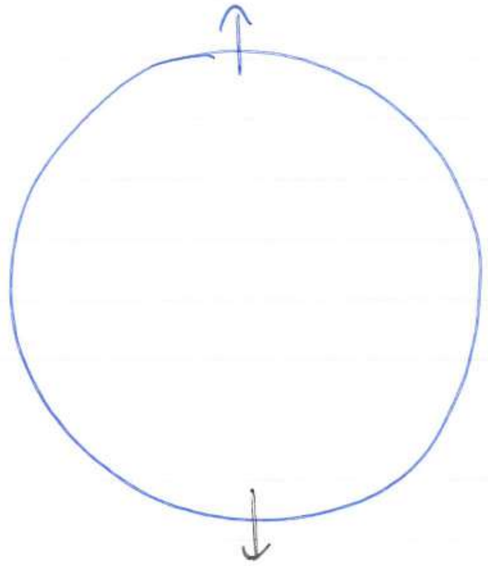
LECTURE 3

- Done repulsive interactions, now attractive interactions with $a < 0$
- Gives BCS superconductivity
- Then look at ongoing research on spin-imbalanced superconductor

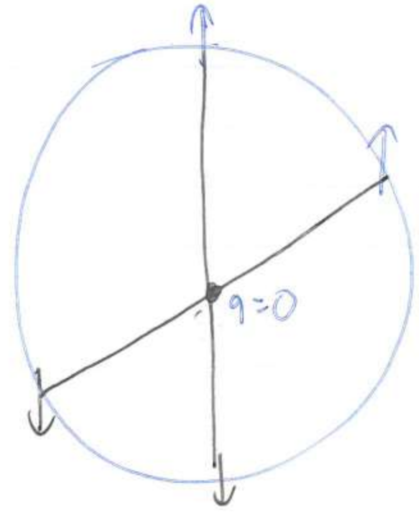
Attractive interactions : start from Fermi surface that is non-interacting



Consider two electrons, opposite spin, on top of Fermi surface

Contact interaction so only opposite spin attract.

COM momentum at $q = 0$ so construct trial wavefunction out of all orbitals sharing zero COM momentum :



KE

$$|\Psi\rangle = \sum_k \alpha_k C^+_{k\uparrow} C^+_{-k\downarrow} |FS\rangle$$

$$\langle FS | C^{\downarrow}_{p\uparrow} C^+_{-p\downarrow} | \hat{T} | \Psi \rangle \quad \hat{T} = \sum_{k\sigma} |k| d\varepsilon'_k \, . C^+_{k\sigma} C_{k\sigma}$$

$$= 2 . \alpha_p . (|p_\uparrow| + |p_\downarrow|) \varepsilon'_p$$

PE

$$\langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | g . \sum C^+_{k\uparrow} C^+_{-k\downarrow} C_{-k'\downarrow} C_{k'\uparrow} | \Psi \rangle$$

$$= g . \sum_k \alpha_k$$

← Sum catches spherical symmetry of the Fermi surface, catching massive degeneracy

Hamiltonian

$$\langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | \hat{H} | \Psi \rangle = \langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | E | \Psi \rangle$$

$$\left(2 (|p_\uparrow| + |p_\downarrow|) \varepsilon'_p - E \right) \alpha_p = g . \sum_k \alpha_k$$

$$\sum_{p=0}^{k_0} \alpha_p = \sum_{p=0}^{k_0} \frac{g \sum_k . \alpha_k}{2(|p_\uparrow| + |p_\downarrow|) \varepsilon'_p - E}$$

