Few-atom approach to many-body physics

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Experimental setup

\[ |F = 1/2, m_F = 1/2 \rangle \Rightarrow \text{Up spin electron} \]

\[ |F = 1/2, m_F = -1/2 \rangle \Rightarrow \text{Down spin electron} \]

\[ \hat{H} = -\frac{\nabla^2}{2} + g n_\uparrow(\vec{r}) n_\downarrow(\vec{r}) \]
Two distinguishable fermions

G. Zürn et al. PRL 108 075303 (2012)
Two distinguishable fermions
Two distinguishable fermions
Two distinguishable fermions
Energy of states

- Weak repulsion
- Strong repulsion
Polaron state

Polaron state

Polaron state

A. Wenz et al., arXiv:1307.3443
Polaron state
<table>
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Stoner Hamiltonian

\[ \hat{H} = -\frac{\nabla^2}{2} + gn_\uparrow(\vec{r}) n_\downarrow(\vec{r}) + V(\vec{r}) \]
Density profiles
Density profiles
Polaron state
Polaron state
Polaron state
Polaron state
Polaron state
Loss region

Fraction of polarized systems [%]

Weak

Strong

\(-1/g_{1D} [a_\parallel, \omega_\parallel]\)
Tunneling probability

![Graph showing tunneling probability]
Why probability of $\frac{1}{2}$?
Why probability of $\frac{1}{2}$?
Why probability of \( \frac{1}{2} \)?

Ferro

\[ |\uparrow\uparrow\rangle + |\downarrow\uparrow\rangle \]

Para

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Two-atom scattering

Repulsive

\[ V(r) \]

\[ r \]

\[ a \]
Two-atom scattering

Repulsive

Attractive
Two-atom scattering

Repulsive

Attractive

Repulsive
Two-atom scattering

- **Repulsive**: The potential $V(r)$ increases as the distance $r$ decreases.
- **Attractive**: The potential $V(r)$ decreases as the distance $r$ decreases.
- **Repulsive**: The potential $V(r)$ decreases as the distance $r$ decreases, typically represented by a U-shaped curve.
Three-atom bound state

Weak

Strong
Three-atom bound state

\[ \frac{E}{\hbar \omega_{||}} \]

Weak

Strong
Three-atom bound state

Weak

Strong
Three-atom bound state
Three-atom bound state

Weak

Strong
Three-atom bound state
Three-atom bound state
Three-atom bound state

- Weak
- Strong

$E/\hbar \omega_{||}$

$-\hbar \omega_{||} a_{||}/g$
Three-atom bound state

Weak

Strong
Tunneling probability

Fraction of polarized systems [%]

$-1/g_{1D} [a_|| \omega_||]$

Weak

Strong
Band crossings

\[ n^{\uparrow}n^{\downarrow} = 4 \]

\[ n^{\uparrow}n^{\downarrow} = 8 \]
Band crossings
BCS Hamiltonian

\[ \hat{H} = -\frac{\nabla^2}{2} + g c_{\mathbf{p}\uparrow}^{\dagger} c_{\mathbf{p}\downarrow} c_{\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\uparrow}^{\dagger} + V(\mathbf{r}) \]
Inhomogeneous pairing
Inhomogeneous pairing

\[ \Delta \rho \]

[Diagram showing inhomogeneous pairing with \( \Delta \rho \) and \( Z \).]
Inhomogeneous pairing

\[ E \]

011  101  110  200  020  002

100  010  001

000
Inhomogeneous pairing
Inhomogeneous pairing

\( \rho_{001} \)

\( z \)
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing

\[ \Delta \rho \]
Inhomogeneous pairing

(2,1) (3,2) (4,3)

(2,1) s (5,1)

(3,1) (4,2)

(2,1) d (5,2)

(4,1)

(2,1) g
Inhomogeneous pairing

Weak interactions

Strong interactions

V
Inhomogeneous pairing

Inhomogeneous pairing

Inhomogeneous pairing
A few-fermion system provides insight into many-body physics

Discretization of energy levels means that losses occur in narrow range of interaction strengths

Observation of Fermi surface and magnetic correlations
Other phenomena

Hund's rules
P.O. Bugnion, J.A. Lofthouse & GJC

Exchange interactions
P.O. Bugnion & GJC
Finding the missing probability

\[ \langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle \]

or

\[ \langle \uparrow \uparrow \rangle \] or \[ \langle \uparrow \downarrow \rangle + \langle \downarrow \uparrow \rangle \]
Finding the missing probability

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]

or

\[ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \]

Weak

Strong
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing
Inhomogeneous pairing