A repulsive atomic gas on the border of itinerant ferromagnetism

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G.J Conduit & E. Altman, arXiv: 0911.2839
Ferromagnetism in iron and nickel

- Typical ferromagnets undergo a *second order* transition.
First order phase behavior — ZrZn$_2$

- At low temperature and high pressure ZrZn$_2$ has a first order transition

Uhlarz et al., PRL 2004
Three body losses

- Three body losses inhibit the stability of the ferromagnetic state

- To reduce three-body losses the interaction strength is ramped rapidly

- In boson systems, three-body scattering can give rise to hard-core interactions and drive the formation of a Tonks-Girardeau gas [Syassen et al., Science 320, 1329 (2009)]

- The experiment is an opportunity to study not only ferromagnetism but also three-body loss and dynamics
Itinerant ferromagnetism in cold atom gases

- Use two $^6\text{Li}$ states to represent pseudo up and down-spin electrons

$$H = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + g \sum_k c_{k\uparrow}^{\dagger} c_{k\downarrow}^{\dagger} c_{k\downarrow} c_{k\uparrow}$$

$$E \approx \sum_{k\sigma} \varepsilon_k n_{\sigma}(\varepsilon_k) + g N_{\uparrow} N_{\downarrow}$$

- A $\Delta E$ shift in the Fermi surface causes:

  1. Kinetic energy increase of $\frac{1}{2} \nu \Delta E^2$
  2. Reduction of repulsion of $-\frac{1}{2} g \nu^2 \Delta E^2$

- Total energy shift is $\frac{1}{2} \nu \Delta E^2 (1 - g \nu)$ so a ferromagnetic transition occurs if $g \nu > 1$

Cold atomic gases — spin

• Two fermionic atom species have a *pseudo-spin*:

\[ ^{6}\text{Li} \quad m_F=1/2 \quad \text{maps to} \quad \text{spin } 1/2 \]

\[ ^{6}\text{Li} \quad m_F=-1/2 \quad \text{maps to} \quad \text{spin } -1/2 \]

• The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

\[ (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2} \quad S=0, \ S_z=0 \quad \text{Non-magnetic state} \]

\[ |\uparrow\uparrow\rangle \quad S=1, \ S_z=1 \quad \text{State not possible as } S_z \text{ has changed} \]

\[ |\downarrow\downarrow\rangle \quad S=1, \ S_z=-1 \quad \text{State not possible as } S_z \text{ has changed} \]

\[ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} \quad S=1, \ S_z=0 \quad \text{Magnetic moment in plane} \]

• Ferromagnetism, if favourable, must form in-plane
Experimental evidence for ferromagnetism

- Experimental points display same qualitative behavior but transition at $k_F a = 2.2$

Further key experimental signatures

\[ E_K \propto n^{5/3} \]

\[ \Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow) \]

Synopsis of theoretical analysis

To evaluate the experimental results we

1) Employ a mean-field approximation to expose the consequences of a trapped geometry

2) Consider how fluctuation corrections affect the transition

3) Introduce new formalism that addresses atom loss

4) Analyze how the mutual annihilation of defects inhibits the formation of a ferromagnetic state

Active research on other possibilities

1) Spin pattern formation [Berdnikov et al., PRB 79, 224403 (2009)]

2) Trapped geometry & texture [LeBlanc et al., PRA 80, 013607 (2009)]

3) Domain formation [Babadi et al., arXiv:0908.3483]

4) Other strongly correlated state [Zhai, PRA 80, 051605(R) (2009)]

5) First order transition [Duine & MacDonald, PRL 95, 230403 (2005)]
Mean-field analysis & consequences of trap

- Recovers qualitative behavior\(^1\) but transition at \(k_Fa=1.8\) instead of \(k_Fa=2.2\)

Fluctuation corrections

\[ Z = \int D\psi \exp \left( -\int \int d\tau d\mathbf{r} \sum_{\sigma} \bar{\psi}_\sigma (-i\omega + \epsilon - \mu) \psi_\sigma - g \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right) \]

- Decouple with the average magnetisation gives the Stoner criterion

\[ F = \sum_{\sigma, k} \epsilon_k^\sigma n(\epsilon_k^\sigma) + gN^\uparrow N^\downarrow \propto (1 - g \nu) m^2 \]