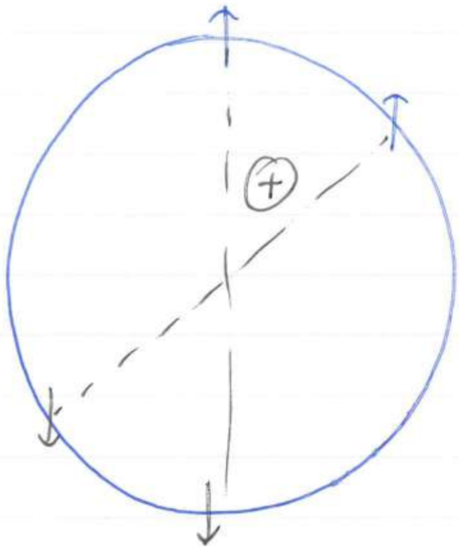
$$1 = g \gamma . \sum_{p=0}^{k_0} \frac{1}{2 . p . \varepsilon' - E}$$

$$E_b = 2 k_0 \varepsilon' . \exp\left(- \frac{2\varepsilon'}{g\gamma} \right).$$

so always bound state, fundamental property of 2D quantum well (here 3D but Fermi surface removes a dimension)

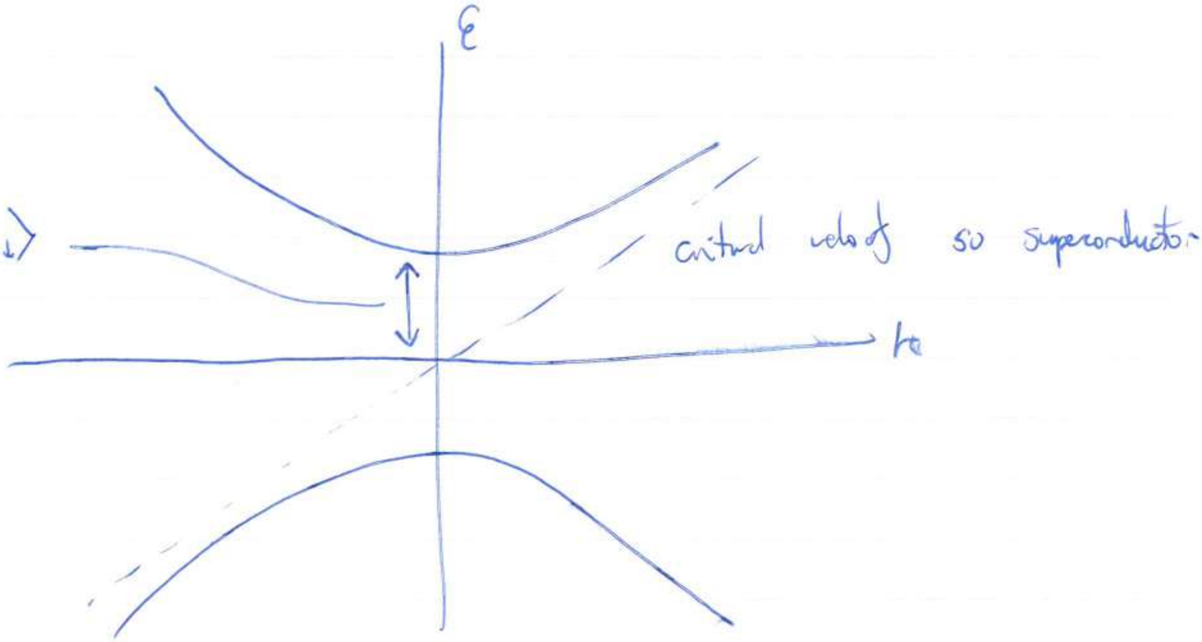
Coope pair ideal building block

Once made Cooper pair can make a superconductor out of coherent state of Cooper pairs



