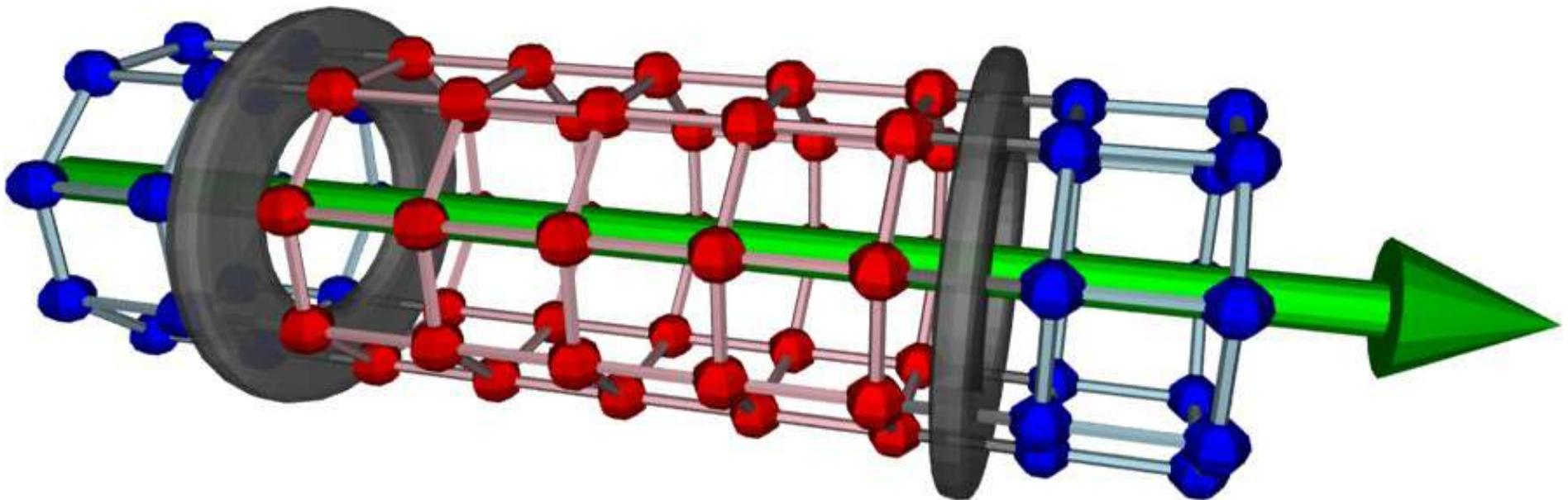


# An *ab initio* study of the Little-Parks effect

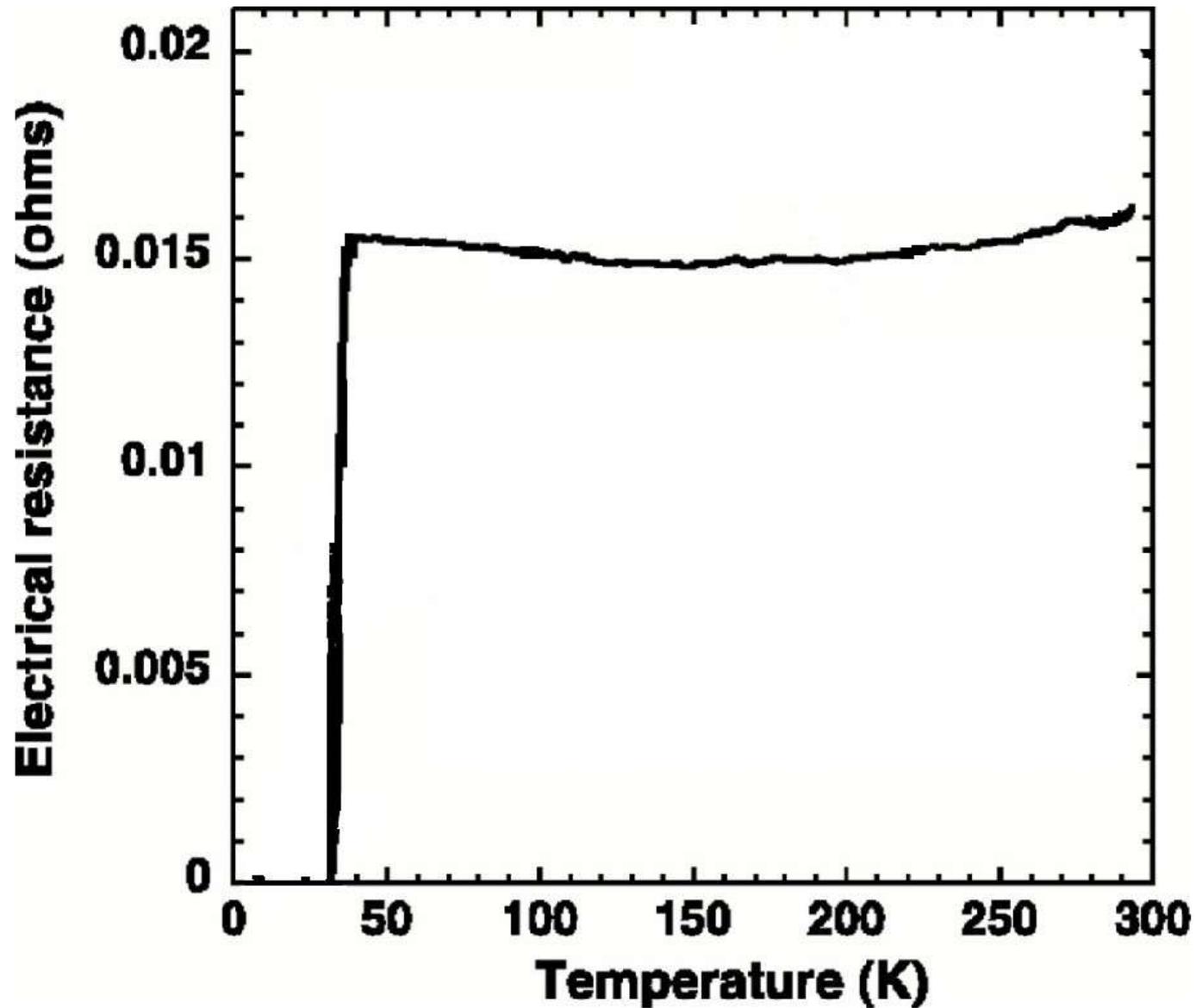


**Gareth Conduit, Yigal Meir**

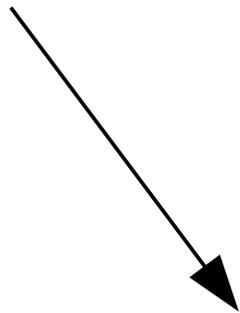
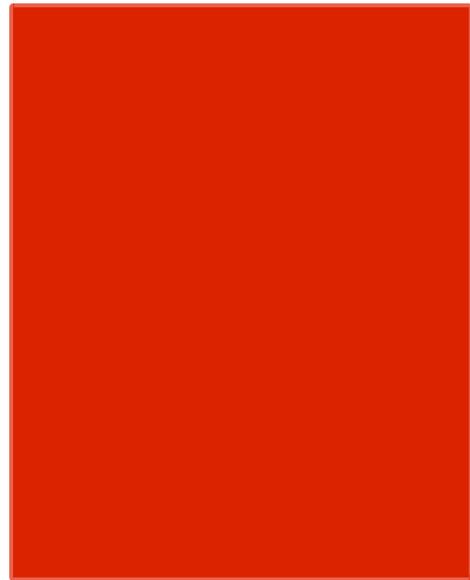
Ben Gurion University

arXiv:1102.1604

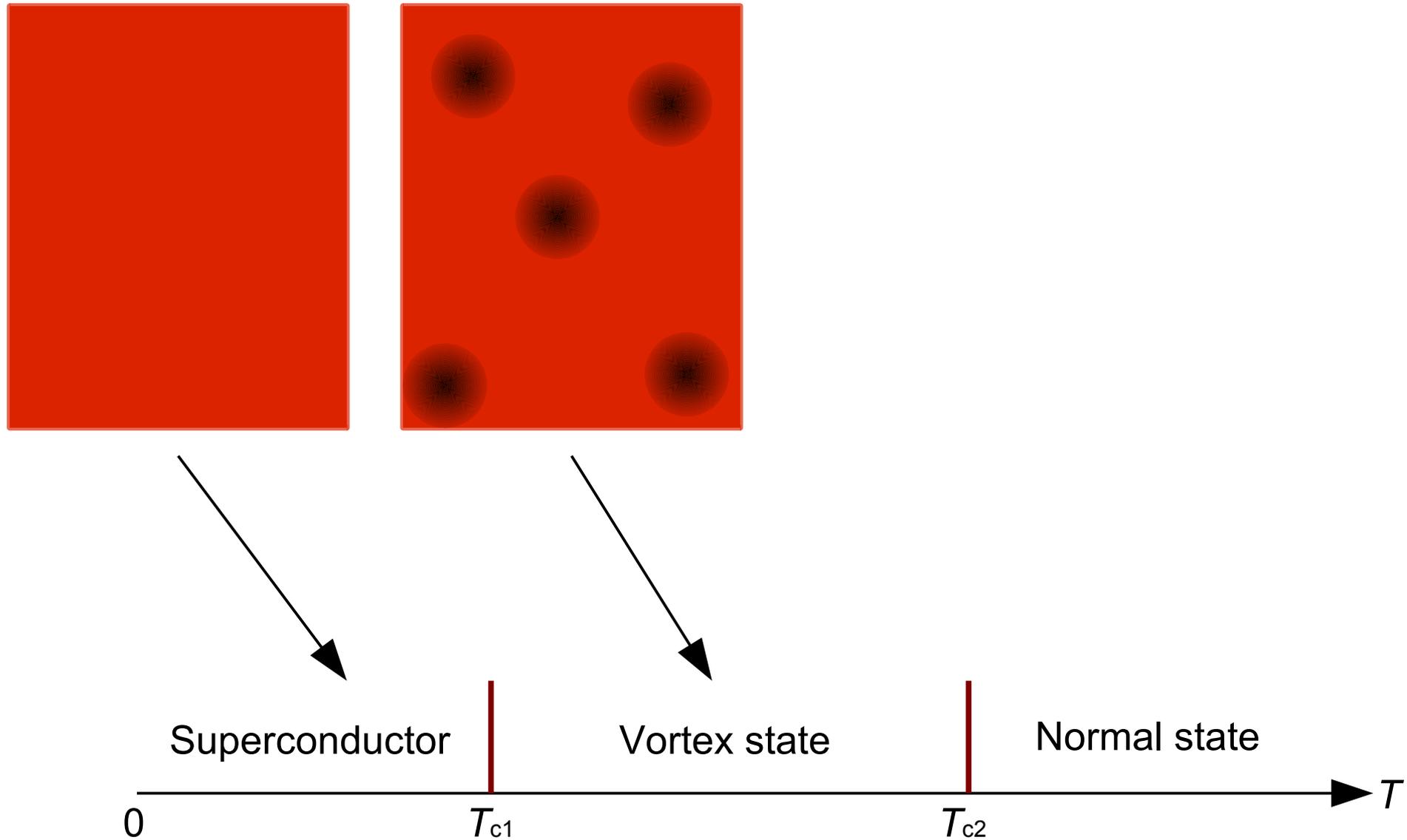
# BCS superconductivity



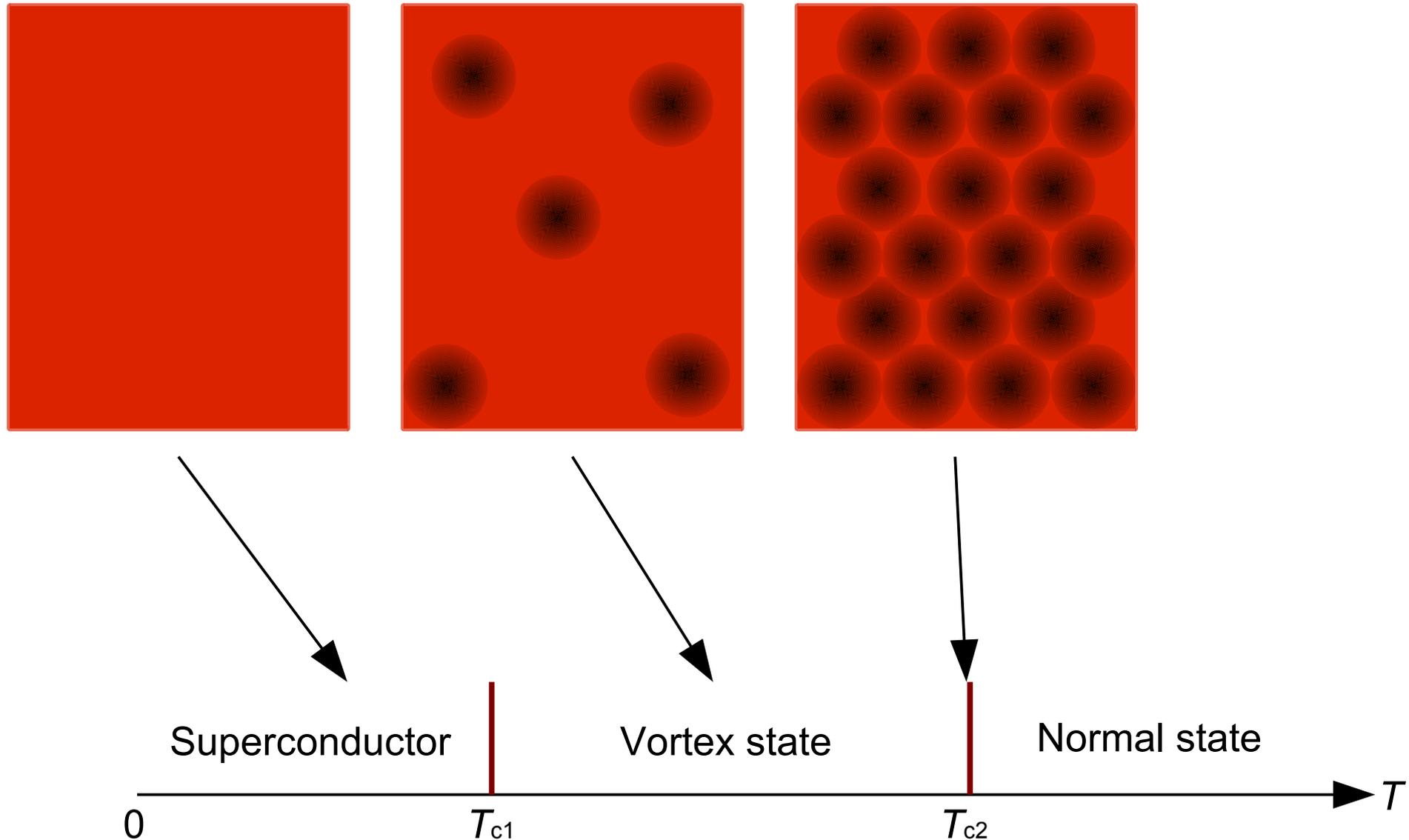
# Kosterlitz Thouless transition



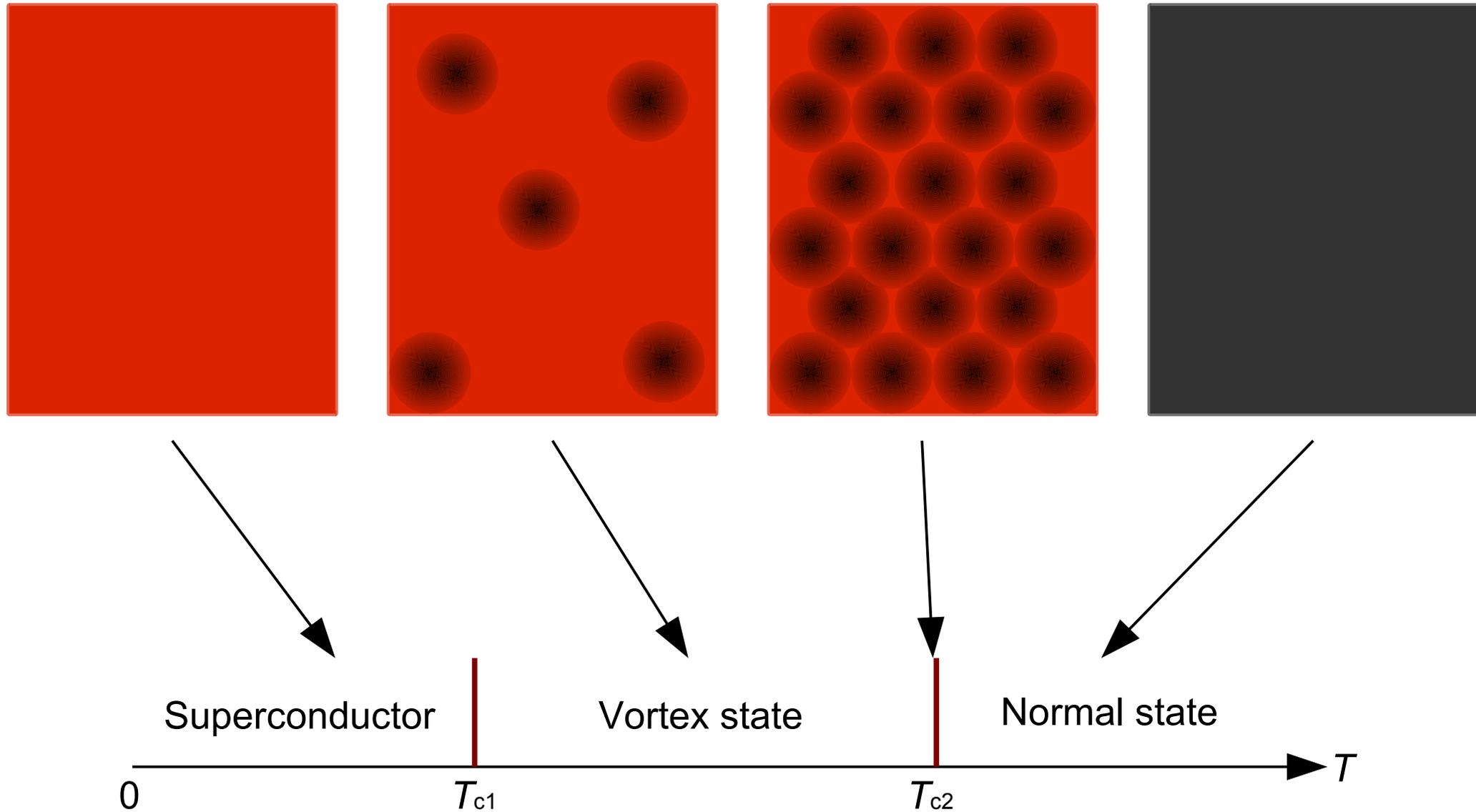
# Kosterlitz Thouless transition



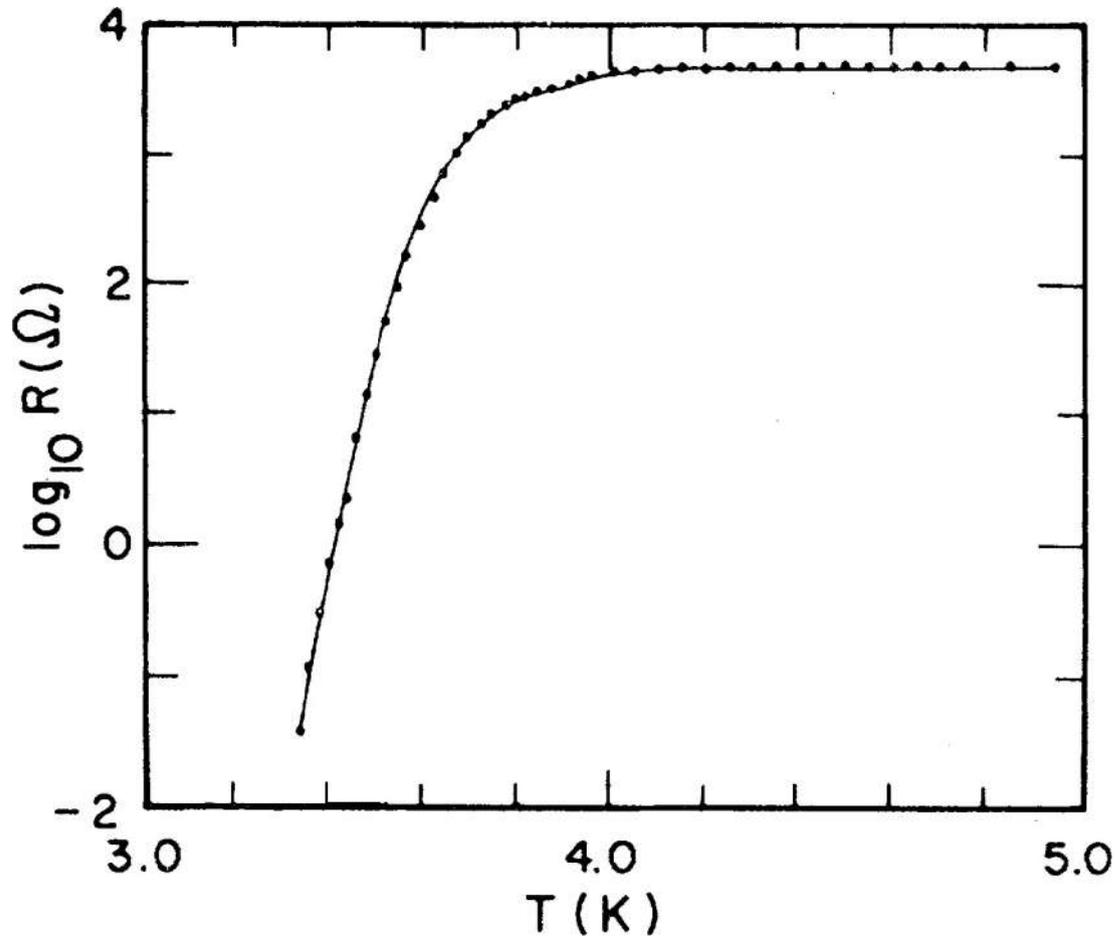
# Kosterlitz Thouless transition



# Kosterlitz Thouless transition



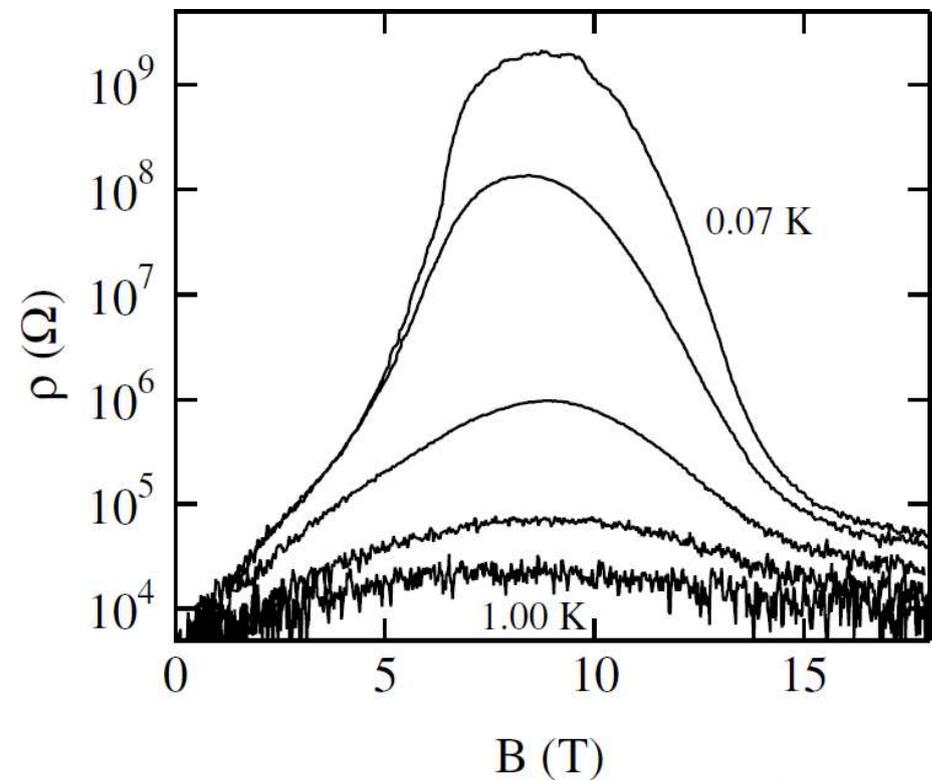
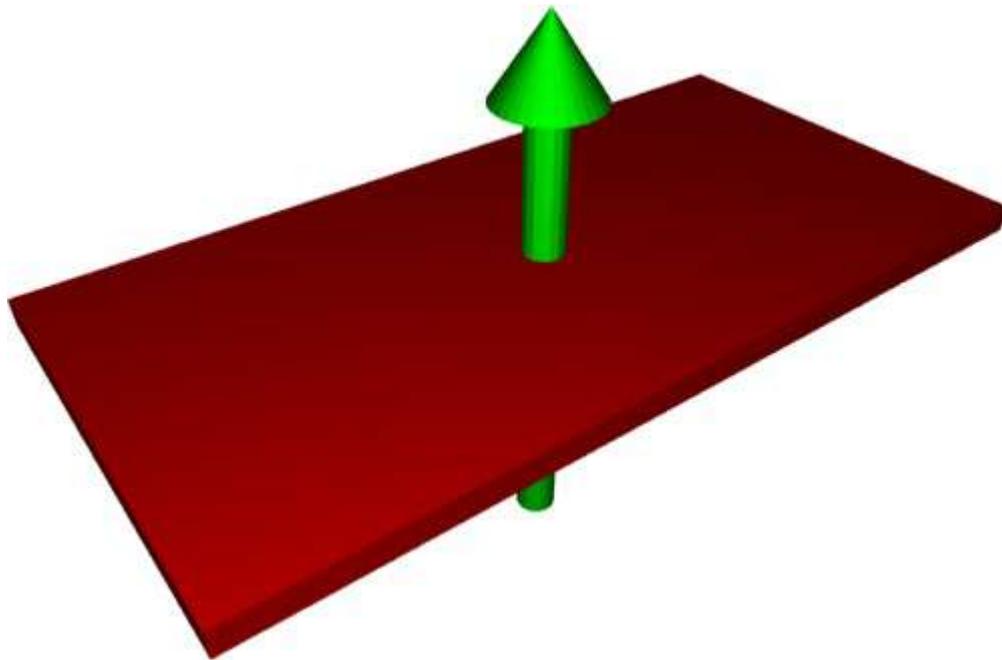
# KT transition conductivity