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength for the free energy\(^1\)

\[ F = \sum_{\sigma, k} \epsilon_k^\sigma n(\epsilon_k^\sigma) + gN^\uparrow N^\downarrow - \frac{2g^2}{V^3} \sum_{k_{1,2,3,4}} \frac{n(\epsilon_{k_1}^\uparrow)n(\epsilon_{k_2}^\downarrow)[n(\epsilon_{k_3}^\uparrow)+n(\epsilon_{k_4}^\downarrow)]}{\epsilon_{k_1}^\uparrow+\epsilon_{k_2}^\downarrow-\epsilon_{k_3}^\uparrow-\epsilon_{k_4}^\downarrow} \]

- Backed up by \textit{ab initio} Quantum Monte Carlo calculations\(^2\)

- Enhanced particle-hole phase space at zero magnetisation\(^2\) leads to an anomalous term\(^3\) \(m^4 \ln|m|\) in the Landau expansion that drives the ferromagnetic transition first order at \(k_F a=1.054\)

\(^3\)Belitz et al. Z. Phys. B (1997)
Fluctuation corrections

- Extend theory through fluctuation corrections

![Graph showing fluctuation corrections](image)

Including atom loss

- Atom loss rate
  \[ \lambda n_{\uparrow}(r)n_{\downarrow}(r)\chi(r-r')[n_{\uparrow}(r') + n_{\downarrow}(r')] \]
  which in second quantized form is
  \[ \lambda c_{\uparrow}(r)c_{\downarrow}(r)c_{\uparrow}(r)c_{\downarrow}(r)\chi(r-r')[c_{\uparrow}(r')c_{\uparrow}(r') + c_{\downarrow}(r')c_{\downarrow}(r')] \]
- With a mean-field approximation, \[ \bar{N} = c_{\uparrow}c_{\uparrow} + c_{\downarrow}c_{\downarrow} \]
  \[ \lambda Nc_{\uparrow}(r)c_{\downarrow}(r)c_{\uparrow}(r)c_{\downarrow}(r) \]
  it appears on same footing as the interaction term
  \[ S_{\text{int}} = (g + i\lambda \bar{N})c_{\uparrow}(r)c_{\downarrow}(r)c_{\downarrow}(r)c_{\uparrow}(r) \]
- Loss damps fluctuations so inhibits transition
  \[ F = \sum_{\sigma, k} \epsilon_{k}^{\sigma} n(\epsilon_{k}^{\sigma}) + gN^{\uparrow}N^{\downarrow} - 2(g^{2} - \lambda^{2} \bar{N}^{2})Y \]
Atom loss has the potential to raise the interaction strength required for a ferromagnetic transition.

Conduit & Altman, arXiv: 0911.2839
Condensation of topological defects

- Defects freeze out from disordered state

- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength

- Defect radius $L \sim t^{1/2}$ [Bray, Adv. Phys. 43, 357 (1994)]
Condensation of topological defects

- Condensation of defects inhibits the transition

Mean-field theory provides a reasonable qualitative description of the transition.

Discrepancy in the interaction strength could be accounted for by:

1) Renormalization of interaction strength due to atom loss
2) The mutual annihilation of defects inhibiting the formation of the ferromagnetic phase
First order phase transition and Quantum Monte Carlo verification

- First order transition into uniform phase with TCP

- QMC also sees first order transition
New approach to fluctuation corrections

\[ Z = \int D\psi \exp \left( -\int \sum_{\sigma} \bar{\psi}_\sigma (-i\omega + \epsilon - \mu) \psi_\sigma - g \int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right) \]

- Analytic strategy:
  1) Decouple in both the density and spin channels (previous approaches employ only spin)
  2) Integrate out electrons
  3) Expand about uniform magnetisation
  4) Expand density and magnetisation fluctuations to second order
  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure
Analytical method

- System free energy $F = -k_B T \ln Z$ is found via the partition function

\[ Z = \int D\psi \exp \left(-\int \sum_{\sigma} \bar{\psi}_\sigma (-i \omega + \epsilon - \mu) \psi_\sigma - g \int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right) \]

- Decouple using only the average magnetisation $m = \bar{\psi}_\downarrow \psi_\uparrow - \bar{\psi}_\uparrow \psi_\downarrow$ gives $F \propto (1 - g \nu) m^2$ i.e. the Stoner criterion


\[ F = \frac{1}{2} \left( |\omega|/\Gamma_q + r + q^2 \right) m^2 + \frac{u}{4} m^4 \ln (m^2 + T^2) + \cdots - hm \]