$$T_c = \hbar\omega \varepsilon' \exp\left(-\frac{\varepsilon'}{g\nu}\right)$$

Spectrum of excitations



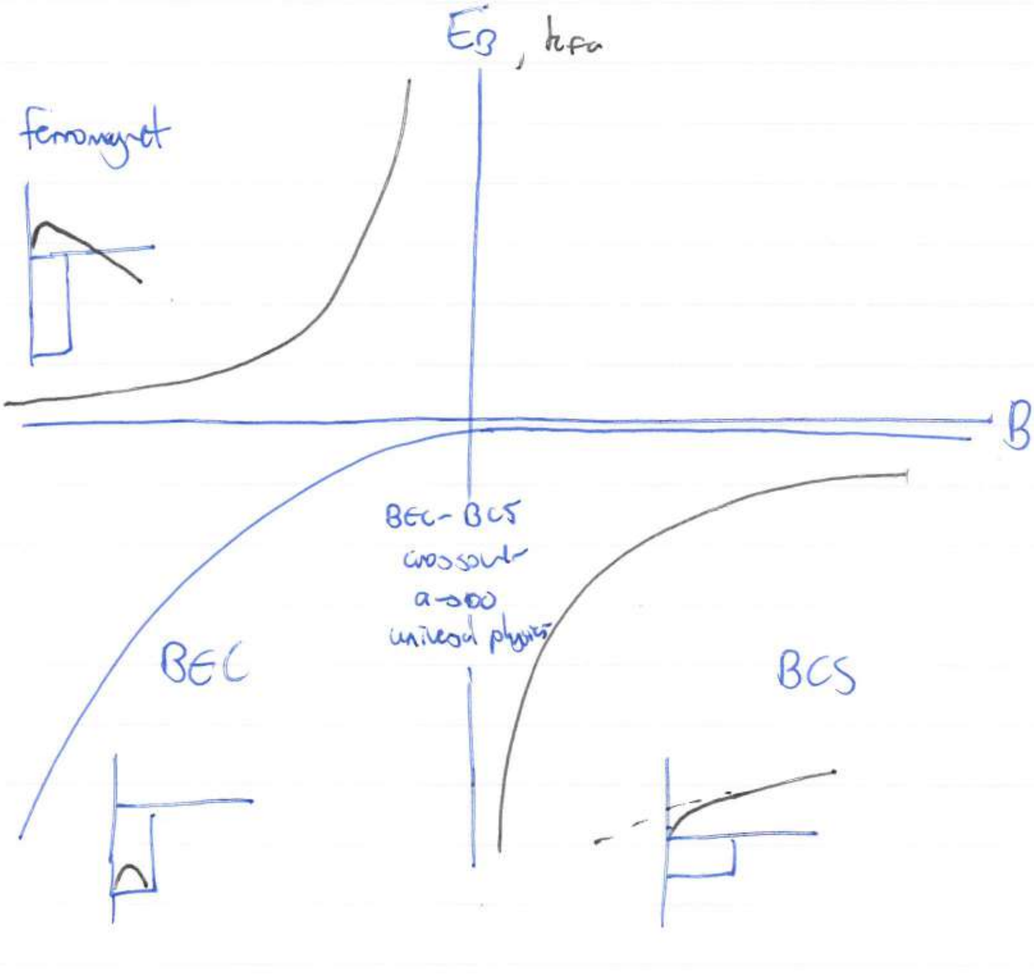
$\Delta = \sum_k \langle c_{k\uparrow} c_{k\downarrow} \rangle$