# Transition in highly disordered systems

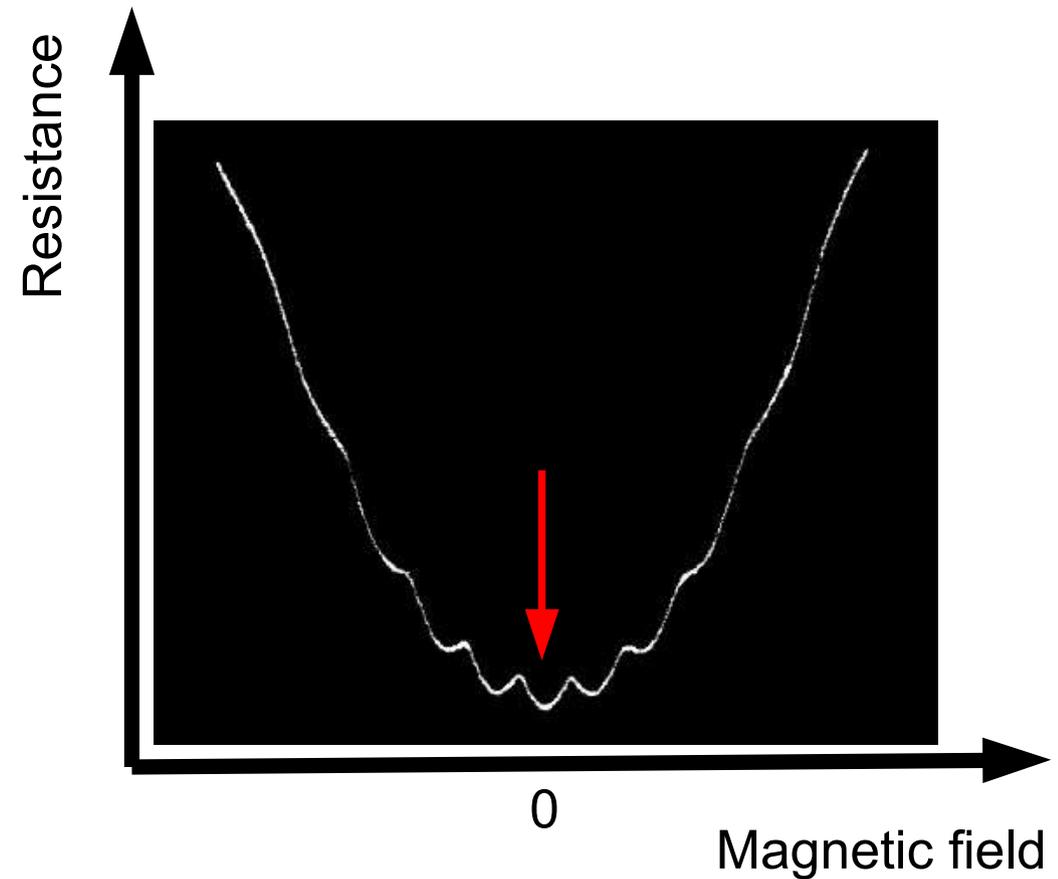
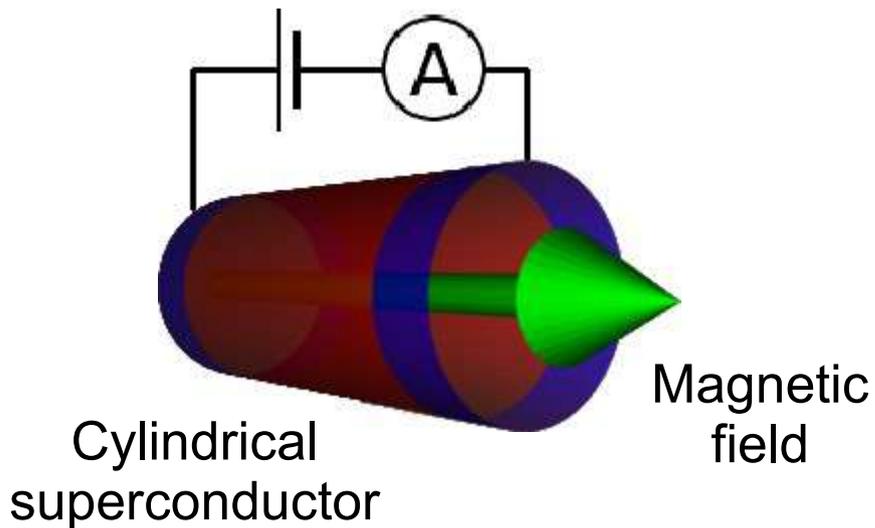
- Magnetoresistance peak [Sambandamurthy 04]

Perpendicular  
magnetic field  $B$



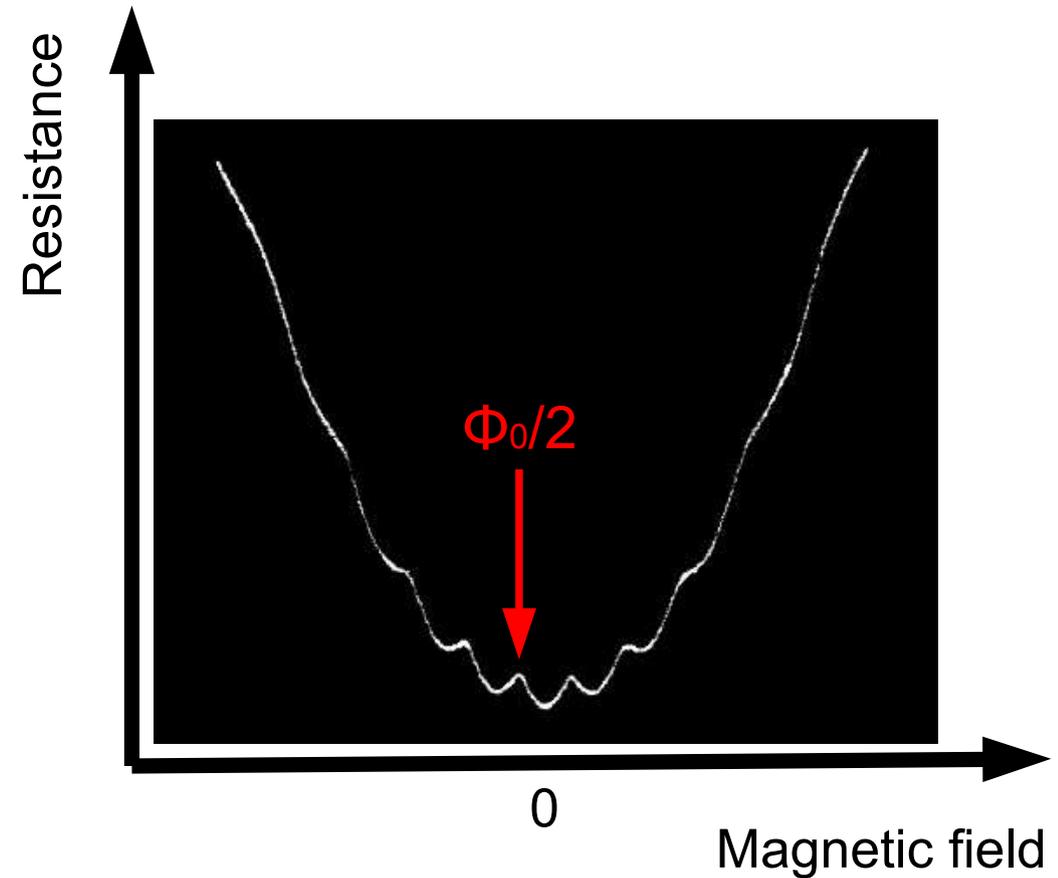
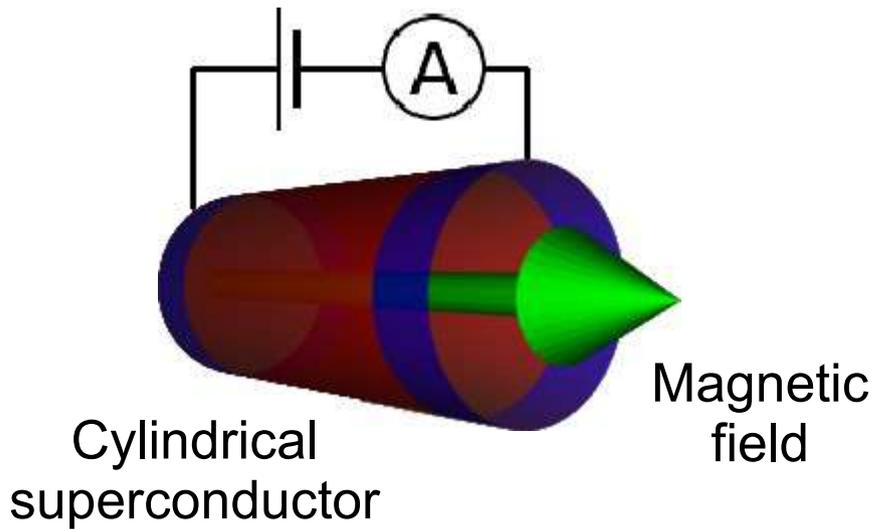
# Little-Parks in a large diameter cylinder

- Cylindrical superconductor held at transition temperature and zero threading flux [Little & Parks, PRL 1962]



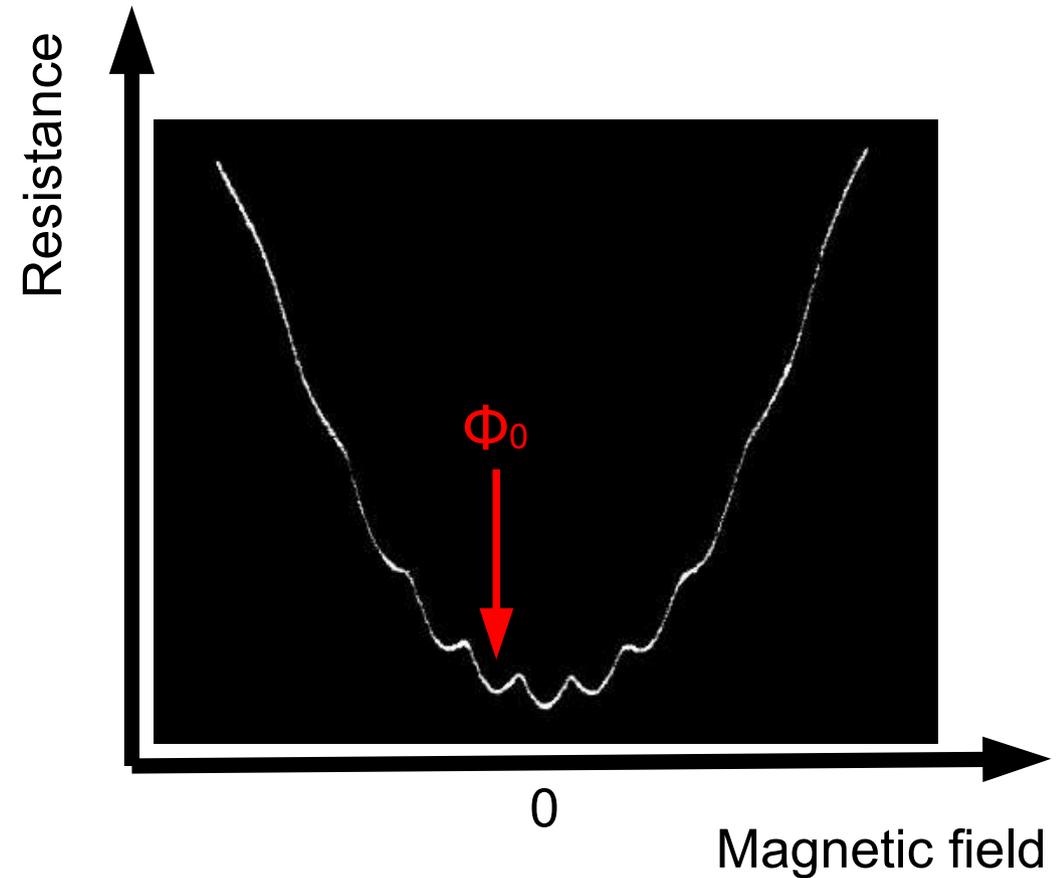
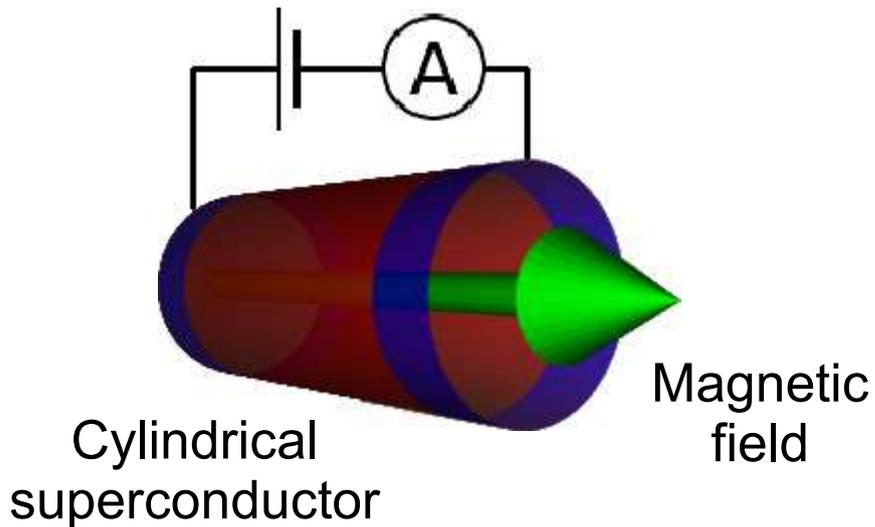
# Little-Parks in a large diameter cylinder

- Cylindrical superconductor held at transition temperature and threading flux is increased [Little & Parks, PRL 1962]



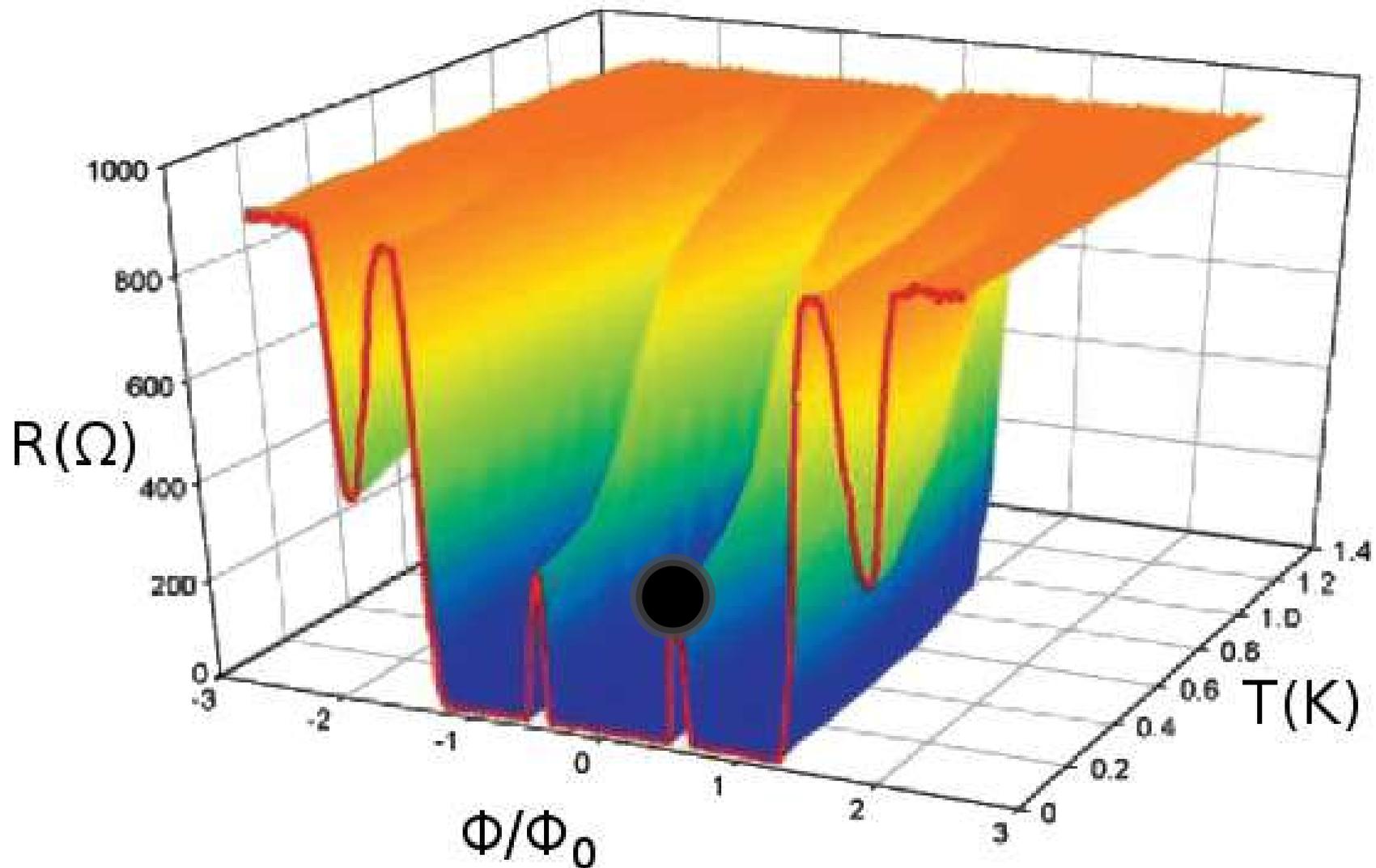
# Little-Parks in a large diameter cylinder

- Cylindrical superconductor held at transition temperature and threading flux is increased [Little & Parks, PRL 1962]



# Little-Parks in a small diameter cylinder

- Reduce cylinder diameter to superconducting correlation length [Liu *et al.*, Science 2001; Wang *et al.*, PRL 2005]



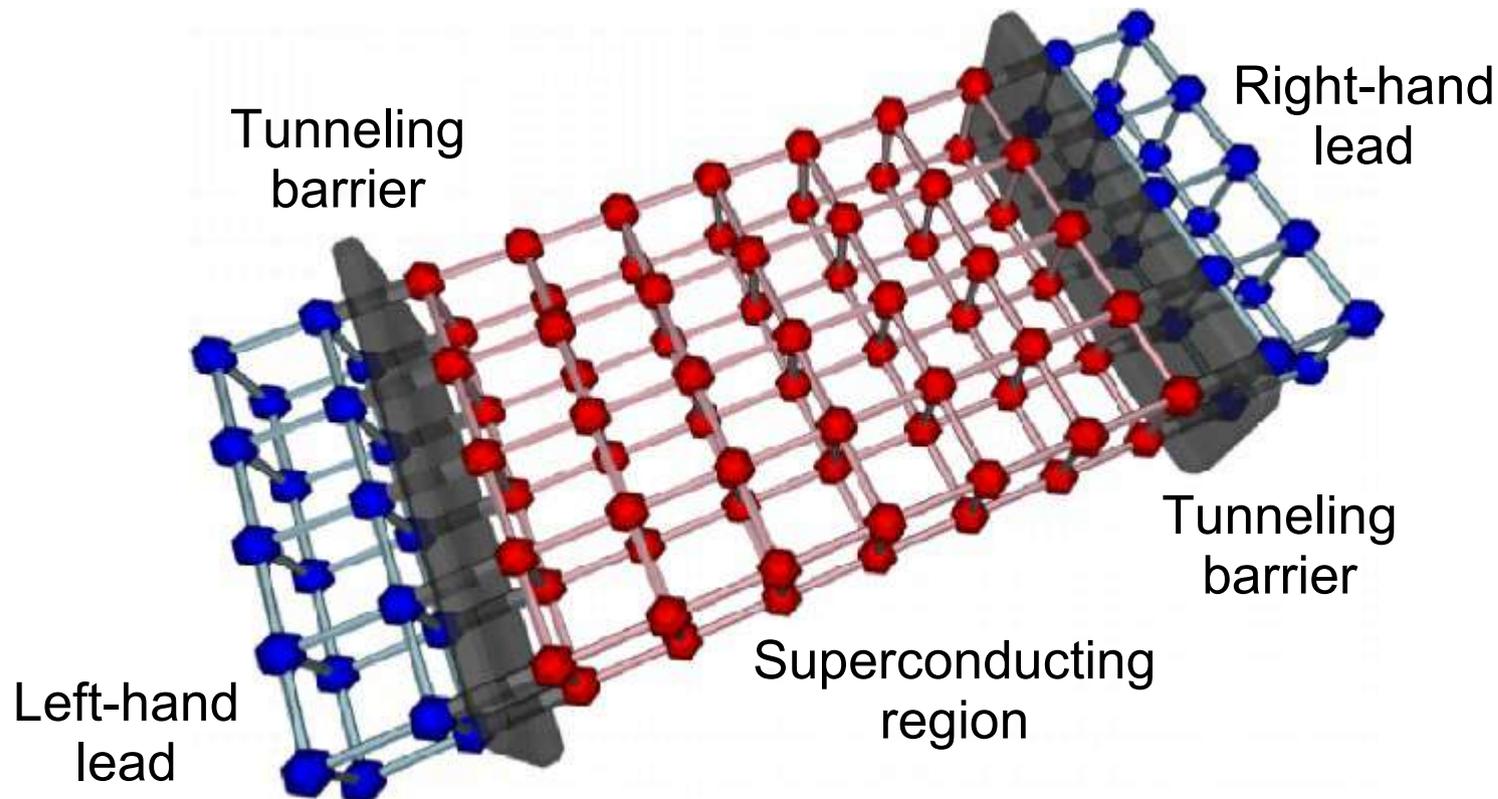
# Strategy to study superconductors

- Develop new formalism to:
  - Calculate exact net current flow
  - Extract the microscopic current flow
  - Account for phase and amplitude fluctuations
  - Develop algorithm that permits access to large systems
- Test the formalism against a series of well-established results
- Study the Little Parks effect and magnetoresistance peak

# How to calculate the current

- General expression for the current [Meir & Wingreen, PRL 1992]

$$J = \frac{ie}{2h} \int d\epsilon \left[ \text{Tr} \left\{ (f_L(\epsilon)\Gamma^L - f_R(\epsilon)\Gamma^R) (G_{e\sigma}^r - G_e^{a\sigma}) \right\} + \text{Tr} \left\{ (\Gamma^L - \Gamma^R) G_{e\sigma}^< \right\} \right]$$



