^{\text{th}}$ bounce. The factor α has value $0 \le \alpha \le 1$.

Part A: How far does the ball travel before coming to rest, given that $\alpha = 0.25$ and $h_0 = 3.0$ m?

Part B: How long does the ball take to cover this distance?

SHM

isaacphysics.org/s/yV9R96

Q: Gravtube

Imagine a tube had been drilled straight through the centre of a uniform spherical planet. The planet has a radius R and the acceleration due to gravity at its surface is g. An object of mass m is released from rest at one end of the tube. From Gauss's theorem, the gravitational force on an object of mass m inside a uniform massive spherical body (in this case, a planet), is given by $F = -\frac{GMm}{r^2}$ where r is the distance of the small mass from the centre of the planet and M is the mass of the planet that exists inside a sphere of radius r (ie. all the mass of the planet that is closer to the planet's centre than the mass m). The force is negative as it acts inwards, towards the point $r = 0$.

Show that the acceleration of an object inside this tube is of the form $a = -\omega^2 r$ and so the object moves with simple harmonic motion.

Part A: What is the time period of the resulting oscillation if $g = 6.00$ m s^{-2}, and $R = 1200$ km?

A satellite is placed in a circular orbit around the same planet, so that it is orbiting just above the ground (ie, at a radius R). The centripetal acceleration of an object of mass m, in a circular orbit at a radius r, is given by $a_\text{c} = \omega^2 r$, where ω is the angular velocity of the mass in orbit.

Part B: Find an expression for ω and calculate the time period of the orbit at this radius?

Angular Motion

isaacphysics.org/s/4k2no4

Q: Rising Hoop

Two beads, each of mass m, are positioned at the top of a frictionless hoop of mass M and radius R, which stands vertically on the ground. The beads are released and slide down opposite sides of the hoop.

What is the smallest value of $\frac{m}{M}$ for which the hoop will rise up off the ground at some time during the motion?