$= e^{-\varepsilon/g\nu}$

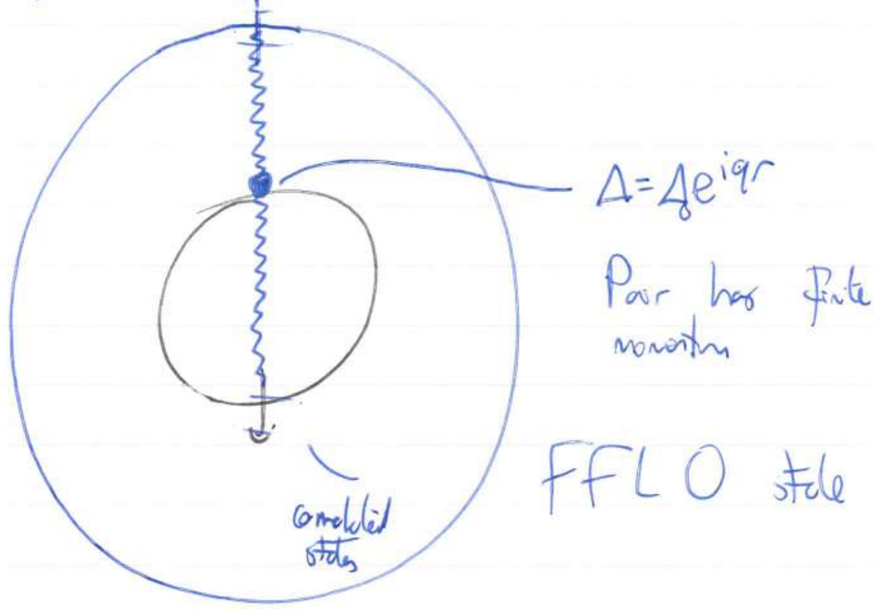
BEC - BCS crossover

BCS: coherent state of weakly bound Cooper pairs

BEC: strongly bound pairs form bosons: (why: size much smaller than average separation of pairs)
Then bosons condense into same q = 0 state



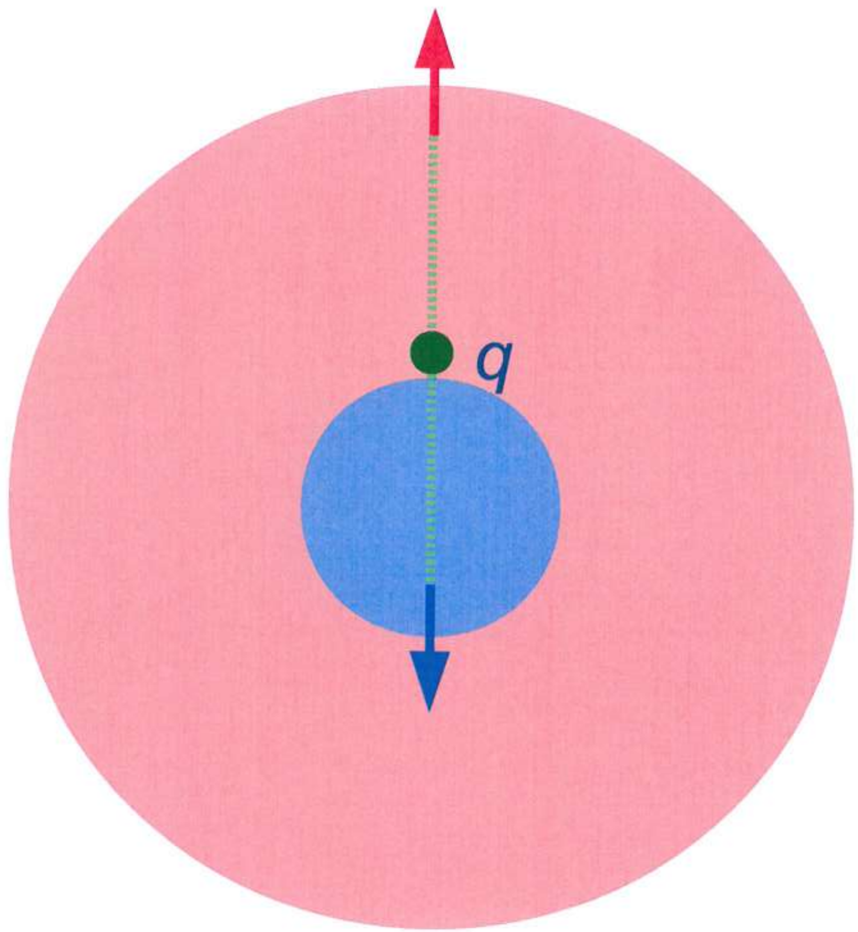
Imbalanced Fermi seas
Go through different options for pairing