# Decoupling the interactions

- Negative  $U$  Hubbard model

$$\hat{H}_{\text{Hubbard}} = \sum_{i,\sigma} \epsilon_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} - \sum_i U_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow} \\ - \sum_{\langle i,j \rangle, \sigma} \left( t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + t_{ij}^* c_{j\sigma}^\dagger c_{i\sigma} \right)$$

- Decouple Hamiltonian in density and Cooper pair channels

$$\hat{\mathcal{H}}_{\text{BdG}} = \sum_{i,\sigma} (\epsilon_i + \rho_i) c_{i\sigma}^\dagger c_{i\sigma} - \sum_{\langle i,j \rangle, \sigma} \left( t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + t_{ij}^* c_{j\sigma}^\dagger c_{i\sigma} \right) \\ + \sum_i \left( \Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \bar{\Delta}_i c_{i\downarrow} c_{i\uparrow} \right) + \sum_i \frac{|\Delta_i|^2 + \rho_i^2}{U_i}$$

# Accelerated Metropolis sampling

- To perform thermal sum calculate

$$\langle J \rangle = \sum_{\Delta, \rho} J[\Delta, \rho] e^{-\beta(E[\Delta, \rho] - E_0)}$$

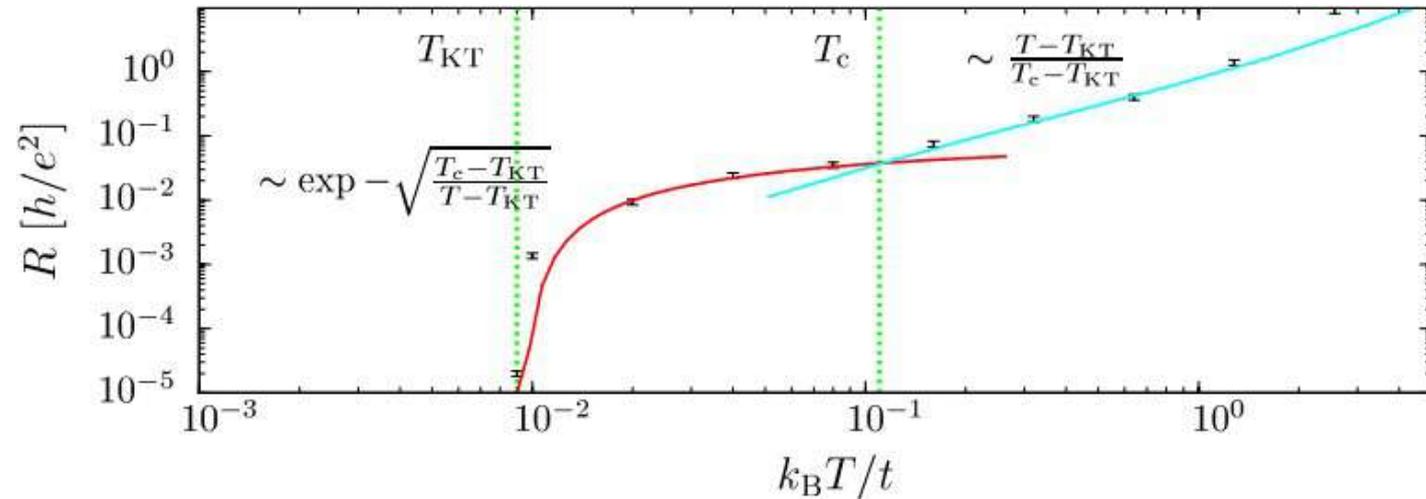
- Propose new configuration of  $\Delta$  and  $\rho$ , accept with probability

$$\exp(\beta E[\Delta_{\text{old}}, \rho_{\text{old}}] - \beta E[\Delta_{\text{new}}, \rho_{\text{new}}])$$

- Calculating  $E[\Delta, \rho]$  costs  $O(N^4)$ , where  $N$  is the number of sites
- New method calculates  $E[\Delta, \rho] - E[\Delta + \delta\Delta, \rho + \delta\rho]$  using a order  $M$  Chebyshev expansion [Weisse 09] in  $O(N^{1.9} M^{2/3})$  time

# Verification

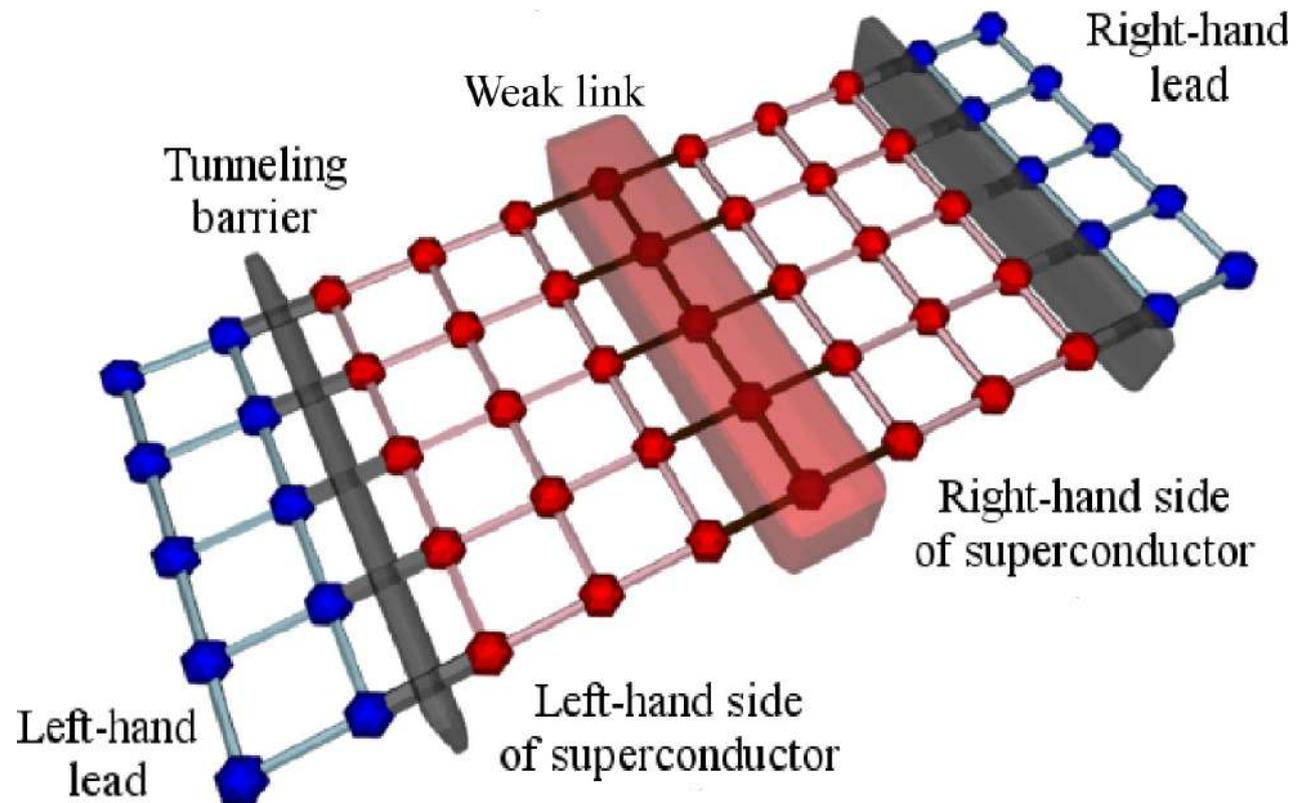
- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



Halperin & Nelson, J. Low Temp. Phys 1979  
Ambegaokar *et al.*, PRB 1980

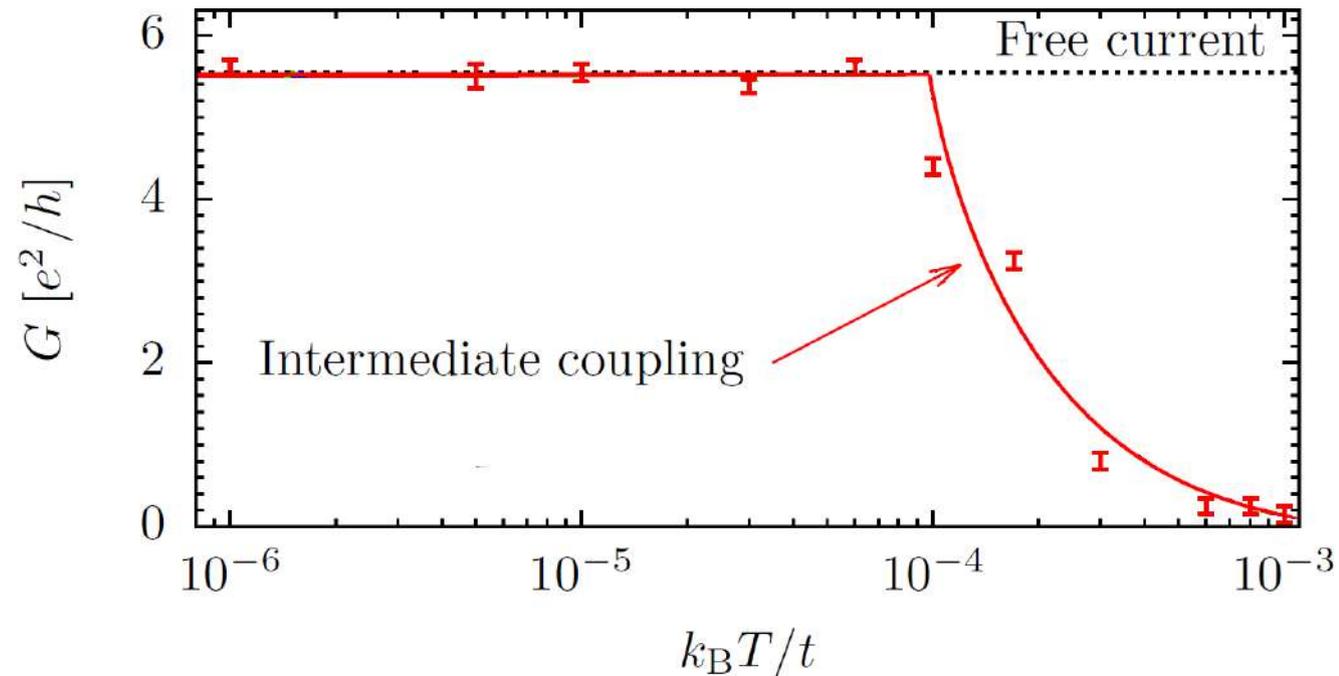
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



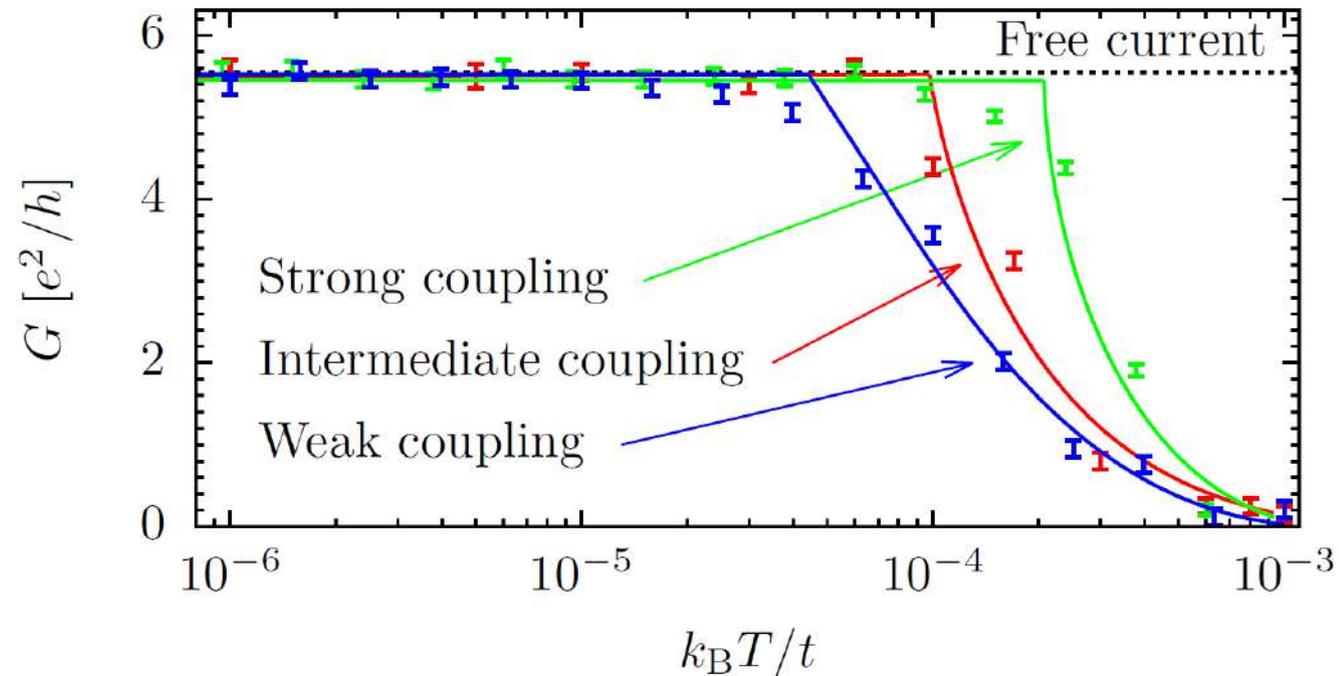
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



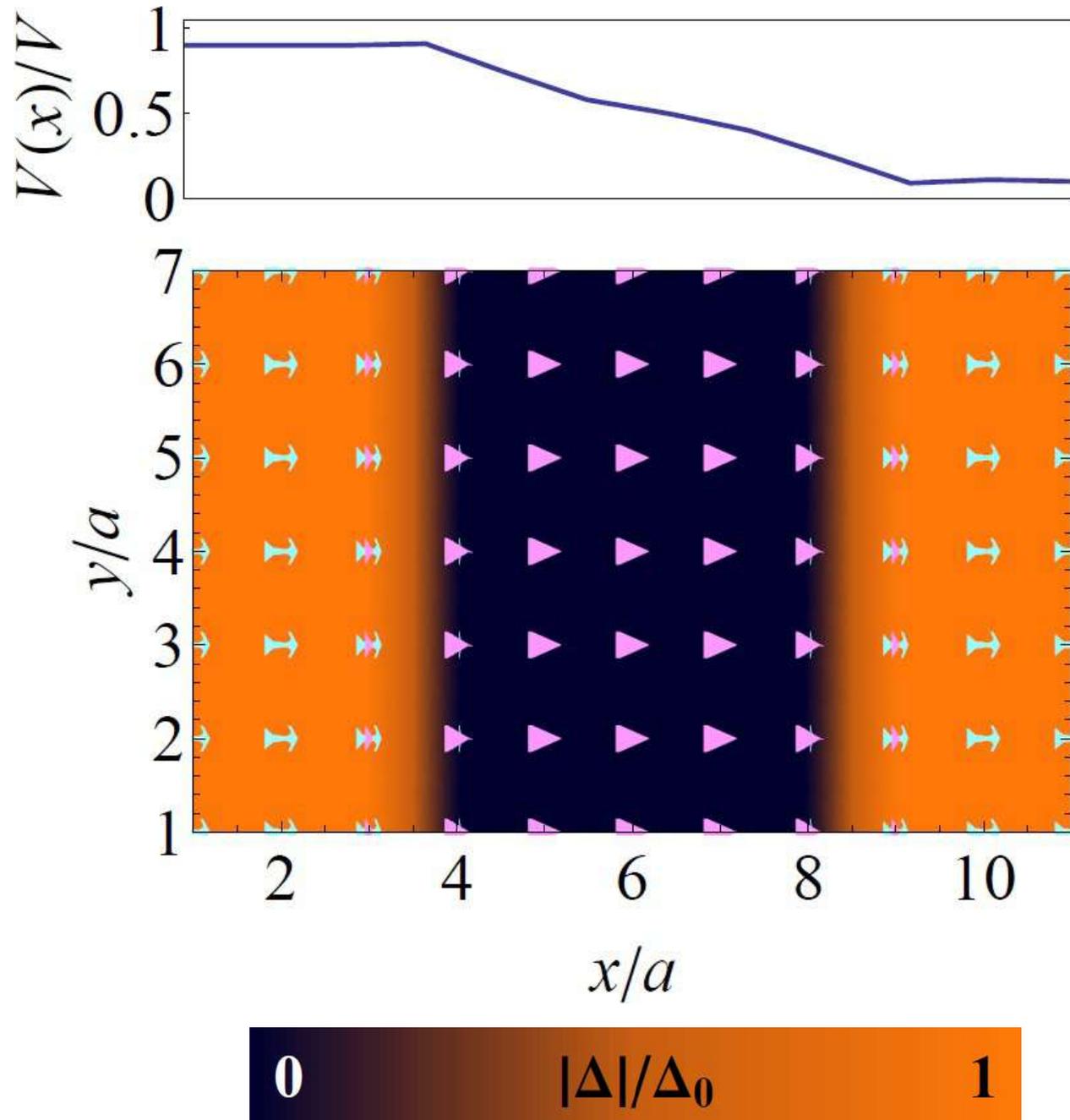
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- **Josephson junction**
- Little-Parks effect in large diameter cylinder



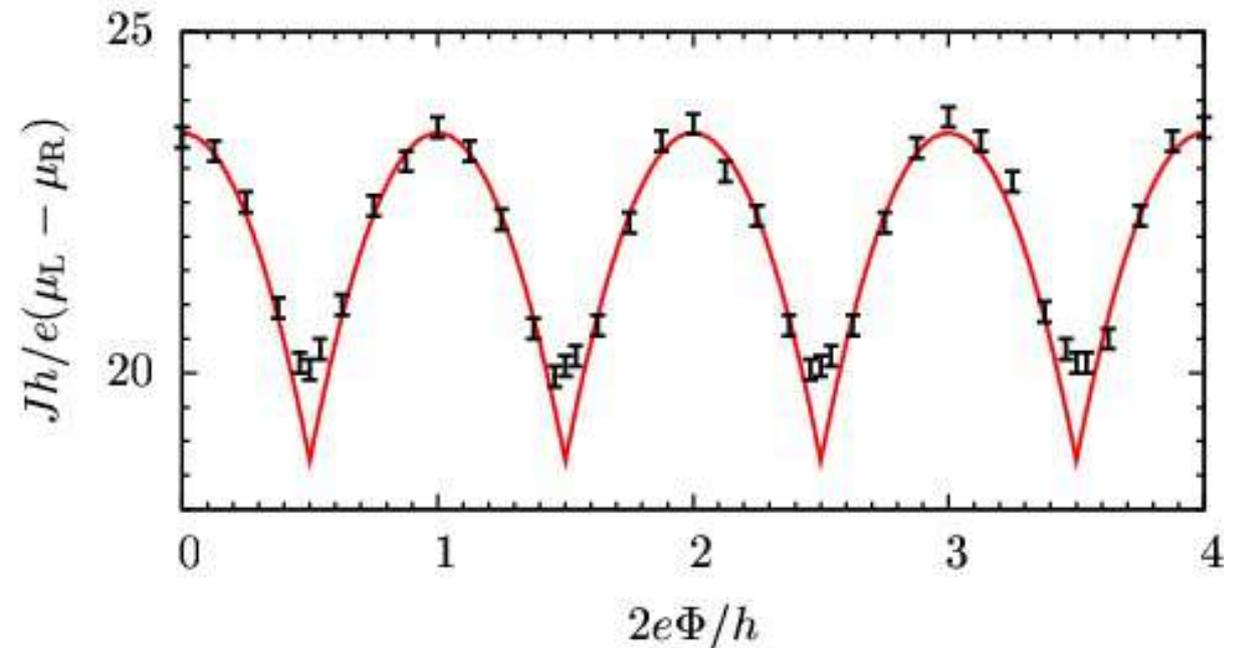
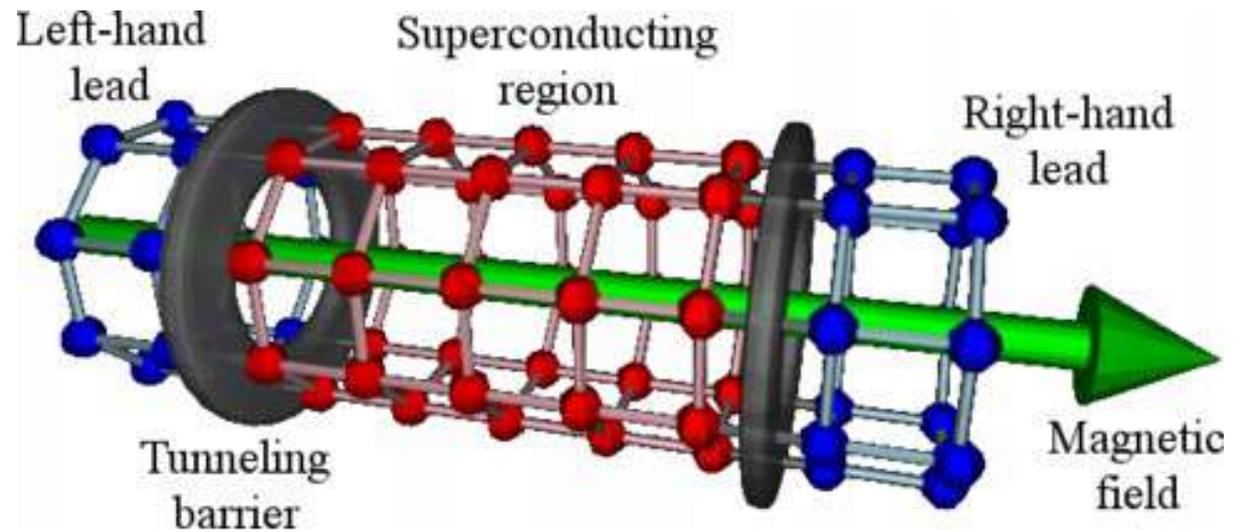
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder

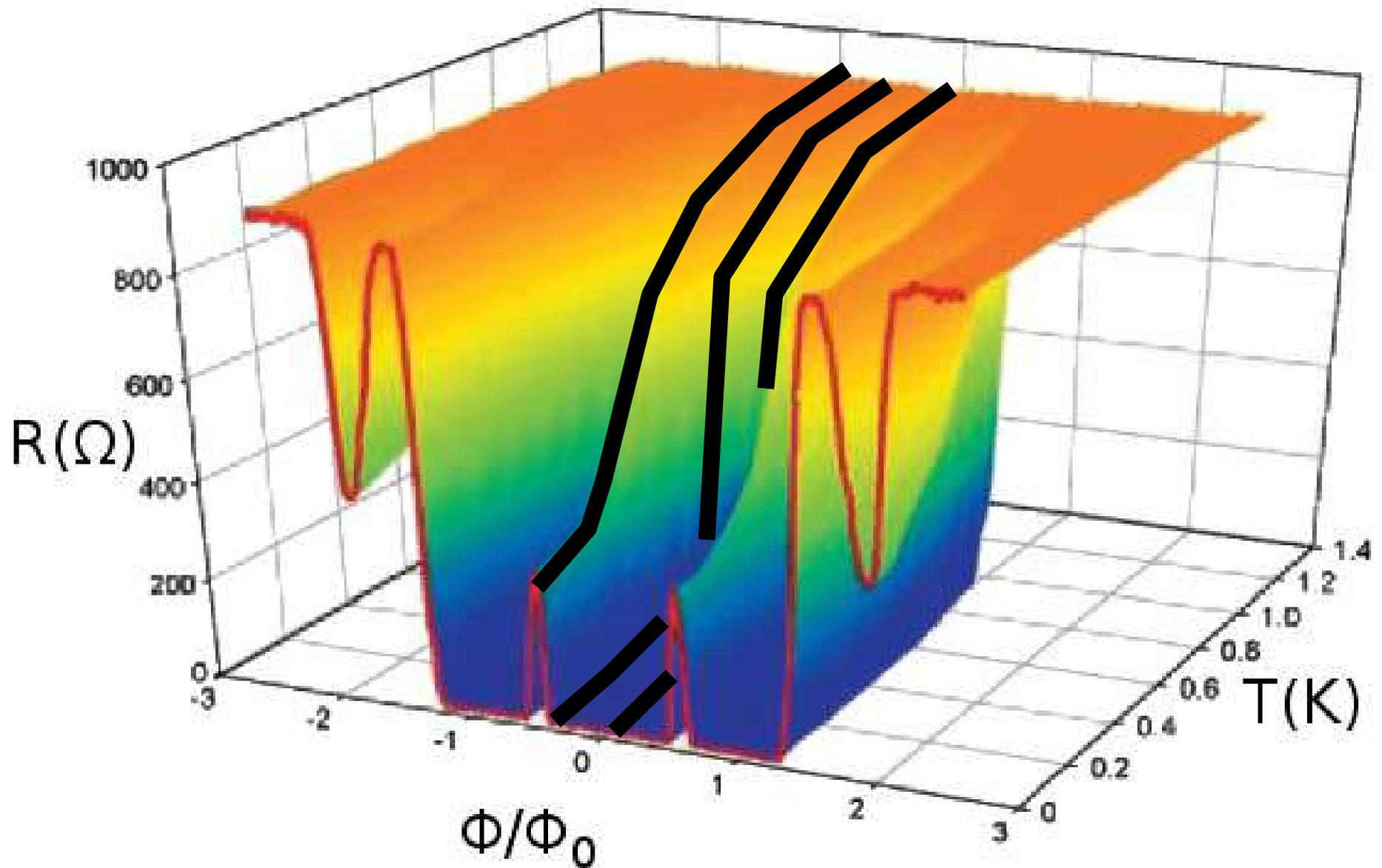


# Verification

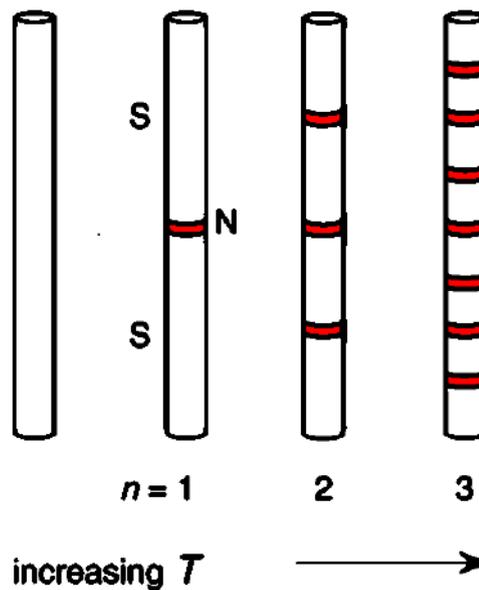
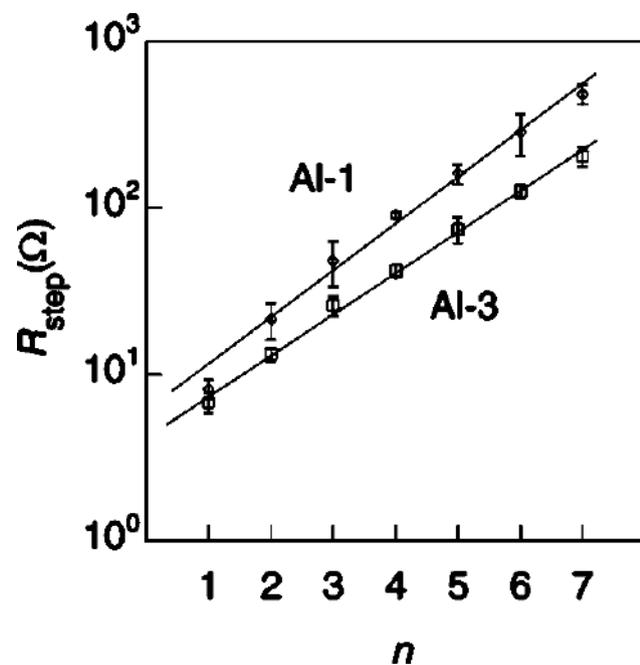
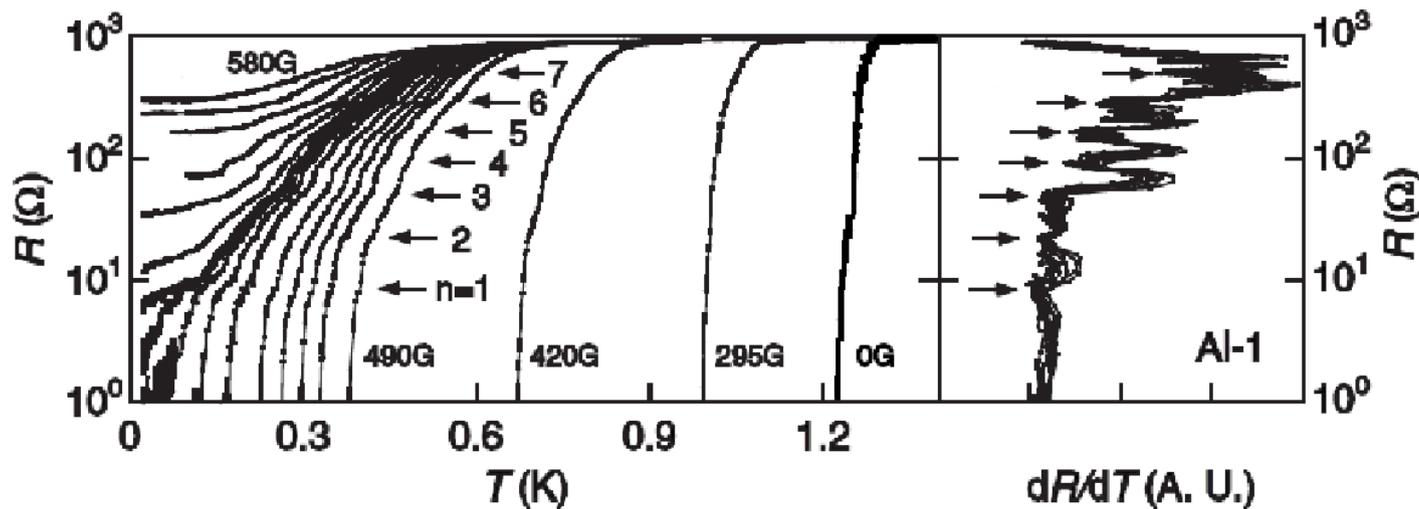
- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



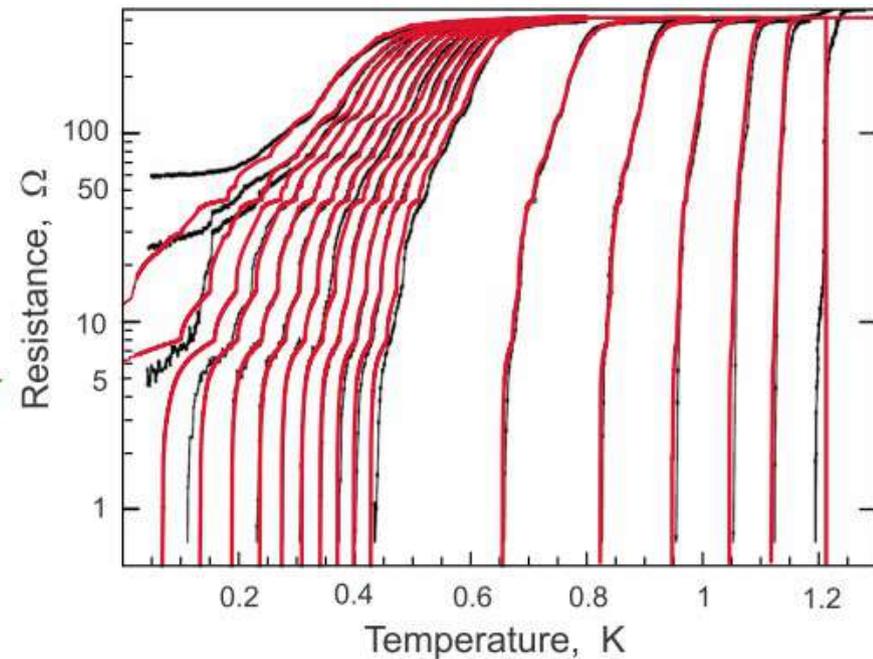
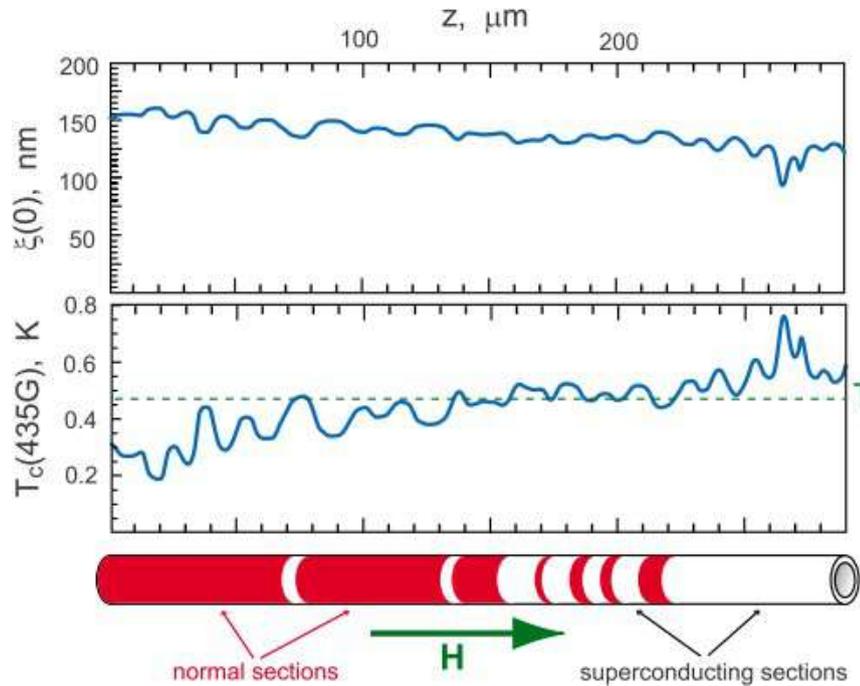
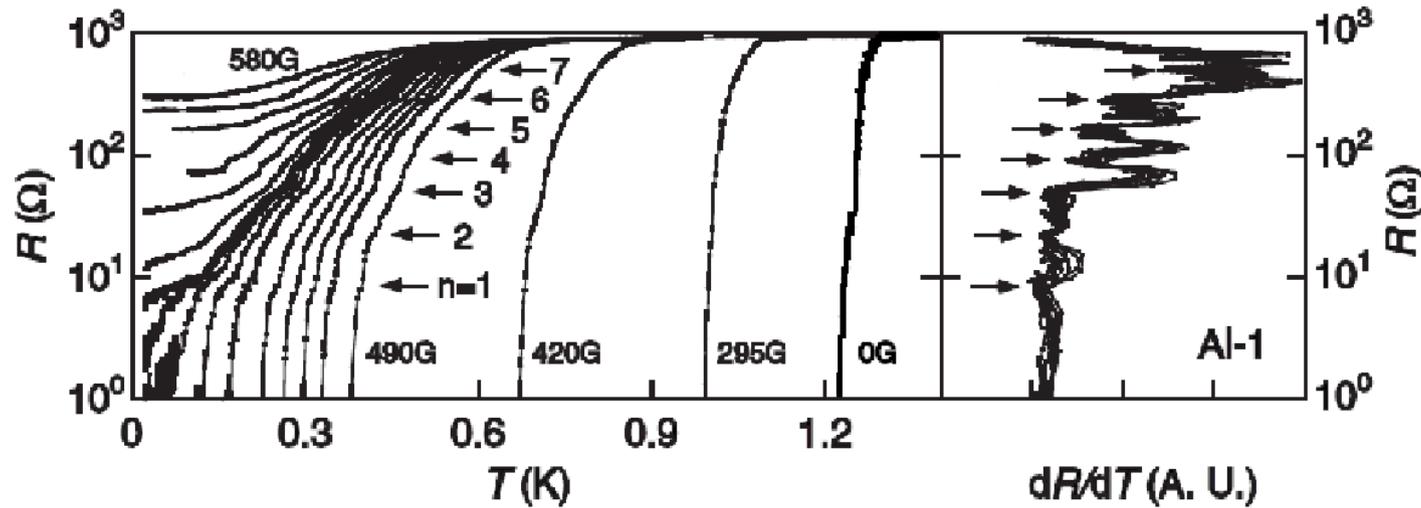
# Little-Parks in a small diameter cylinder



# Quantum phase transition hypothesis

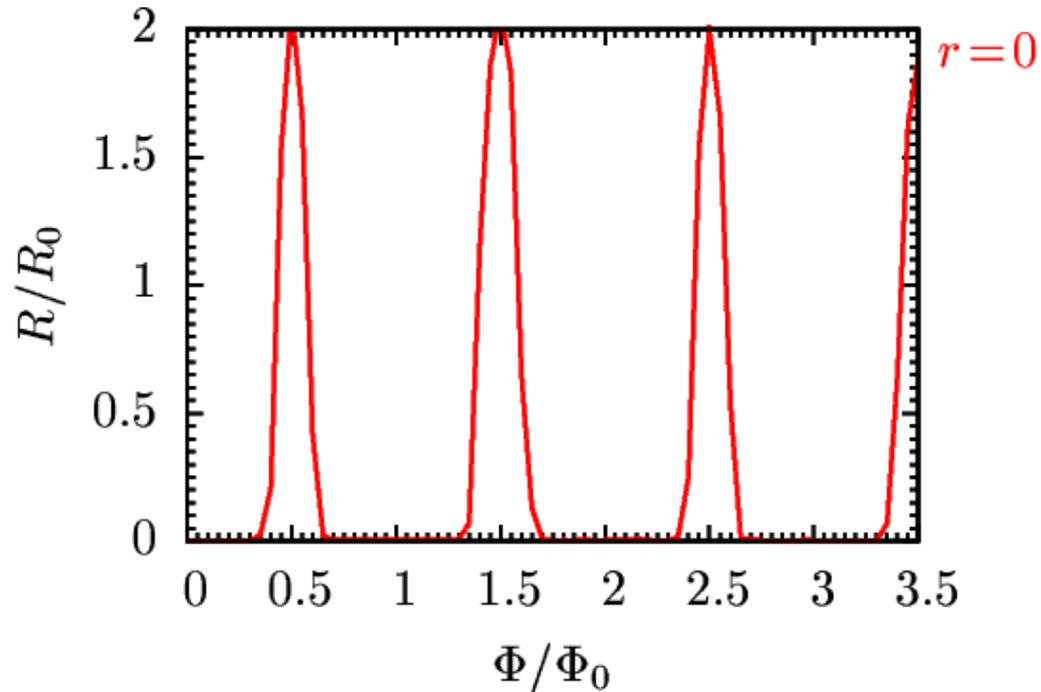


# Mean-field BCS transition hypothesis

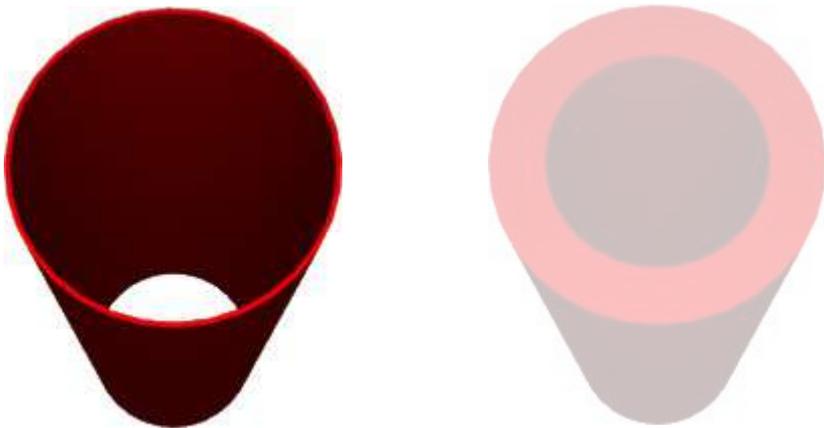
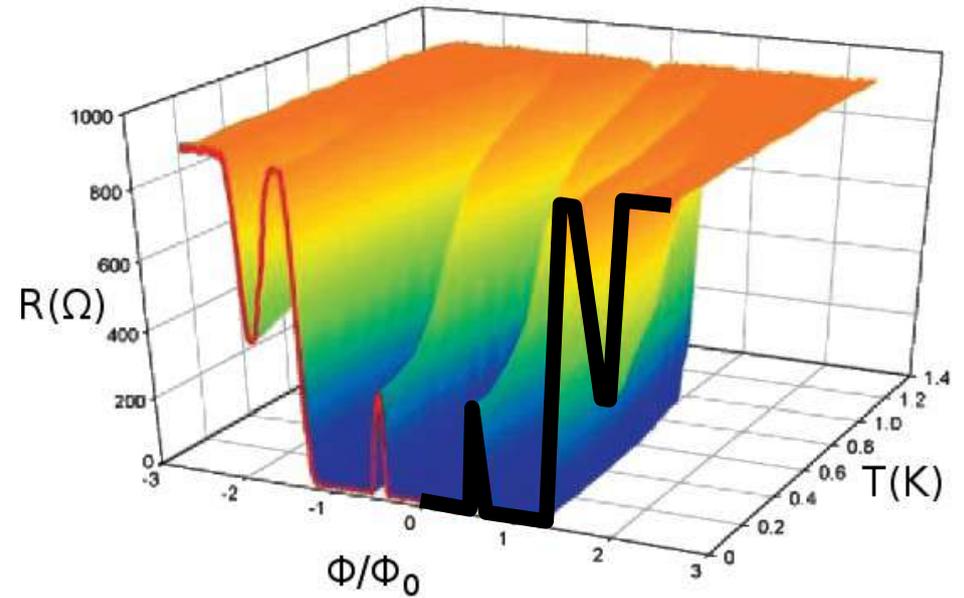


# Little-Parks in a small diameter cylinder

Theory:

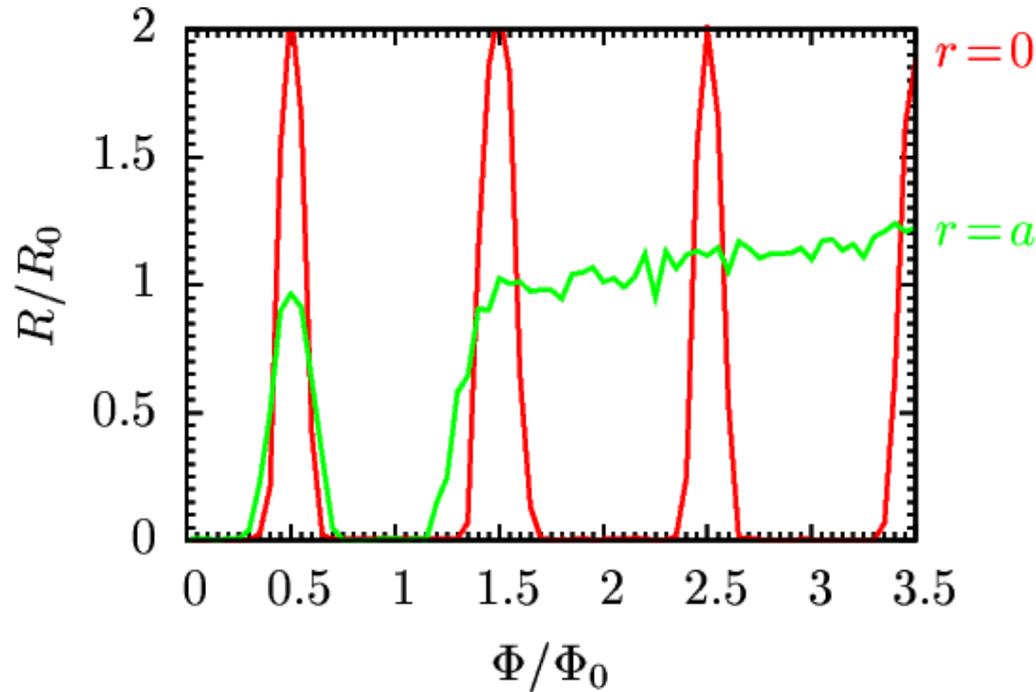


Experiment:

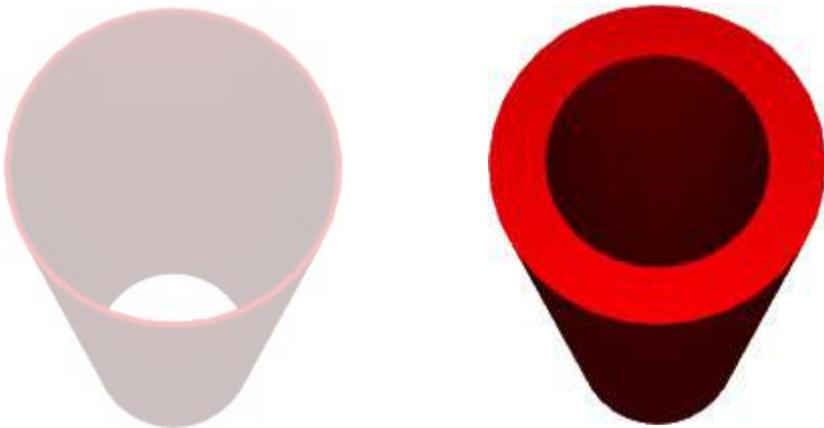
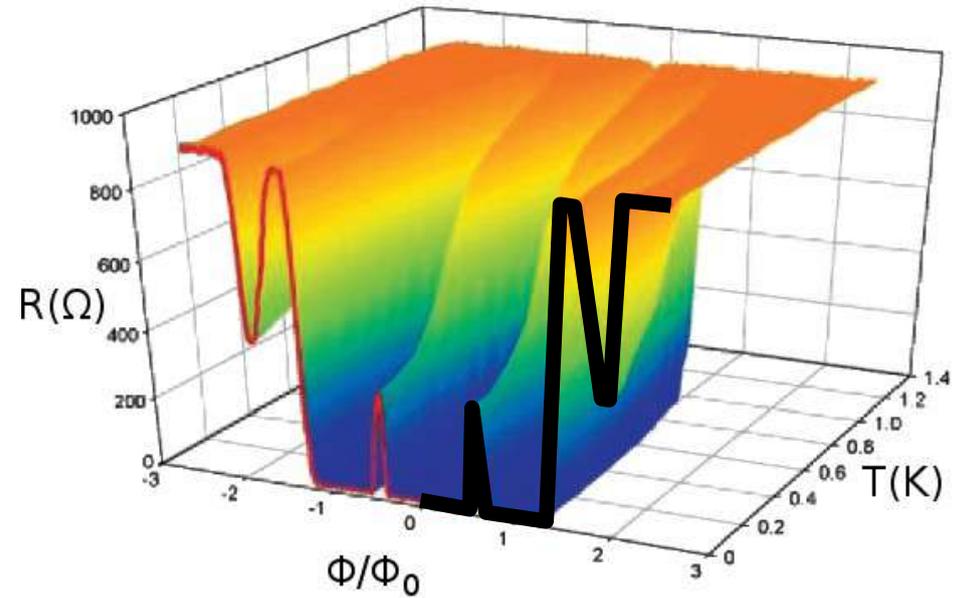


# Little-Parks in a small diameter cylinder

Theory:

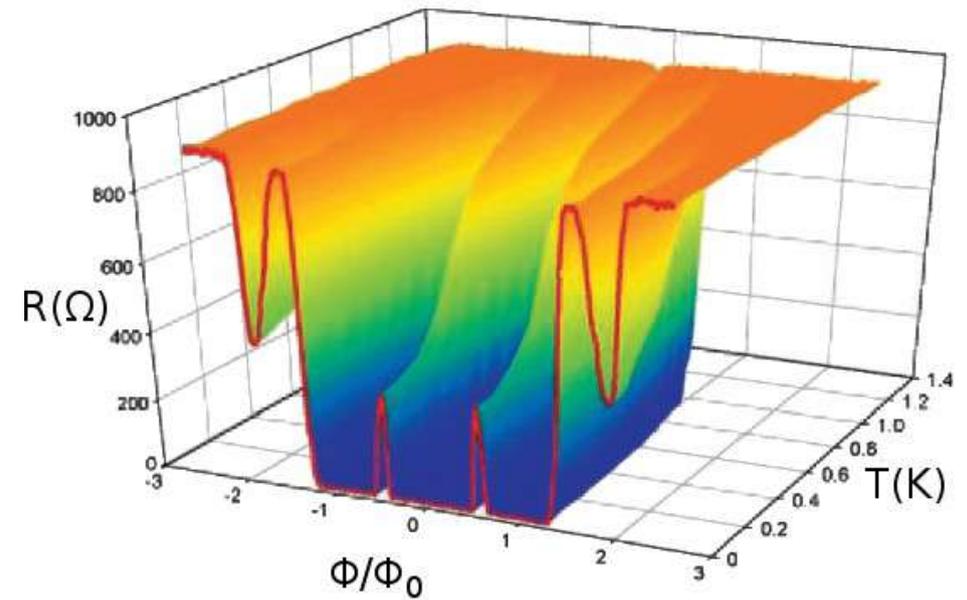


Experiment:

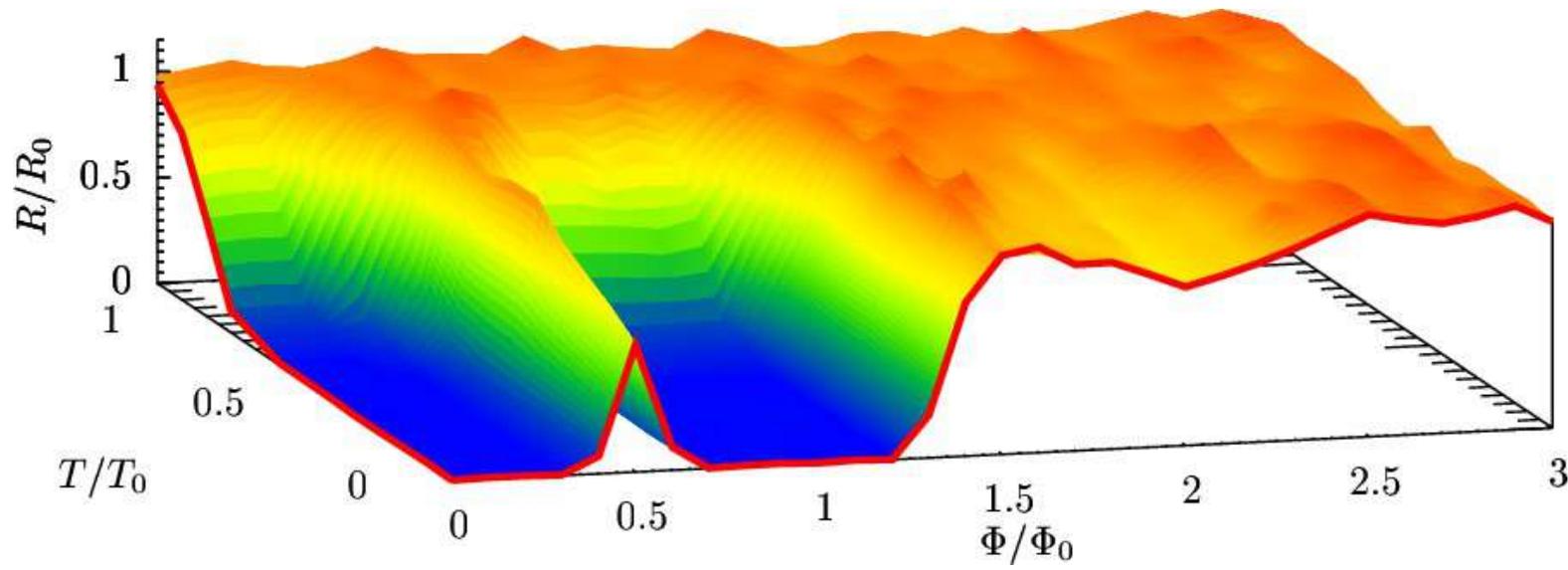


# Little-Parks in a small diameter cylinder

Experiment:

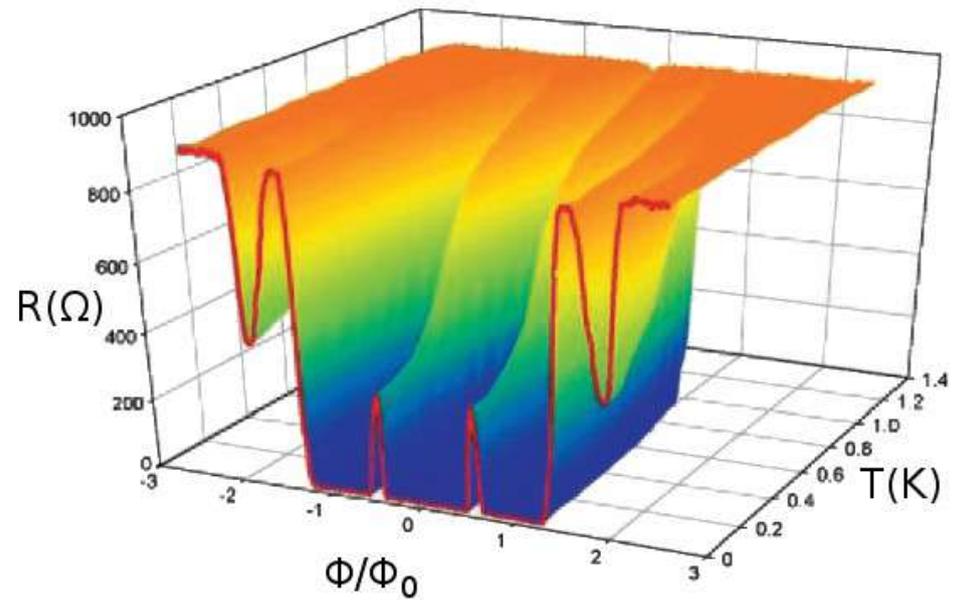


Theory:

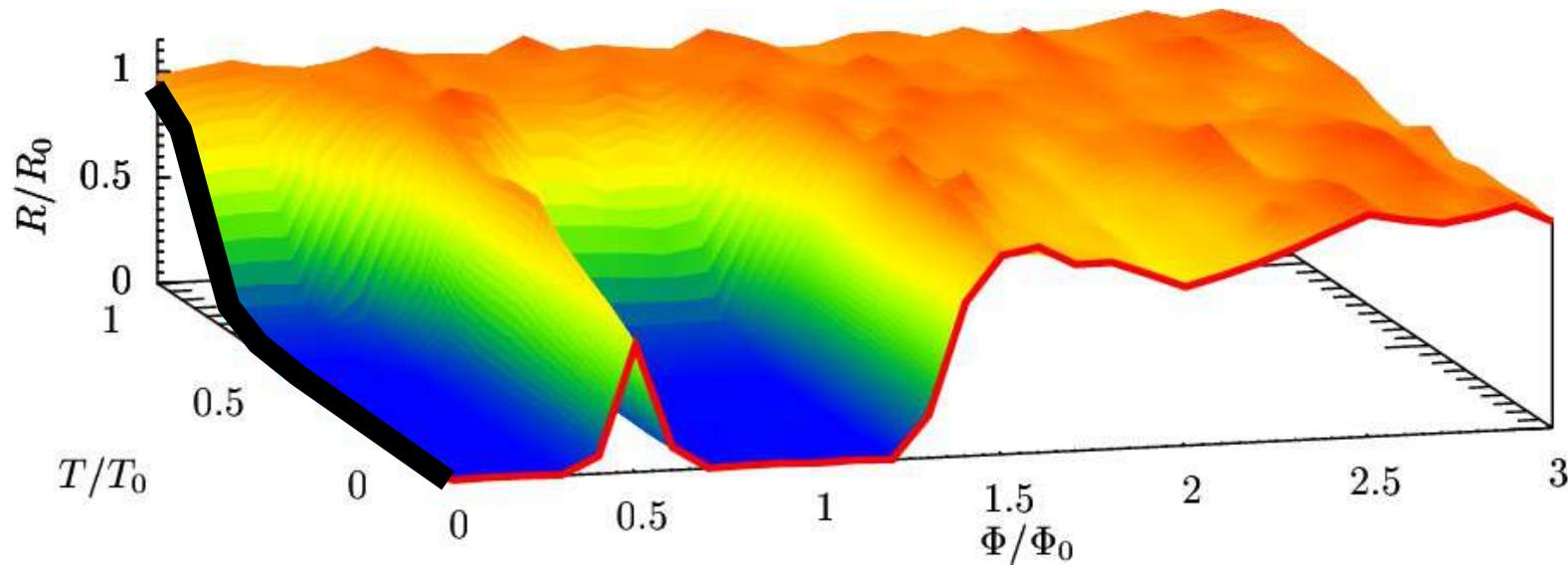


# Little-Parks in a small diameter cylinder

Experiment:

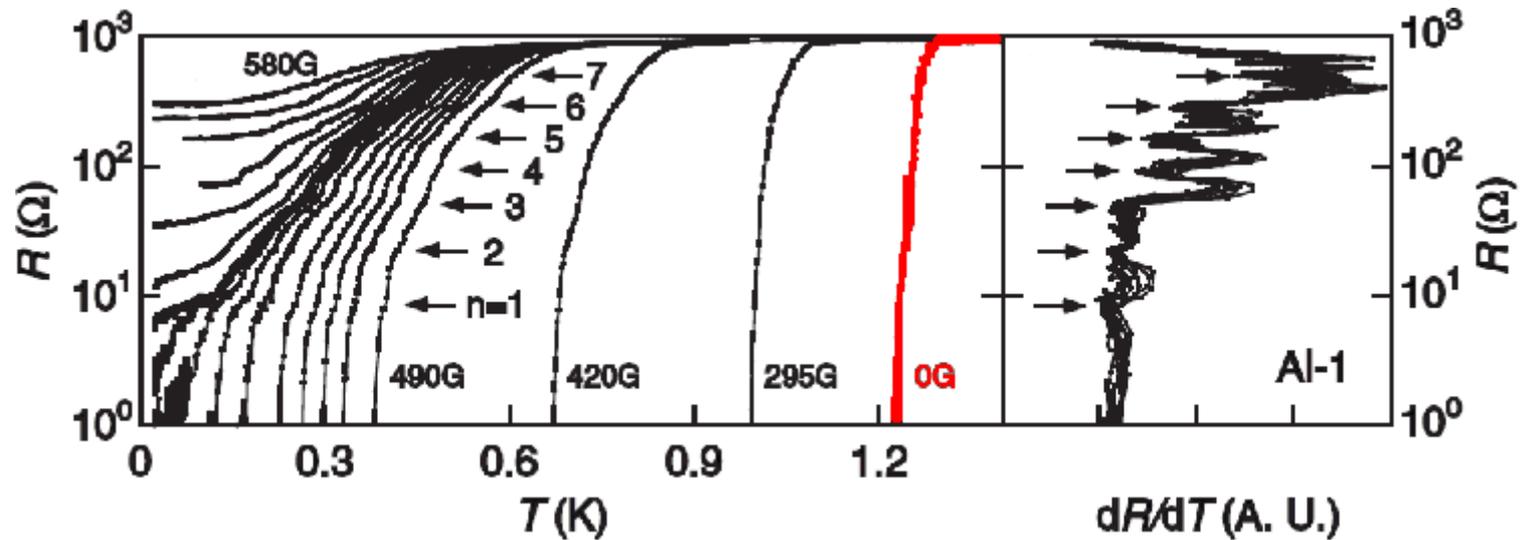


Theory:

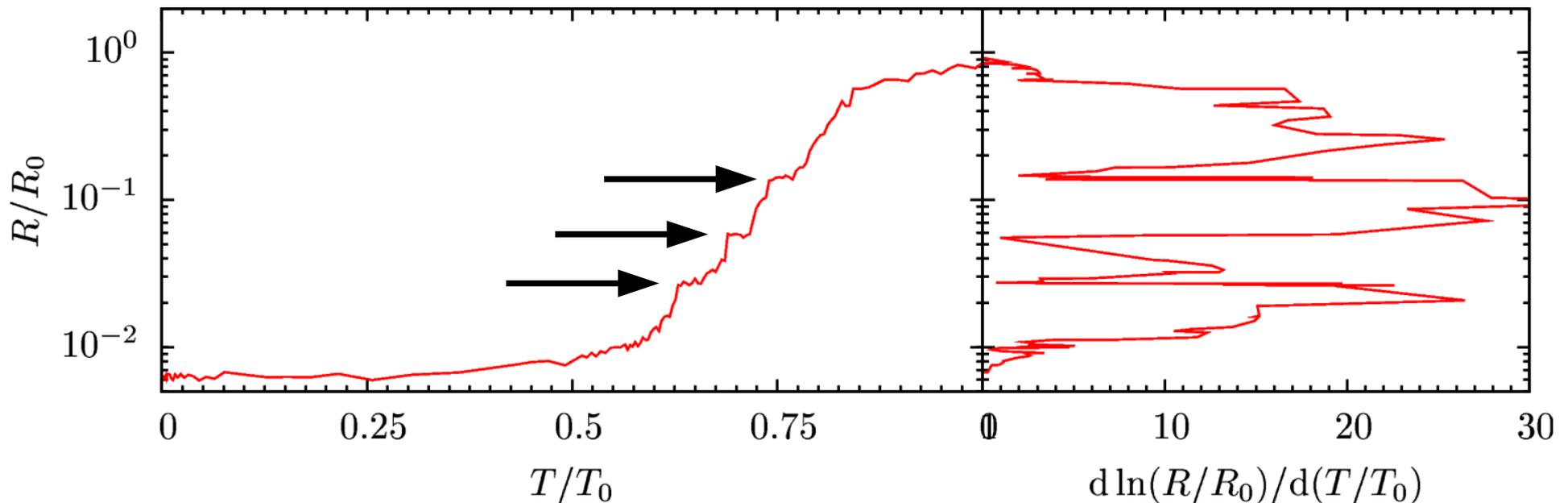


# Evidence of phase reconstruction

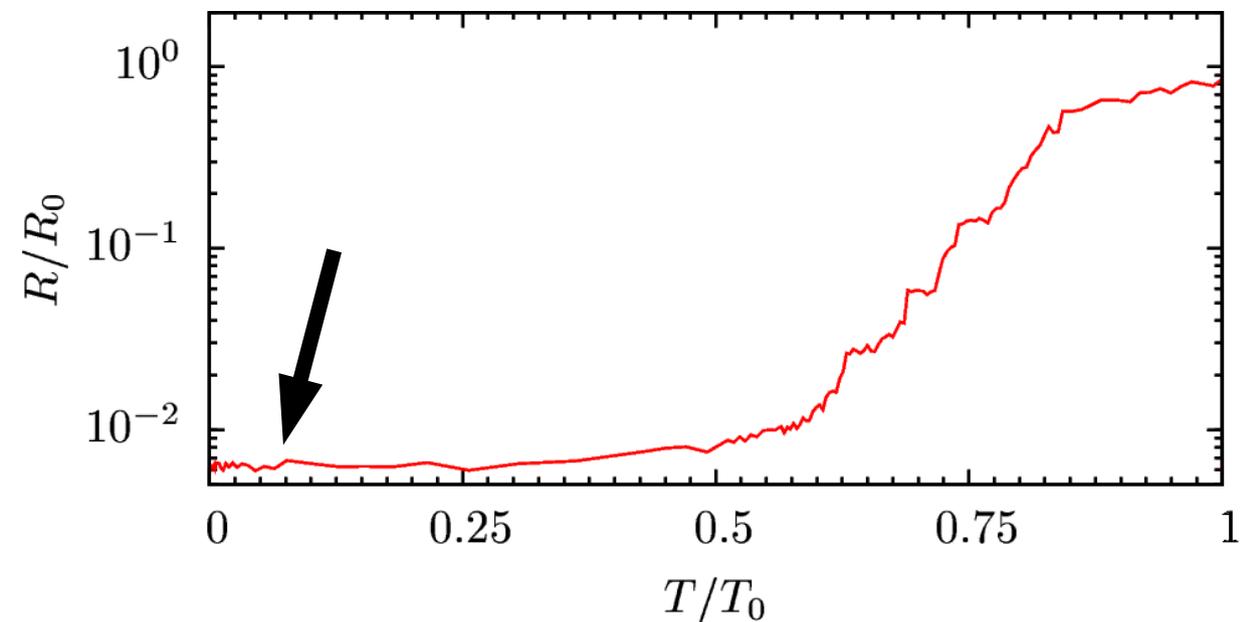
- Experiment:



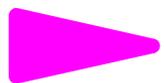
- Theory:



# Completely superconducting



Superconducting current

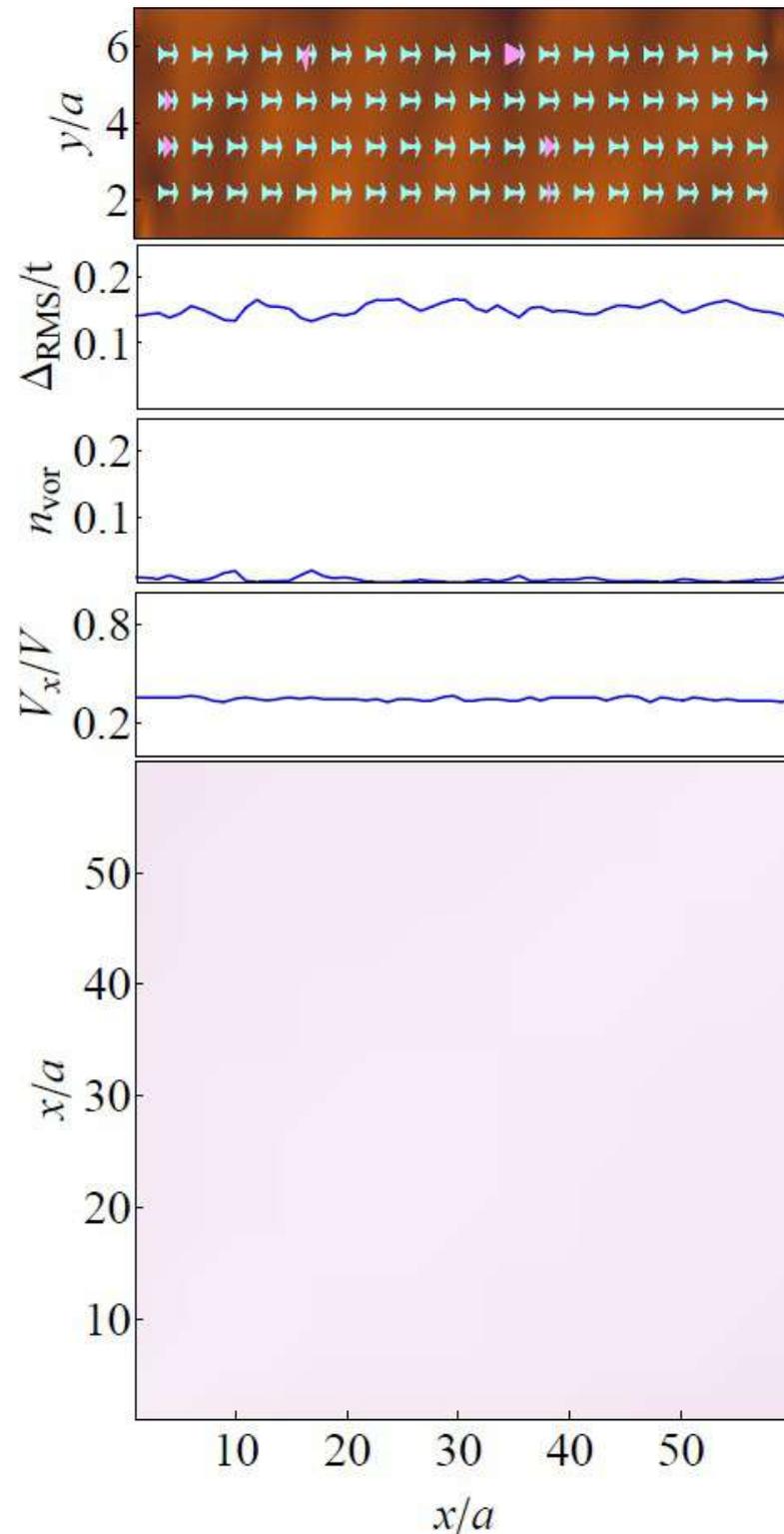


Normal current

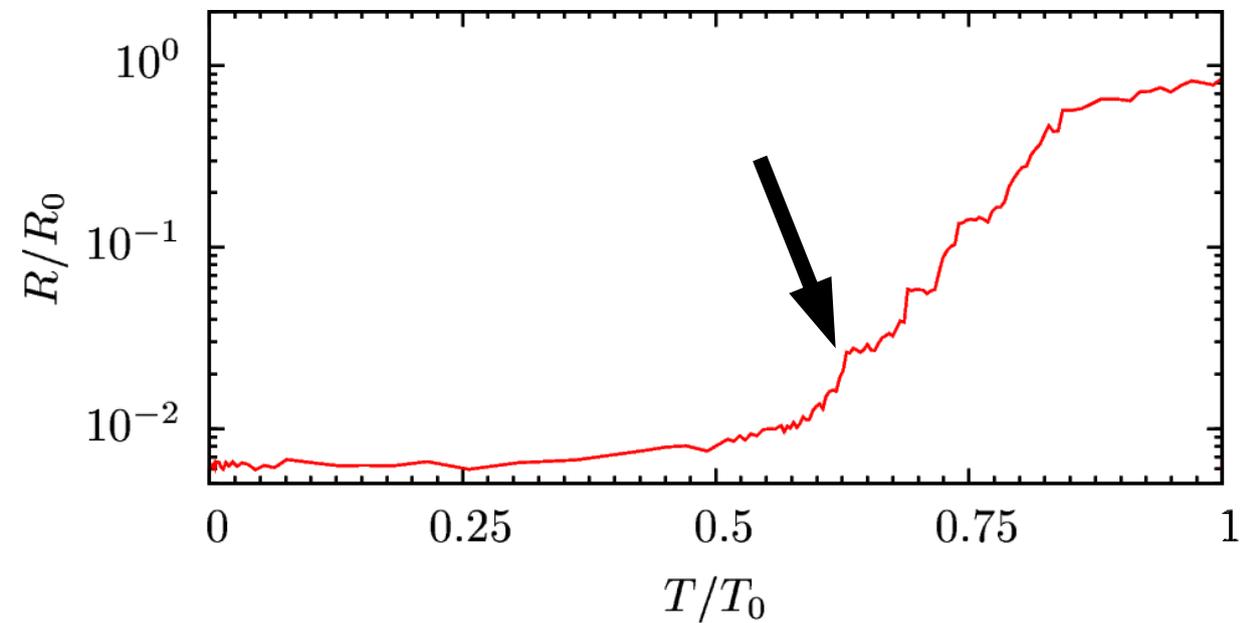
0

$\langle \cos(\theta_1 - \theta_2) \rangle$

1

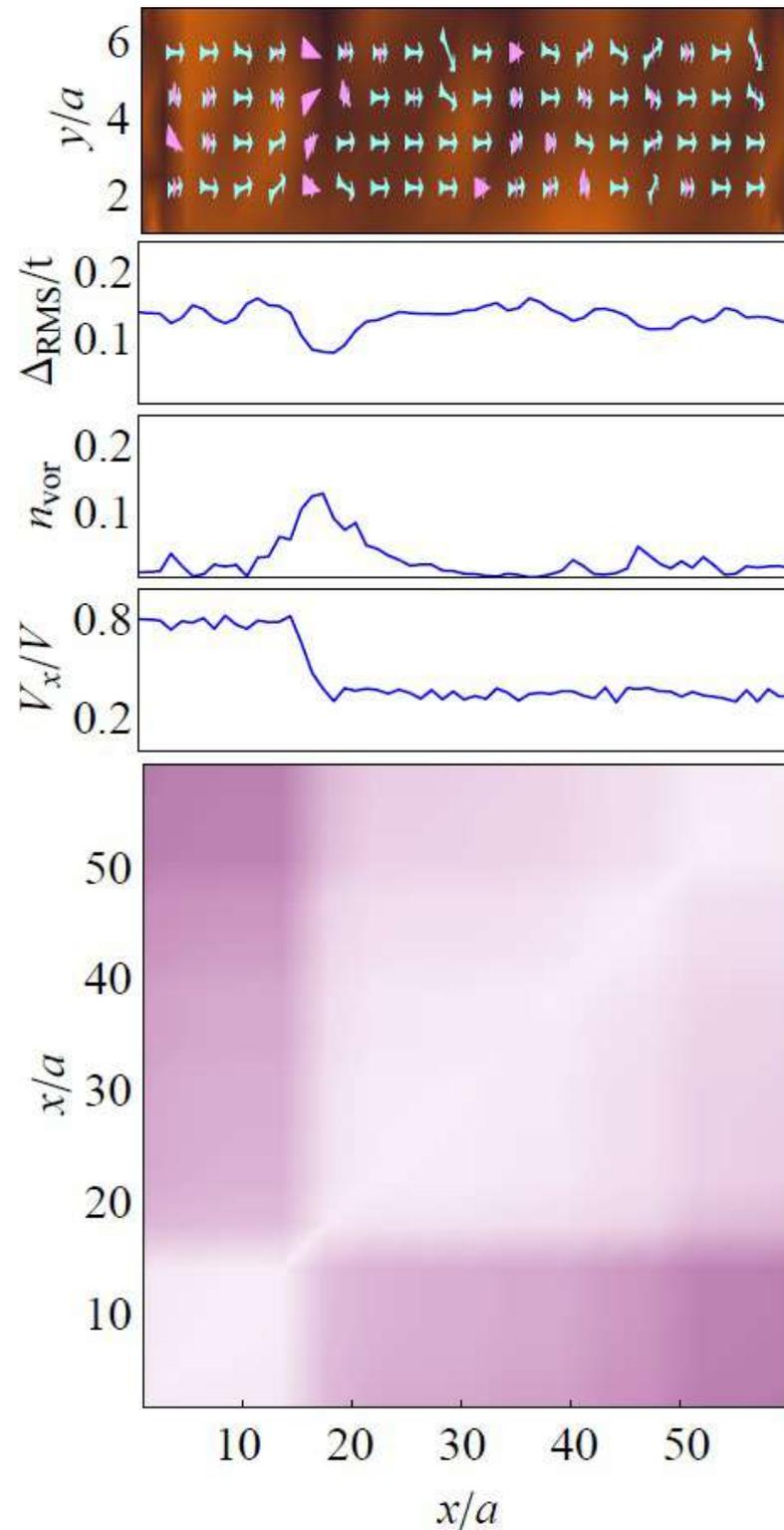


# Two superconducting regions

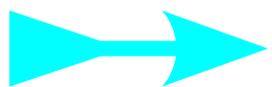
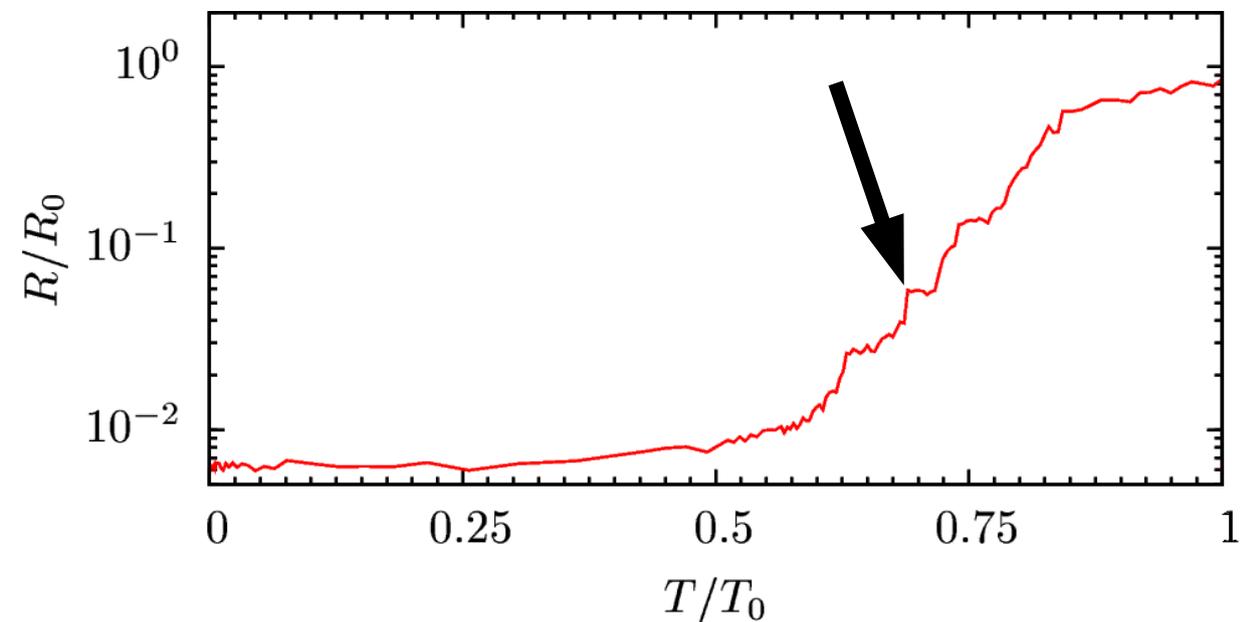


 Superconducting current  
 Normal current

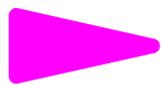
**0**  $\langle \cos(\theta_1 - \theta_2) \rangle$  **1**



# Three superconducting regions



Superconducting current

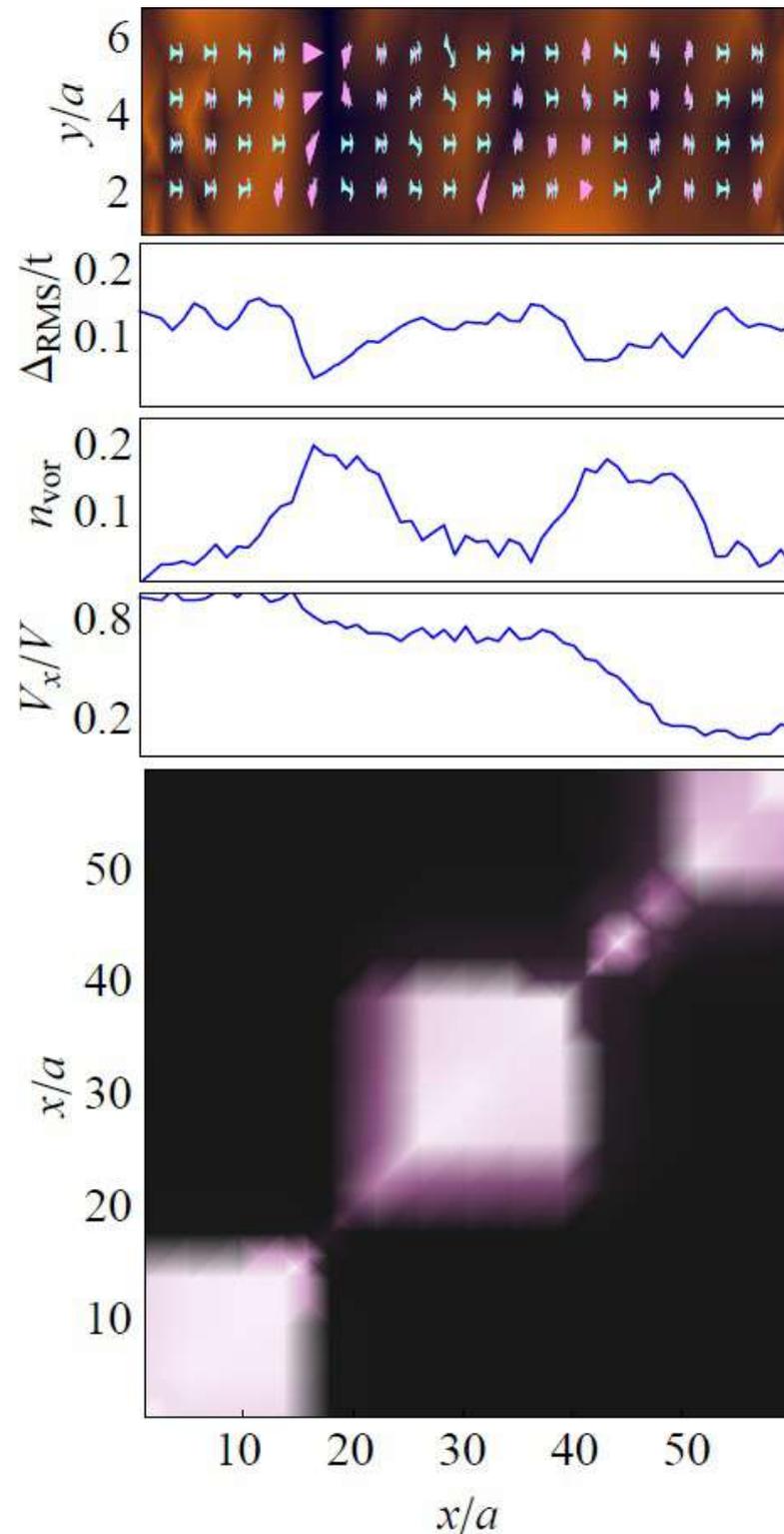


Normal current

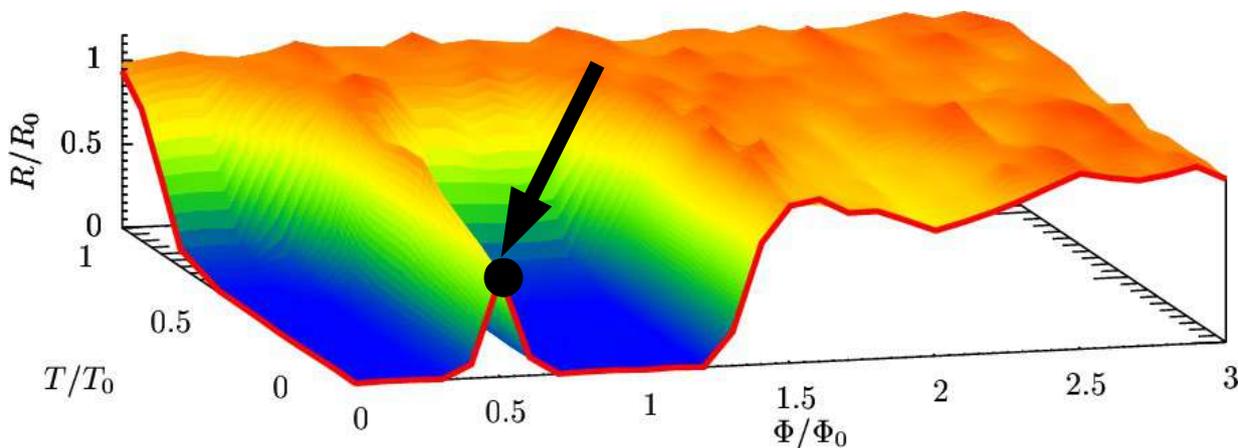
0

$\langle \cos(\theta_1 - \theta_2) \rangle$

1



# Half flux quantum normal state



Superconducting current

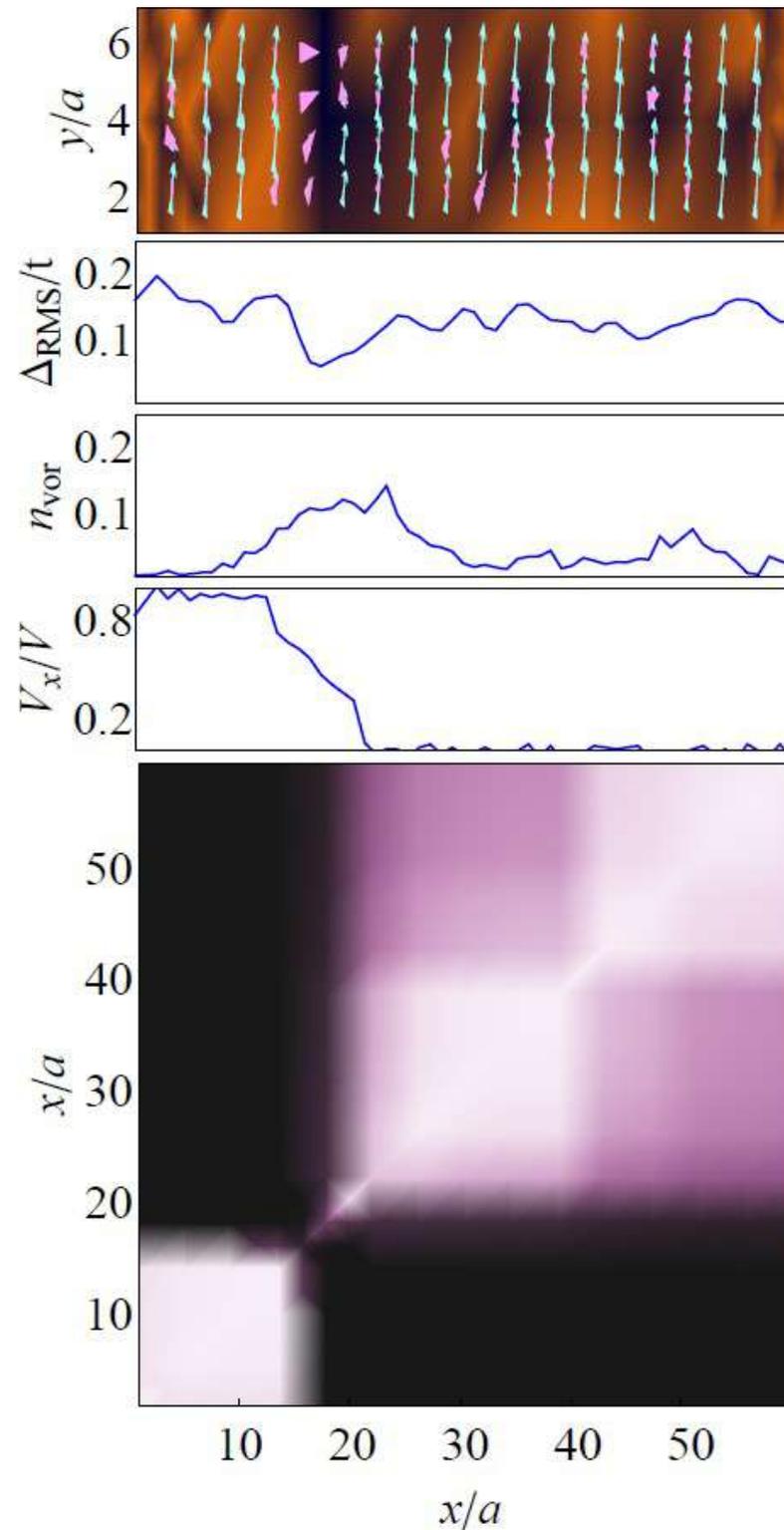


Normal current

0

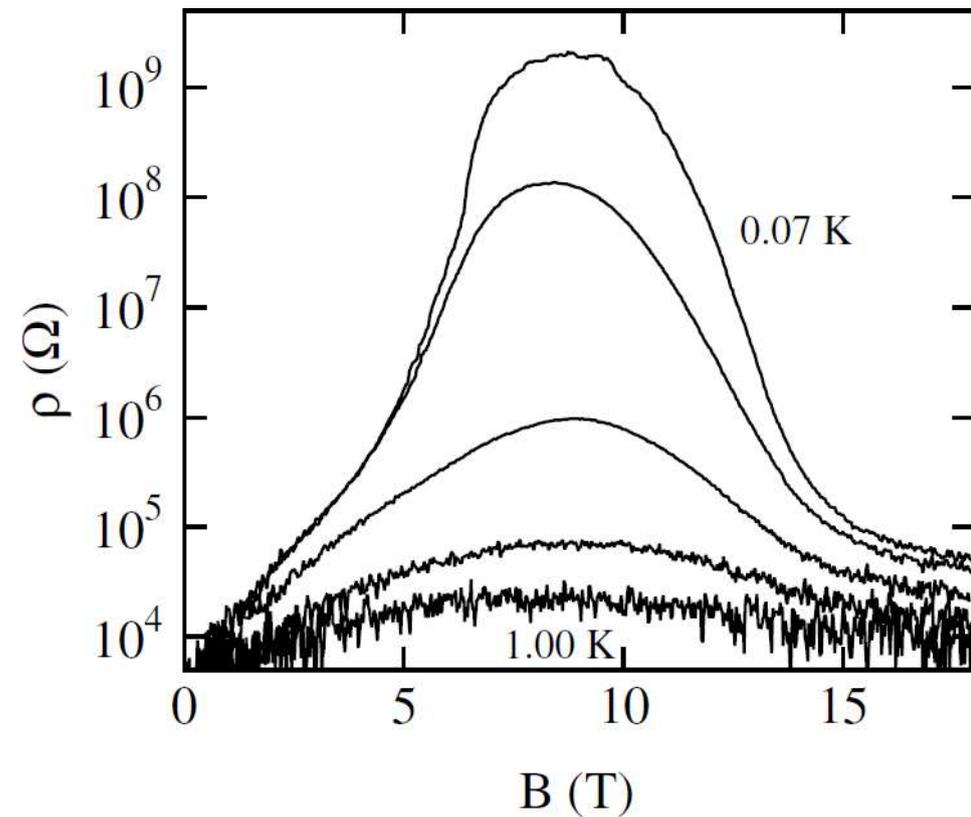
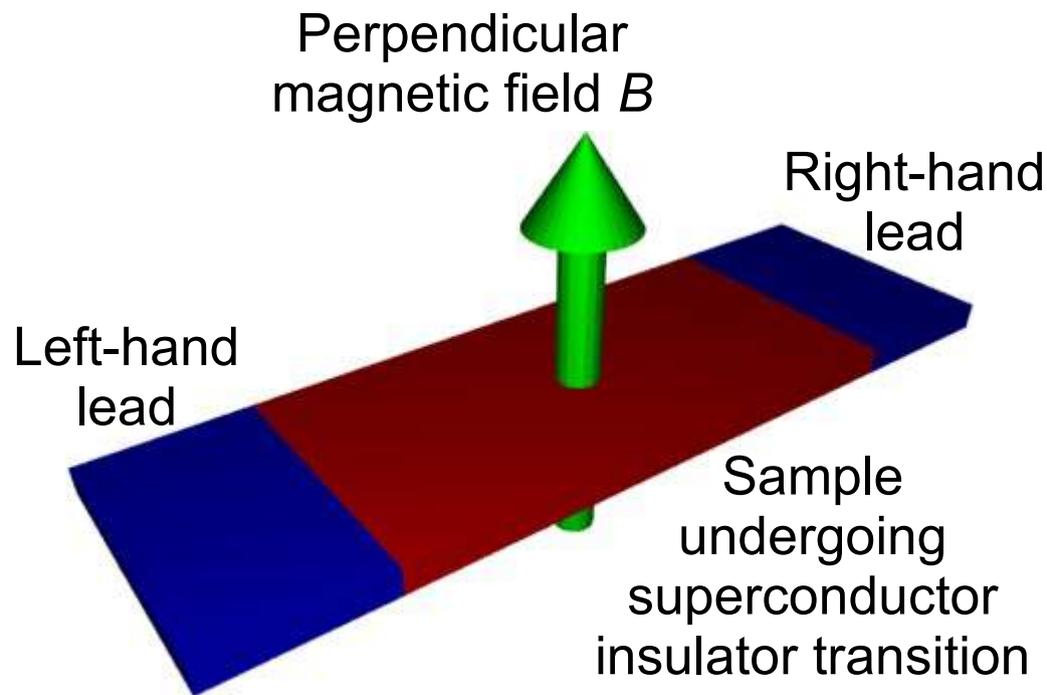
$\langle \cos(\theta_1 - \theta_2) \rangle$

1



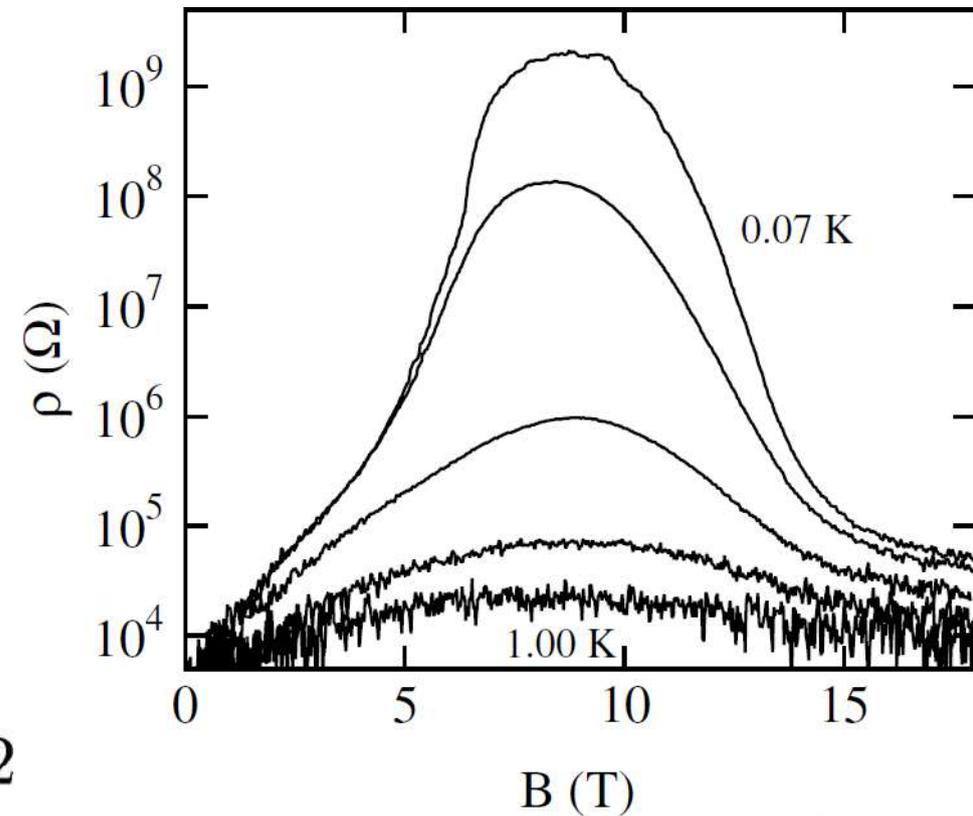
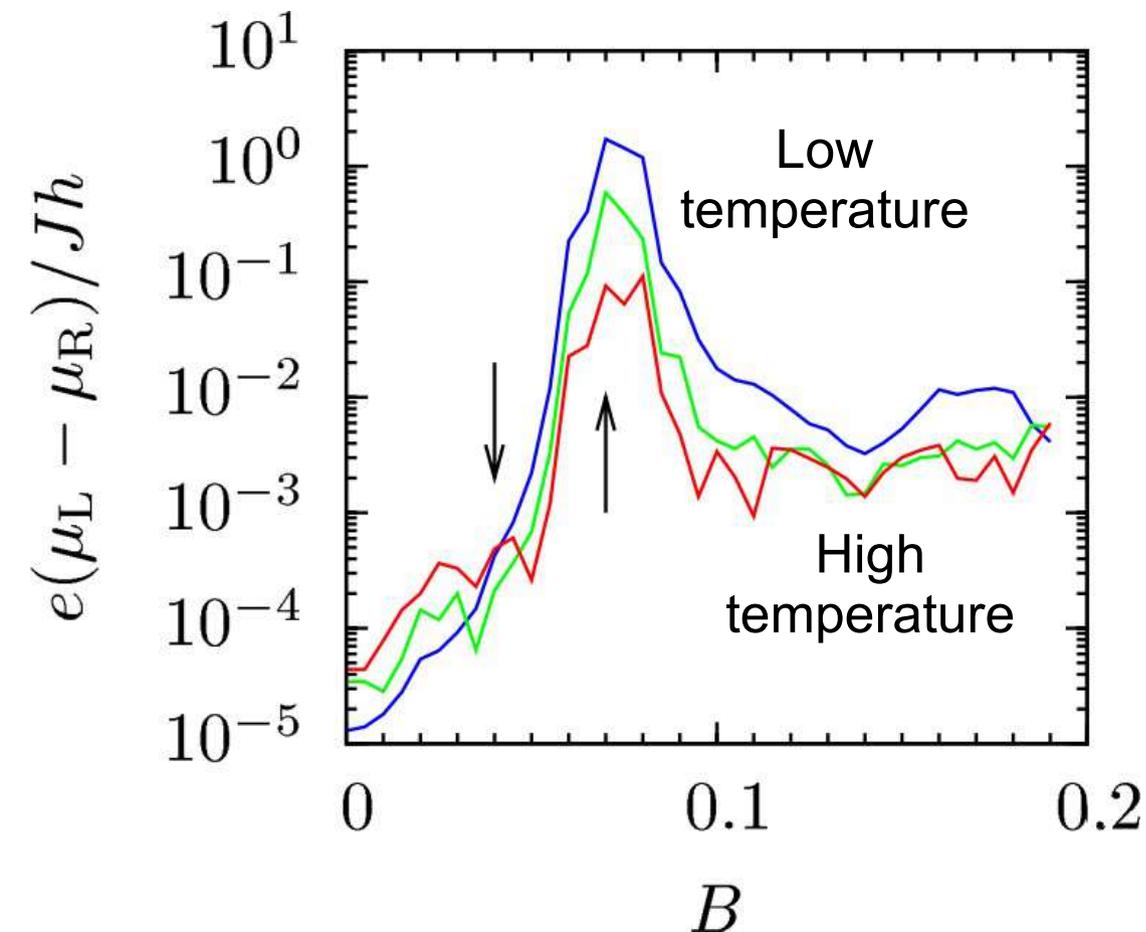
# Magnetoresistance peak

- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field



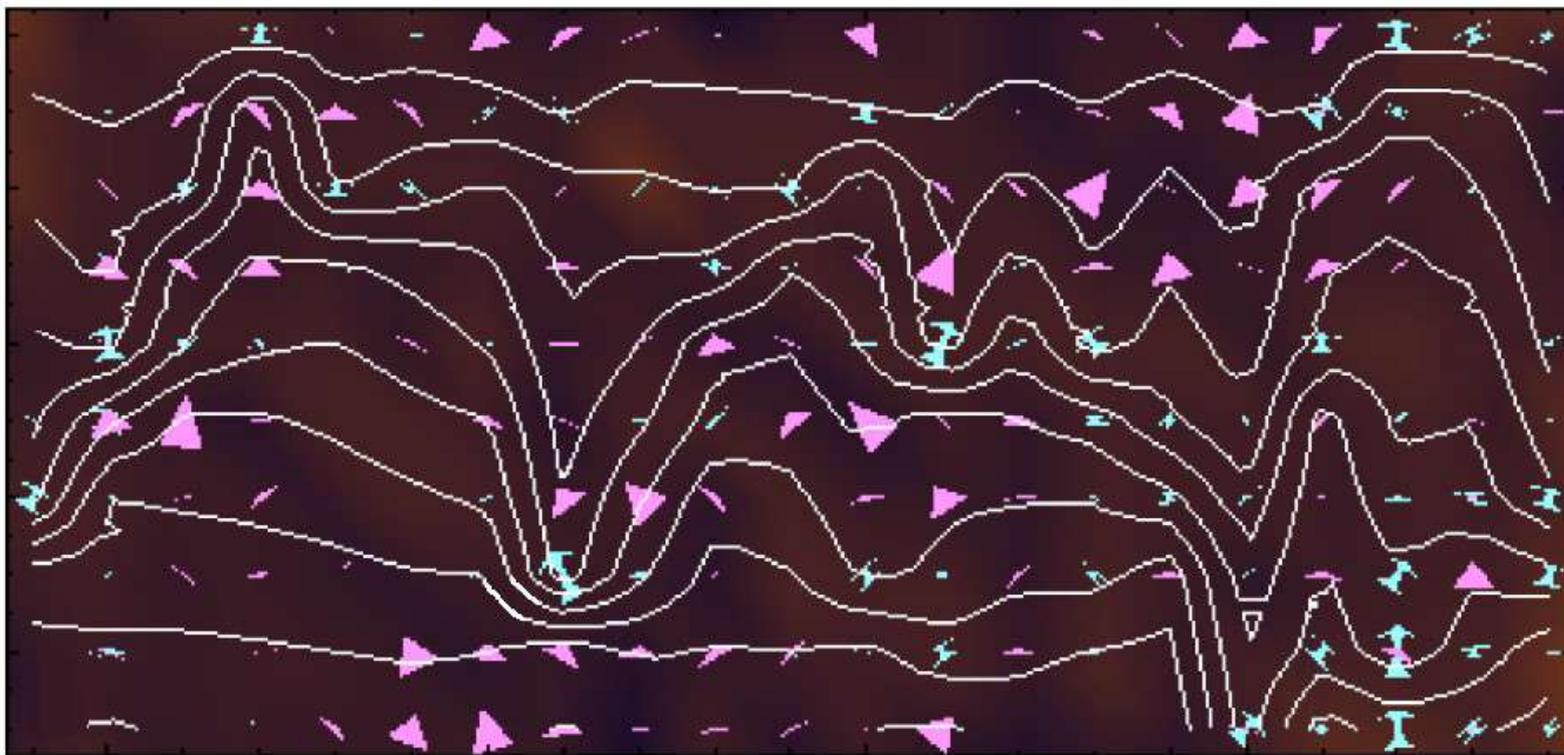
# Magnetoresistance peak

- Study superconductor-insulator transition in dirty sample with perpendicular magnetic field



# Clues: current maps

- Weak links across superconducting puddles

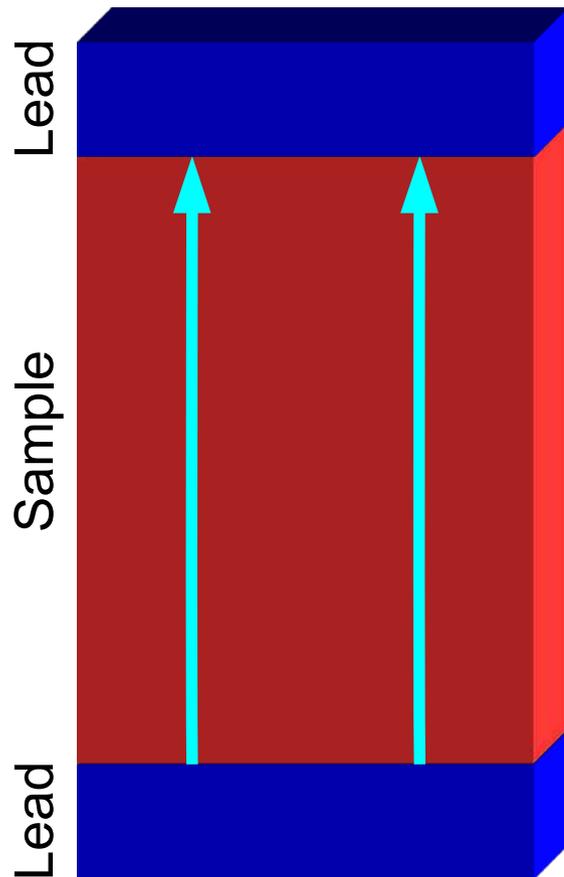


Superconducting current

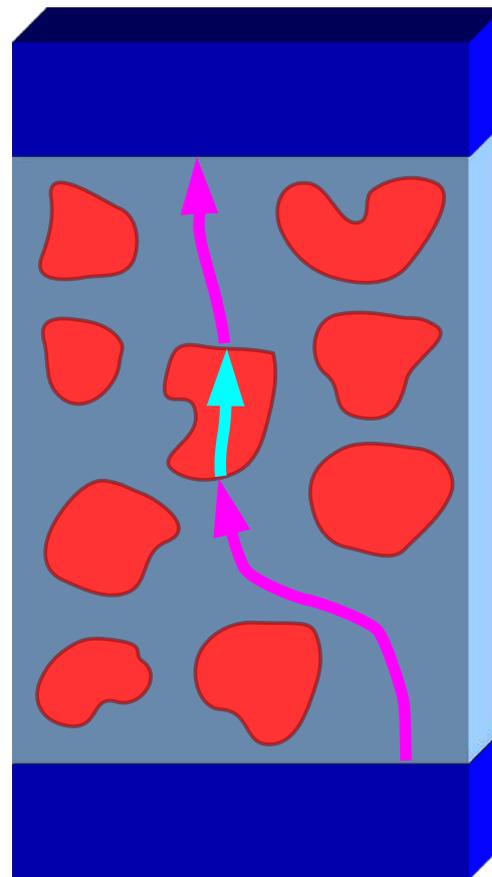


Normal current

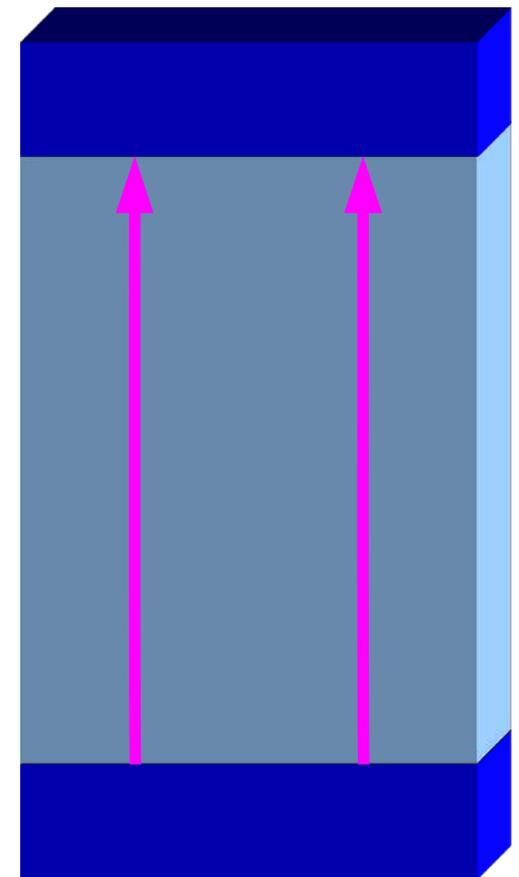
# Working hypothesis



Sample entirely superconducting



Superconducting puddles have a charging energy and a tunneling barrier

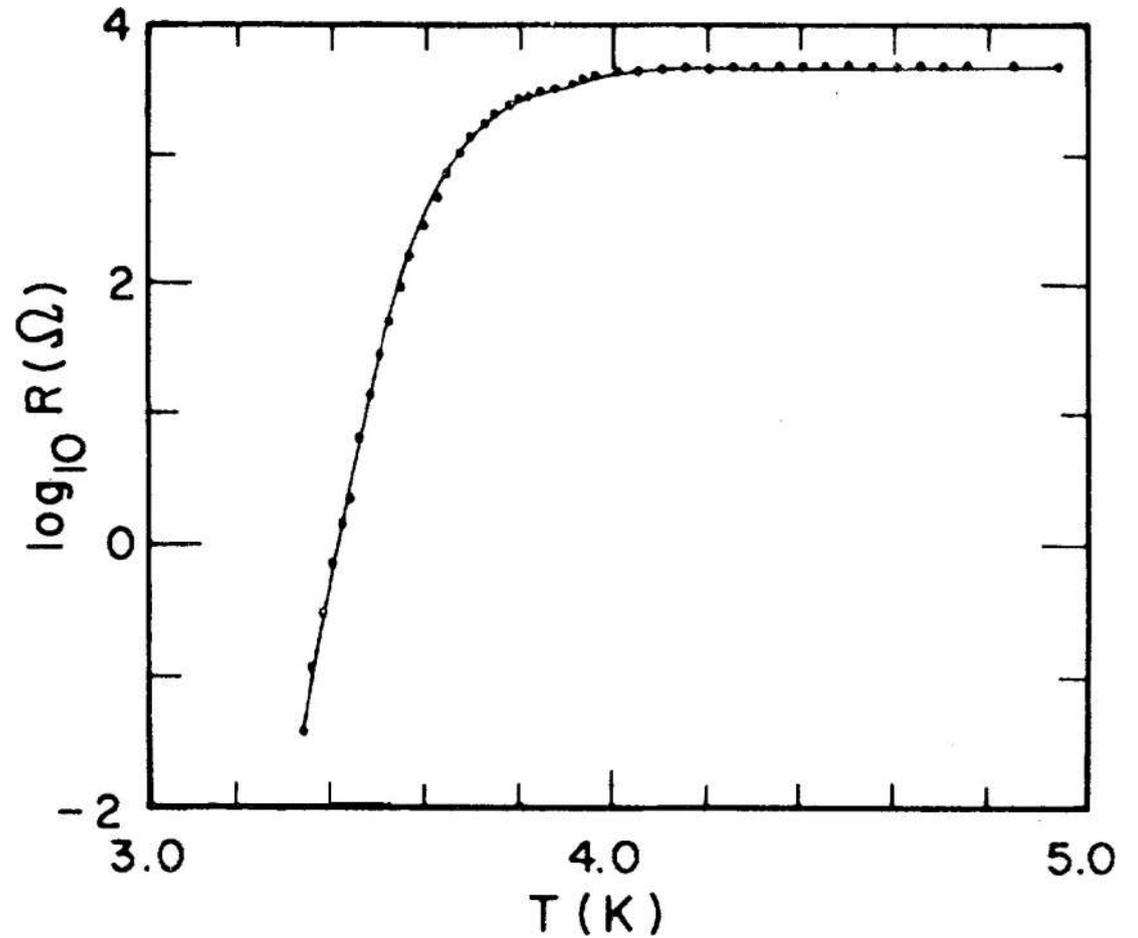
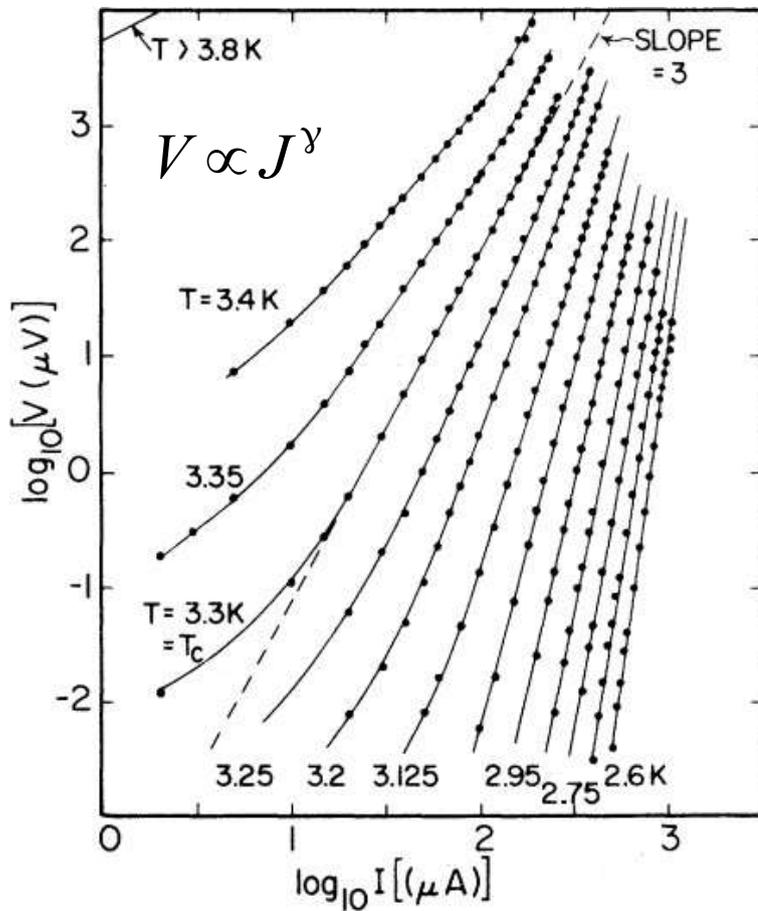


Sample entirely normal

# Summary & future prospects

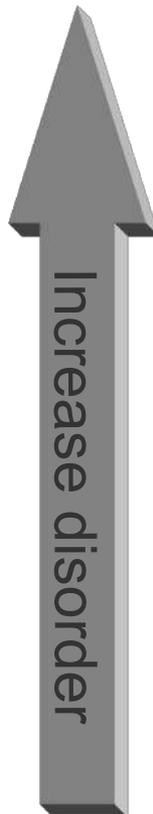
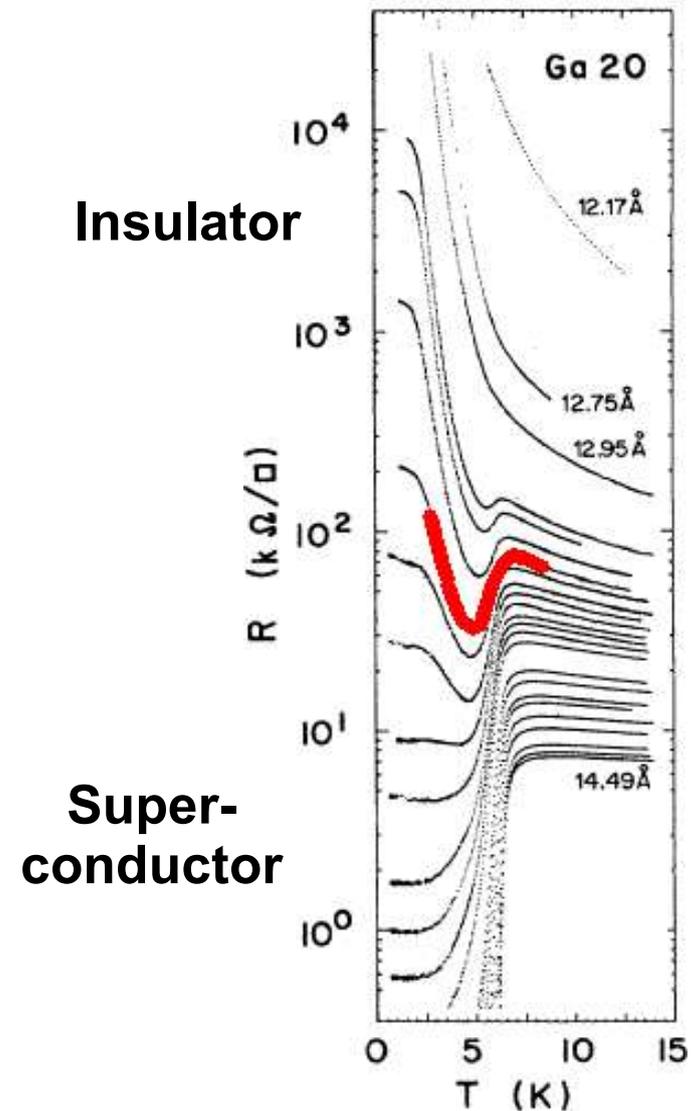
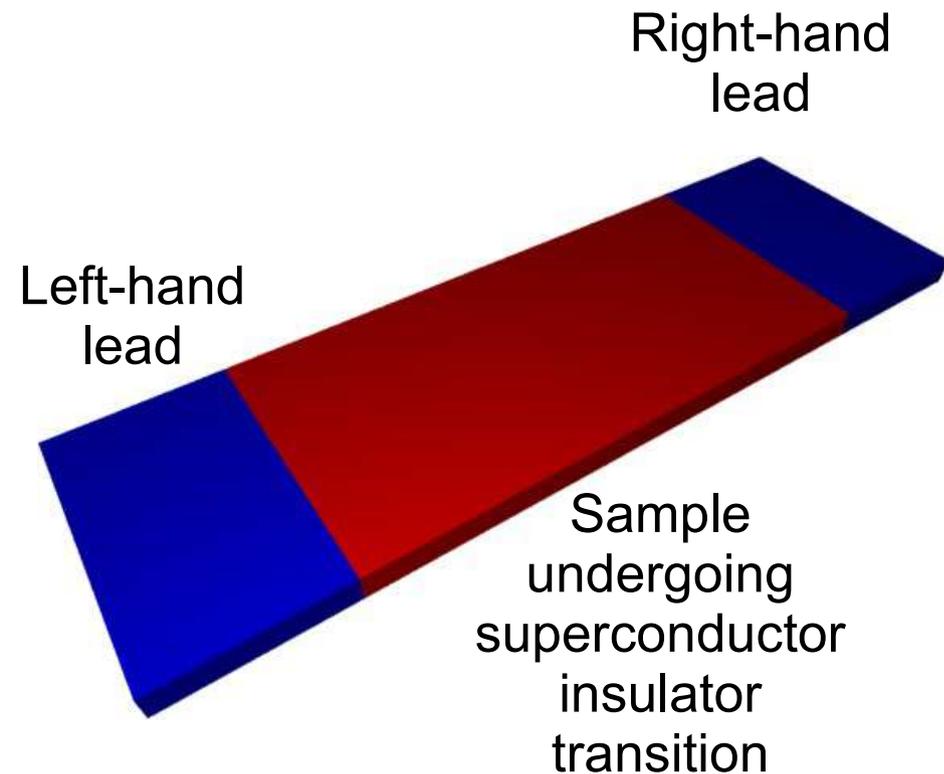
- Developed new formalism that includes thermal phase fluctuations to calculate and probe transport in superconductors
- New numerical techniques permit access to large systems
- Tested formalism against a series of well established results
- Shown that superconductor-insulator transition in small diameter cylinders is driven by phase fluctuations
- Shown that magnetoresistance peak could be driven by condensation of superconducting puddles
- Flexibility allows us to study wide range of unexplained effects

# KT transition conductivity



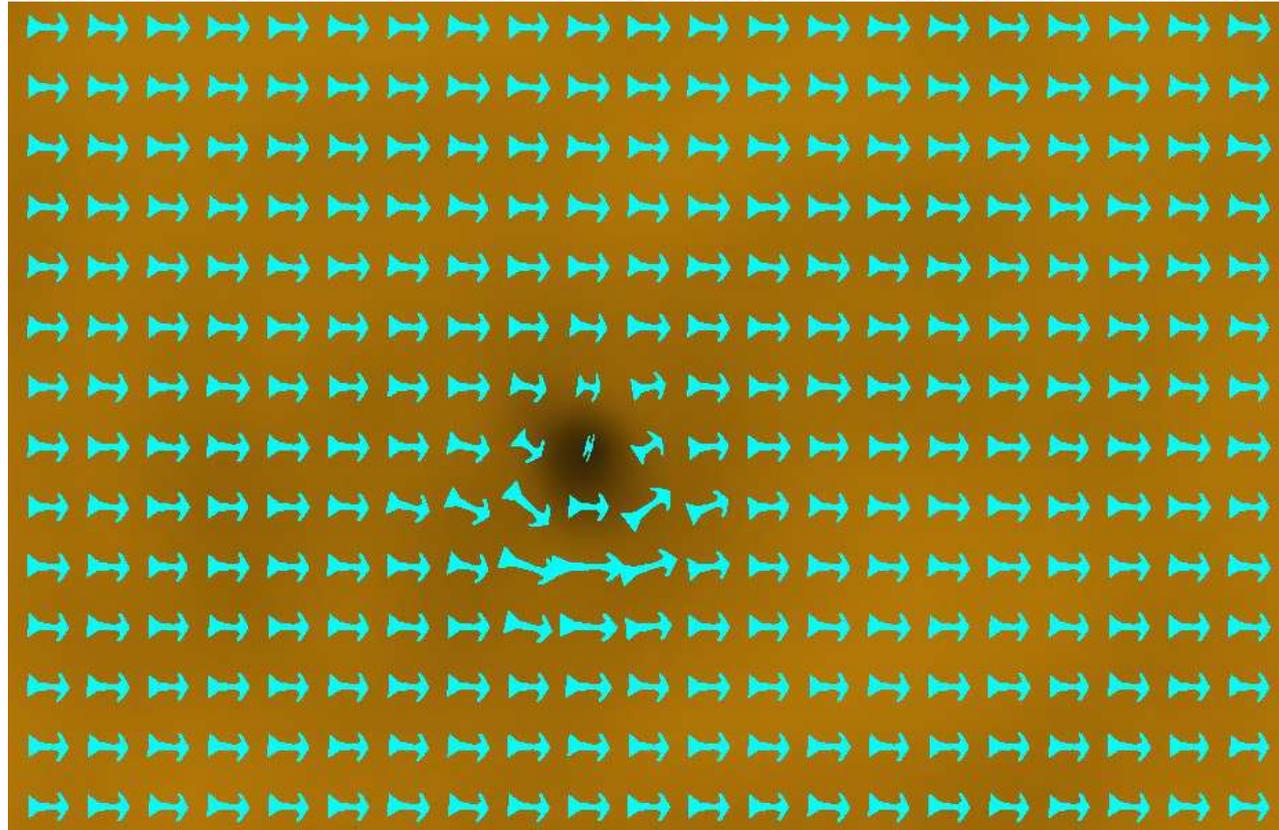
# Superconductor-insulator transition

- Non-monotonic R-T curves [Jaeger 89, Baturina 07]