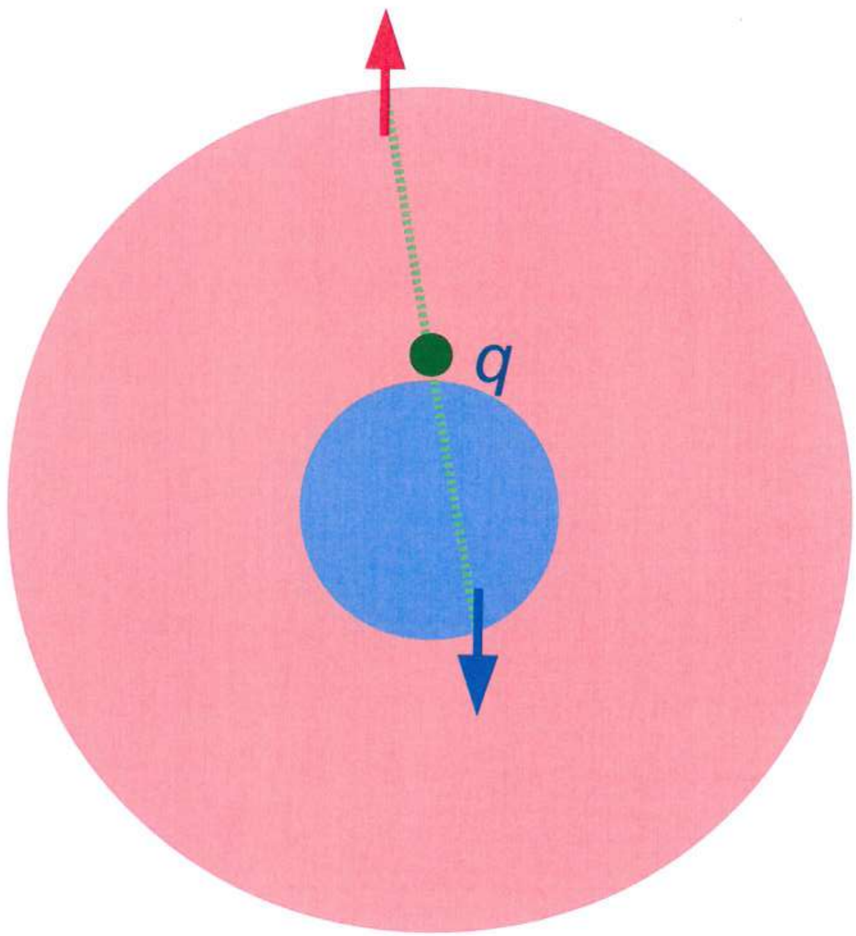
FFLO state

Could look for in cold atom gas, organic SC, heavy fermion materials that contain ferromagnetism and SC.

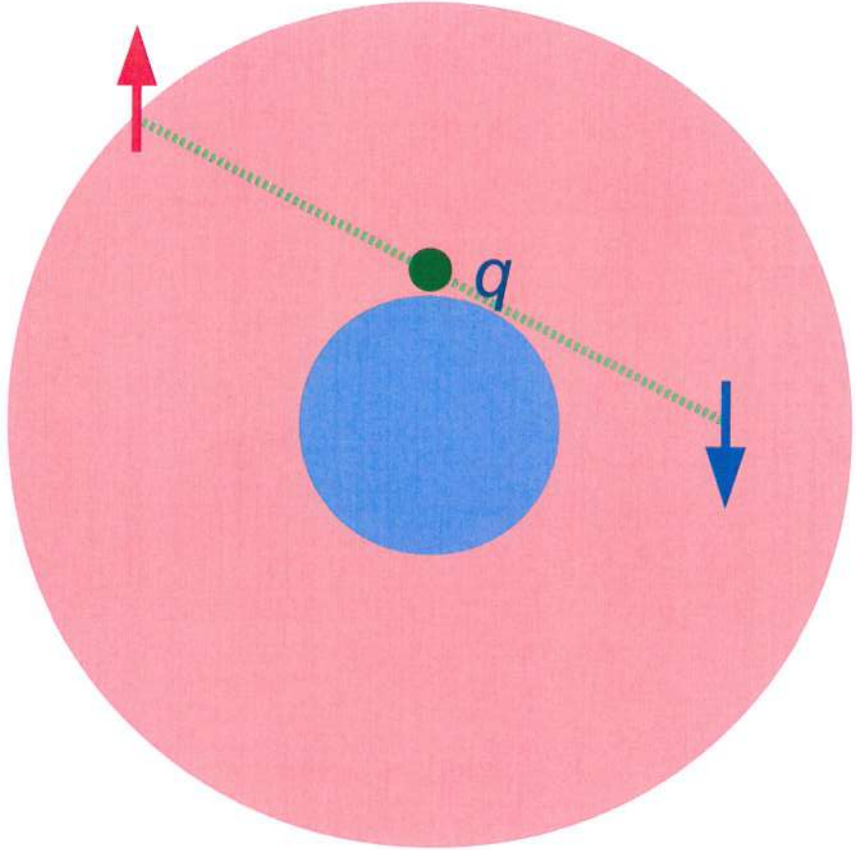
Cooper pair in a spin-imbalanced Fermi sea



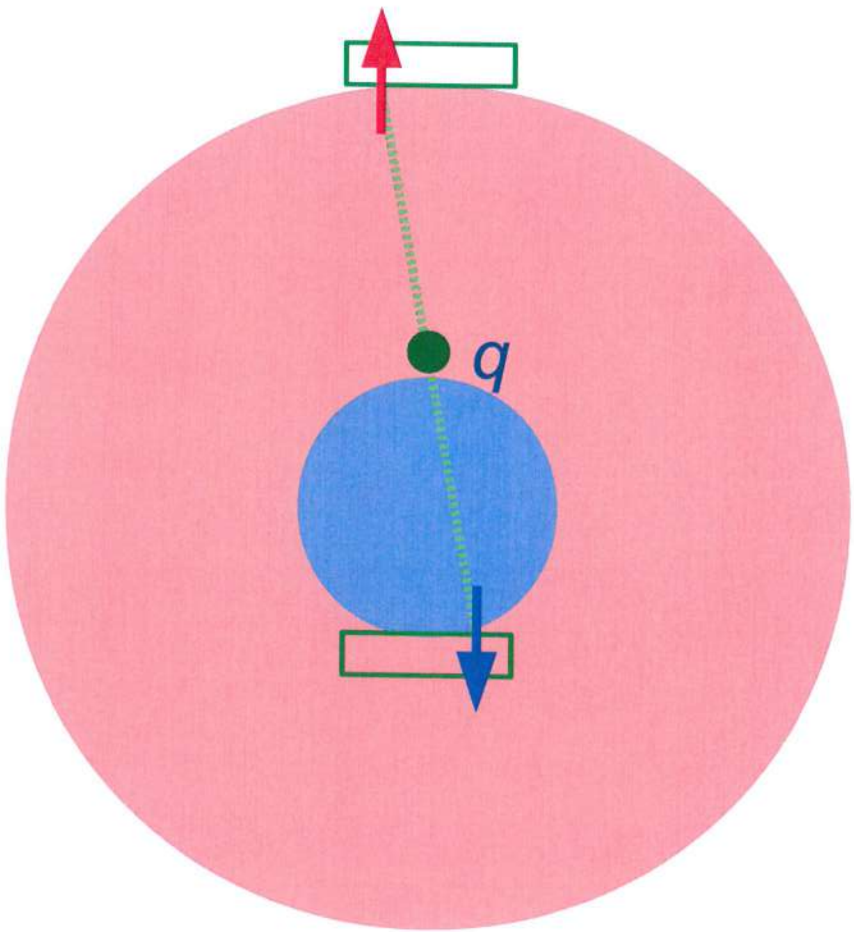
Cooper pair in a spin-imbalanced Fermi sea