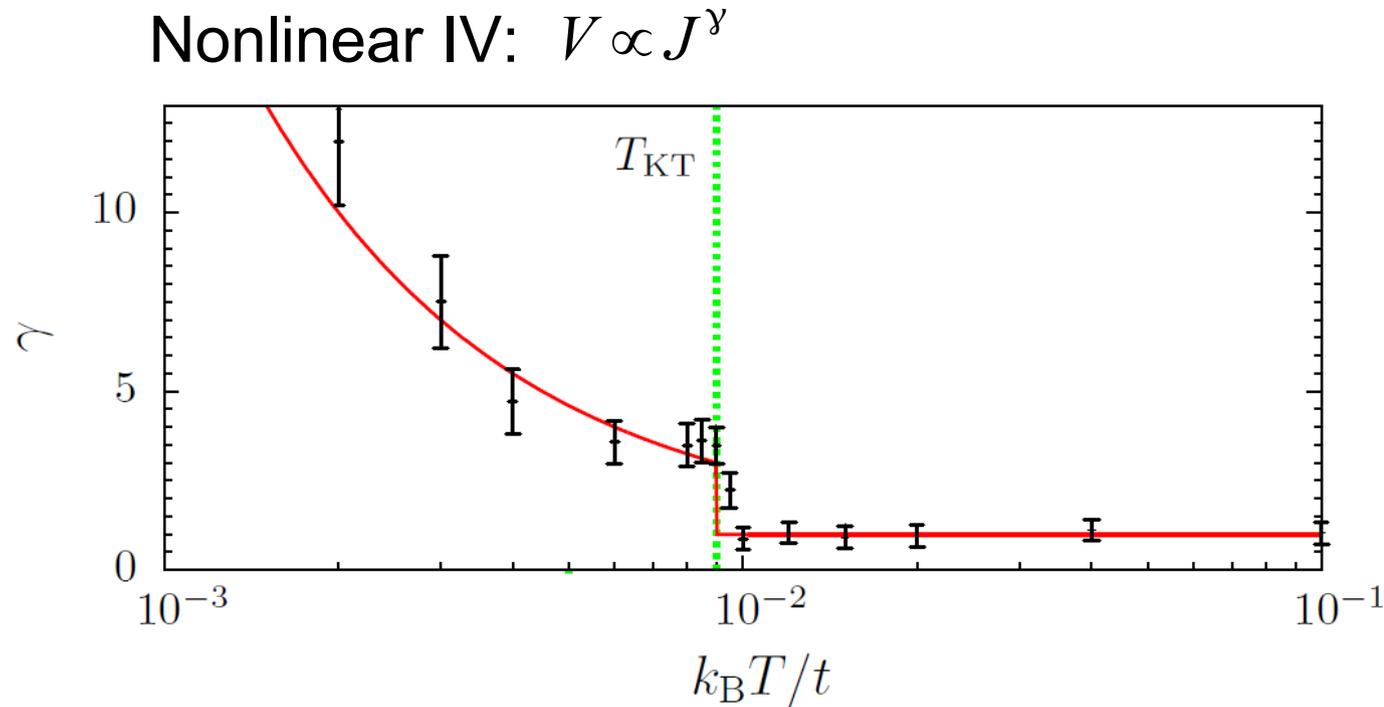
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



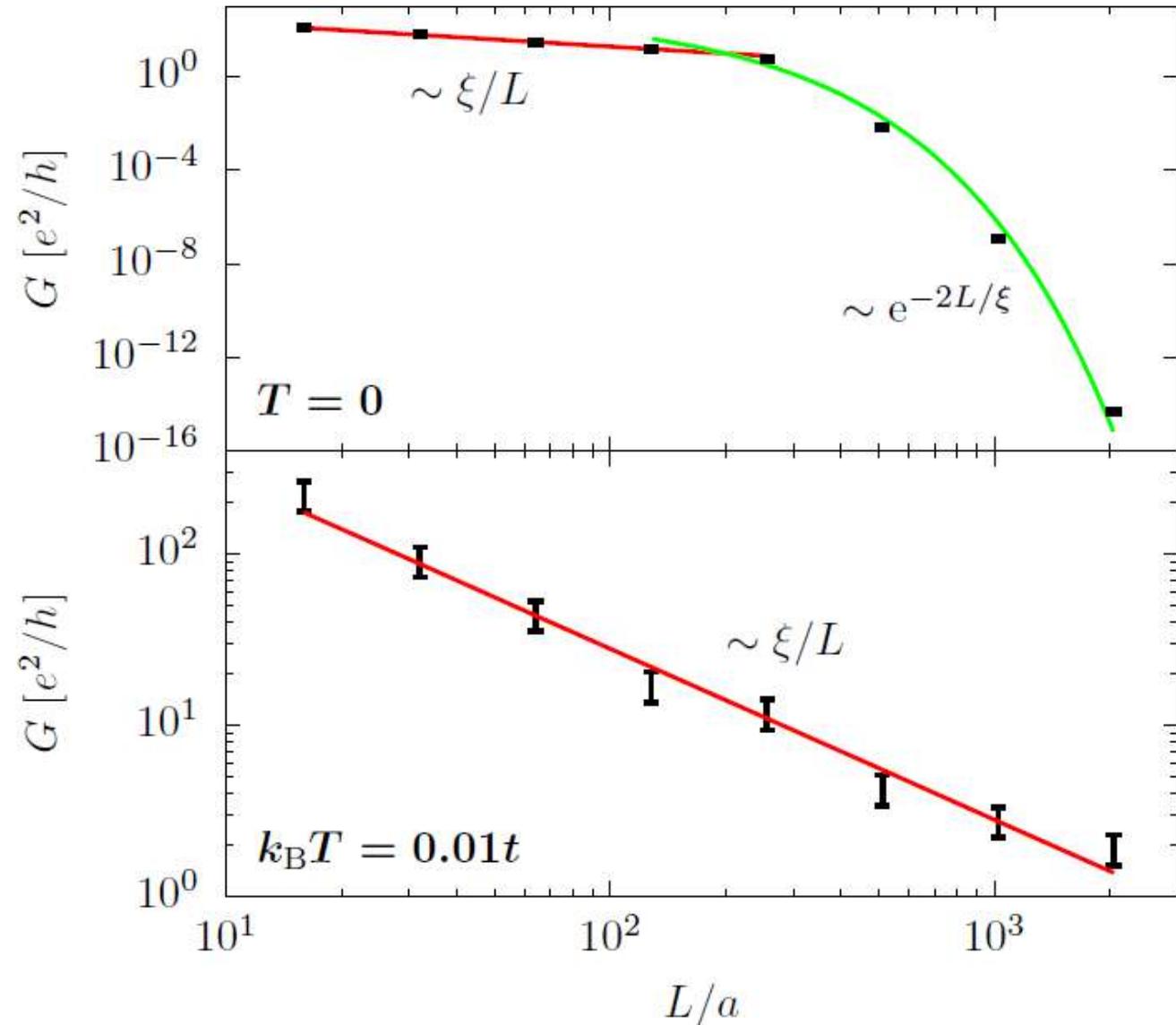
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



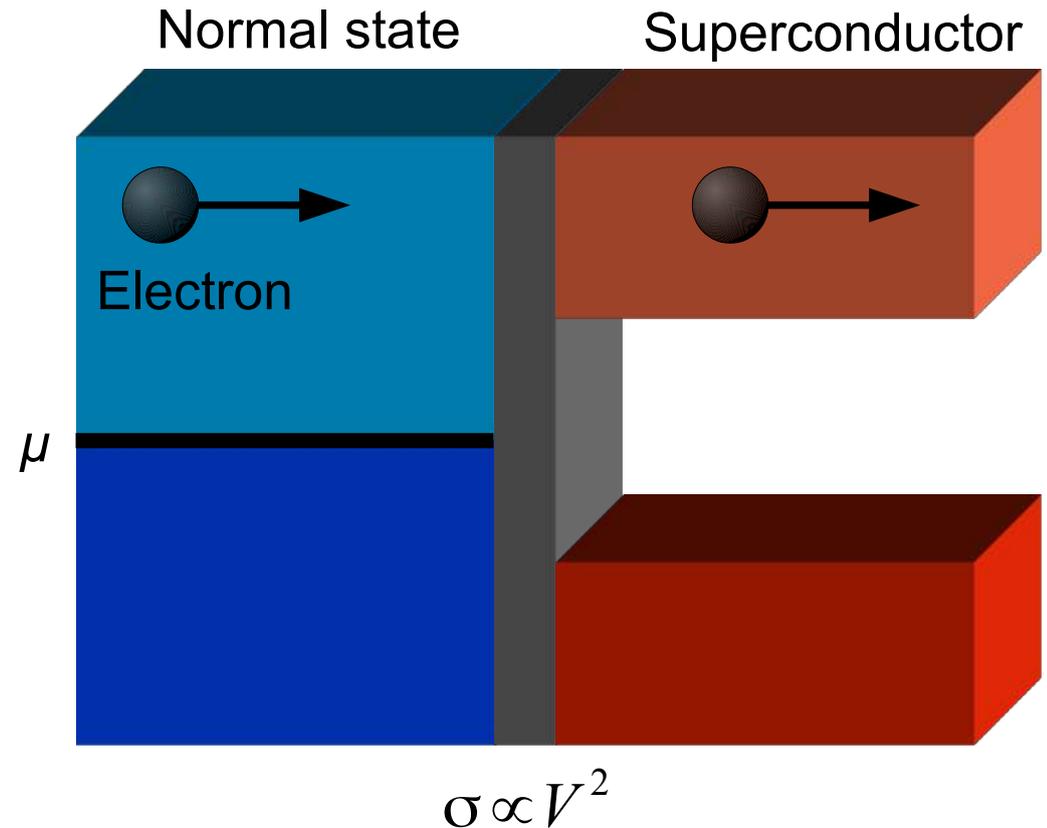
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



# Verification

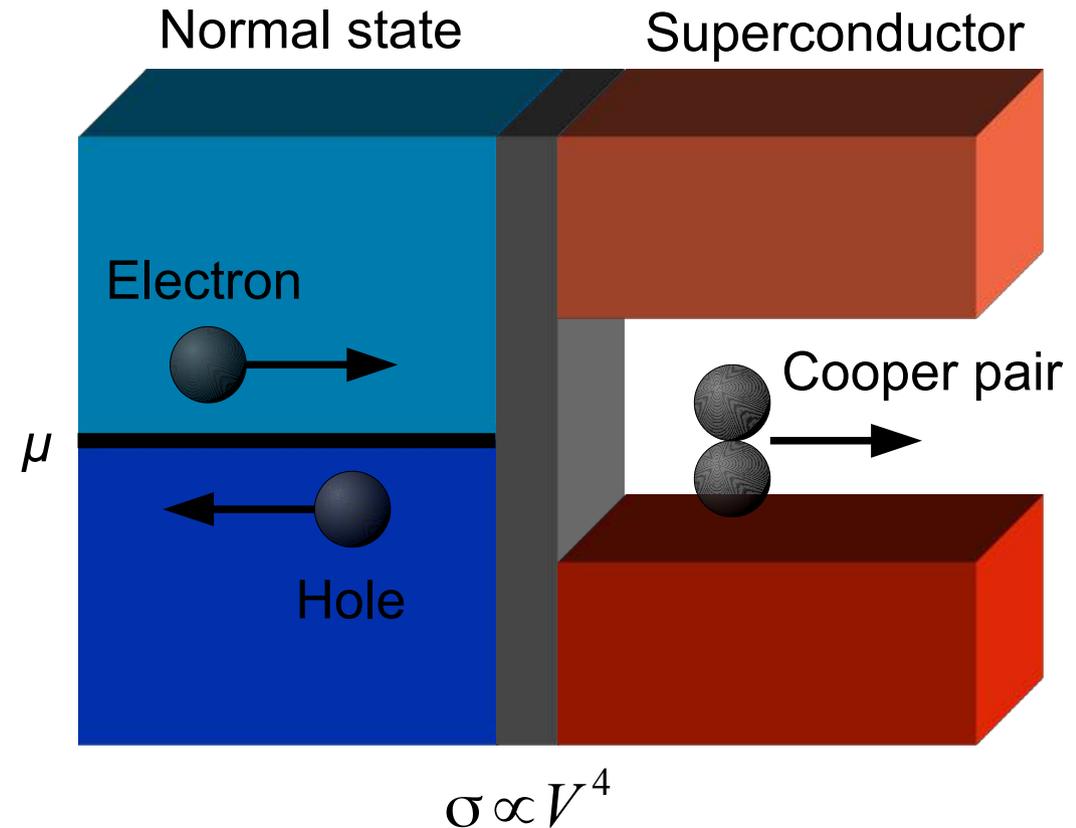
- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- **Andreev reflection**
- Josephson junction
- Little-Parks effect in large diameter cylinder



Andreev, JETP **19**, 1228 (1964)

# Verification

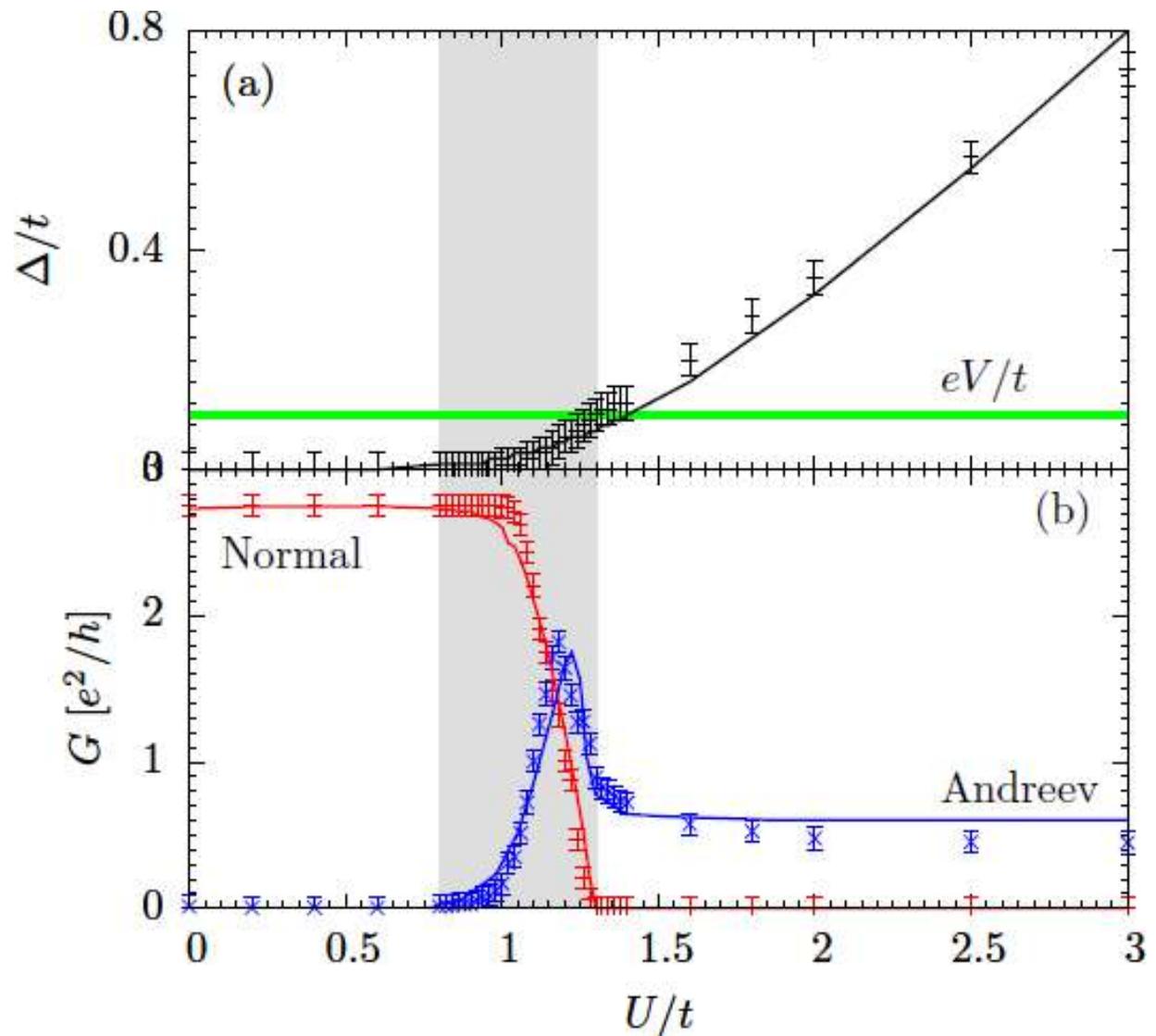
- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- **Andreev reflection**
- Josephson junction
- Little-Parks effect in large diameter cylinder



Andreev, JETP **19**, 1228 (1964)

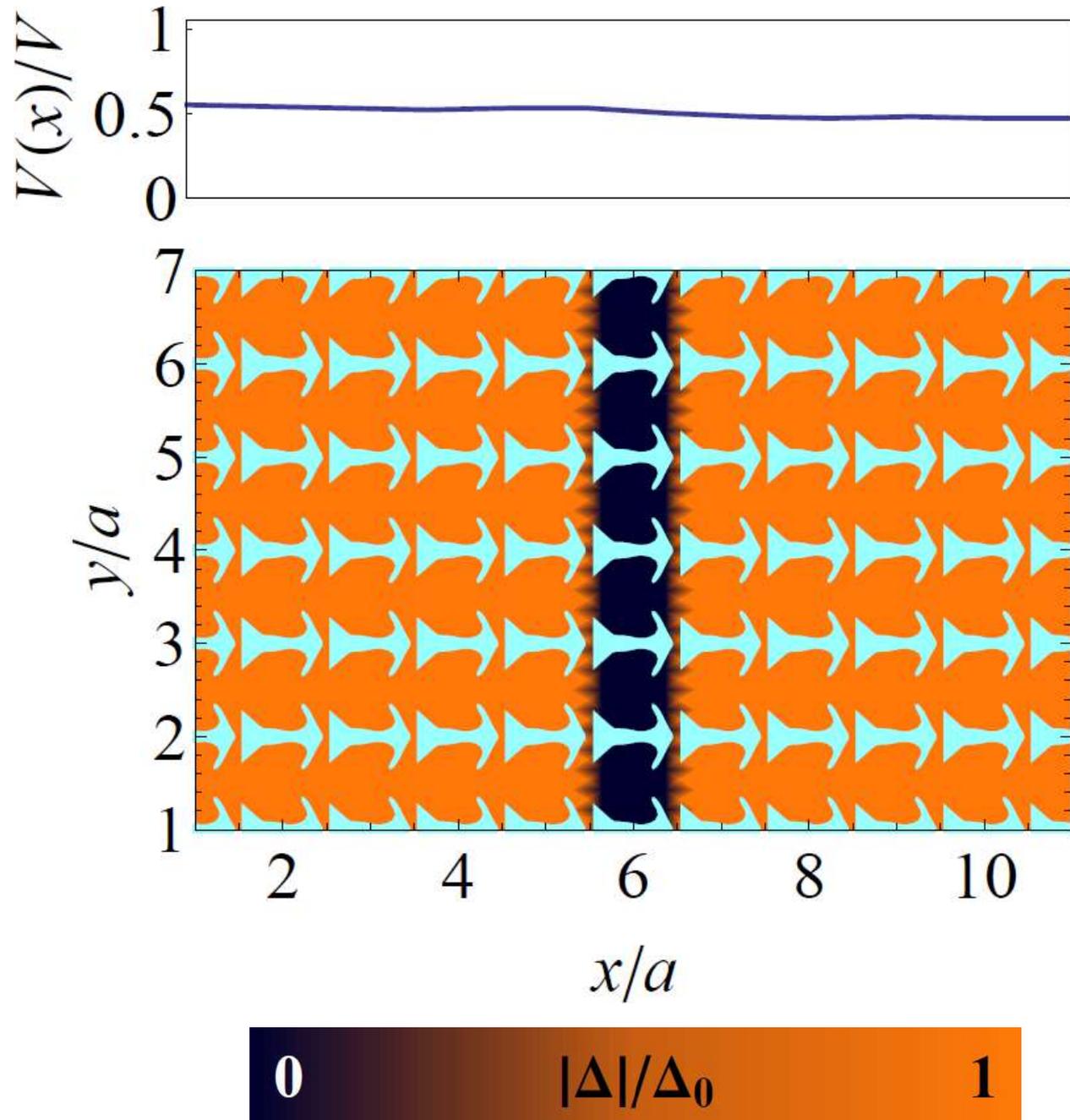
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



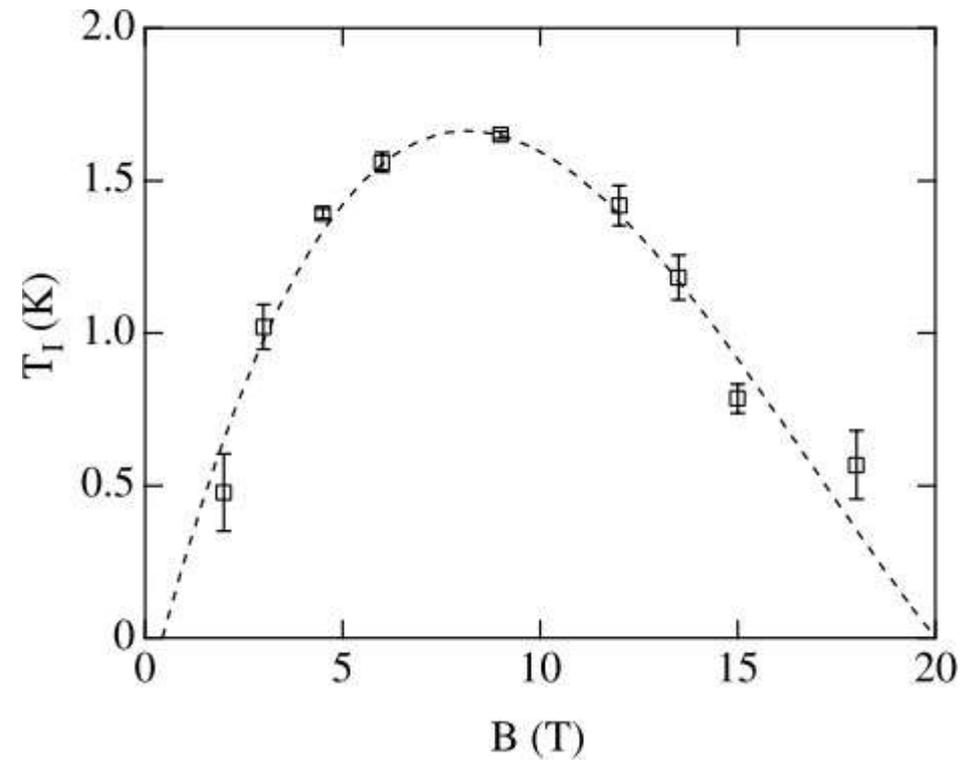
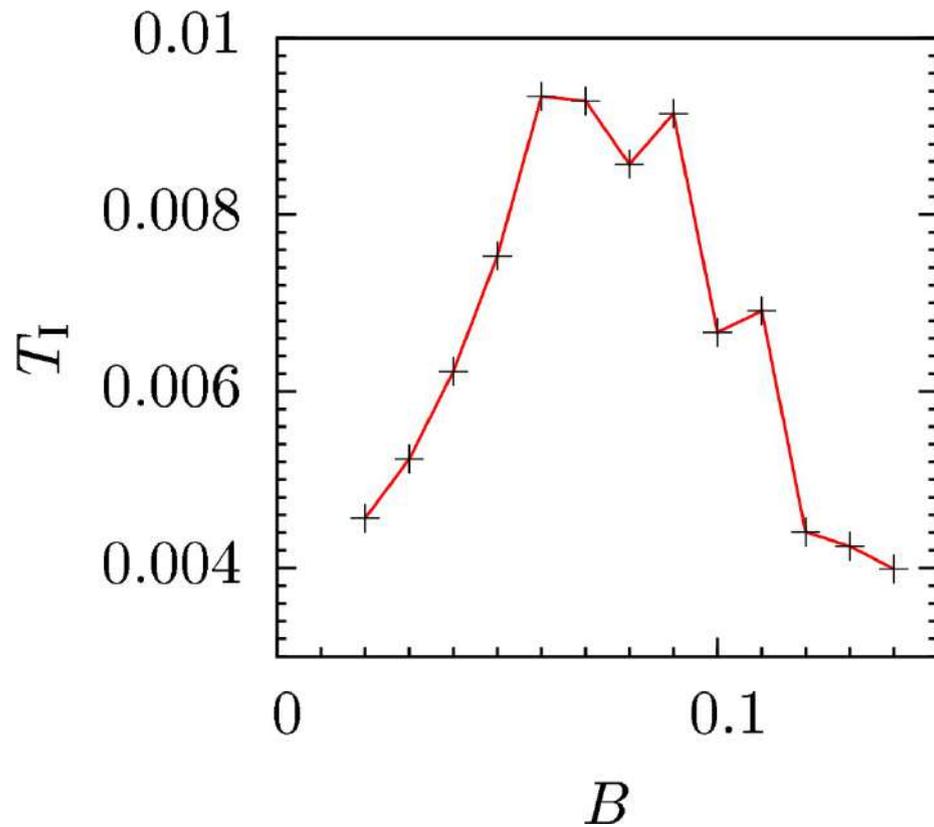
# Verification

- Resistivity at the Kosterlitz-Thouless transition
- Nonlinear  $I/V$  characteristics
- Length dependence of conductivity
- Andreev reflection
- Josephson junction
- Little-Parks effect in large diameter cylinder



# Clues: activated transport

- Activated transport  $\rho = \rho_0 e^{T_1/T}$



# Model one-dimensional systems