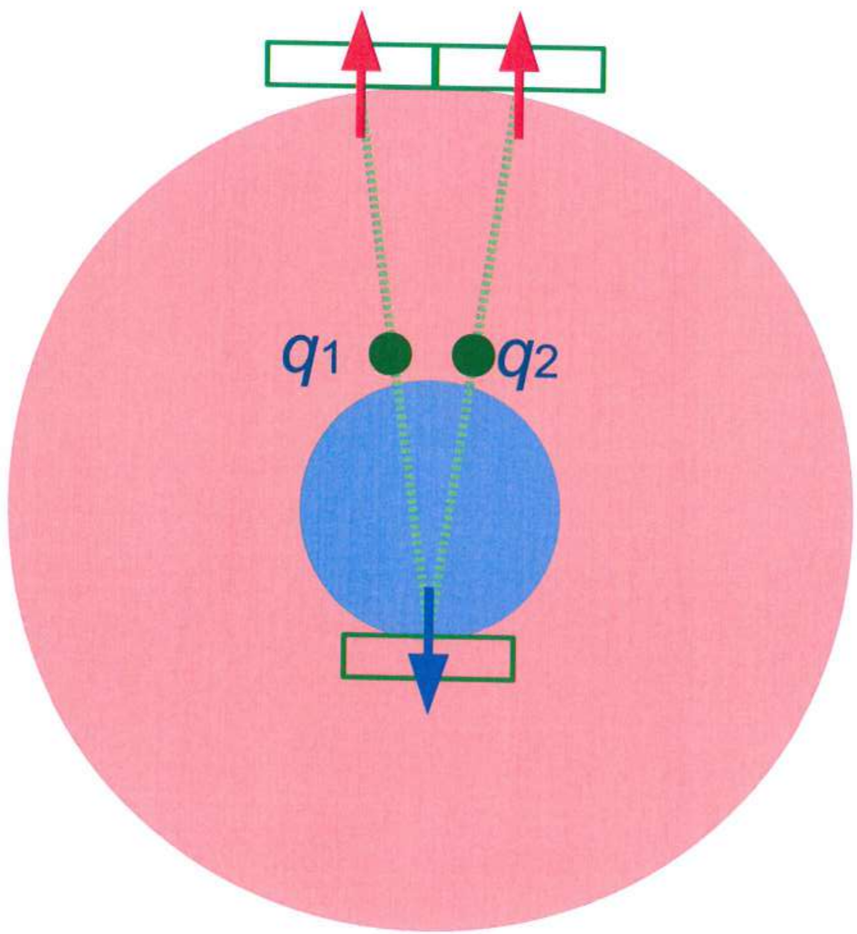
Cooper pair in a spin-imbalanced Fermi sea



States included in the wave function



Multiple majority spins in the instability



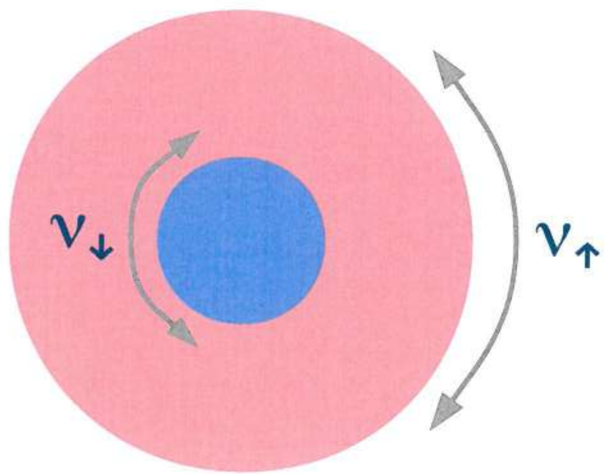
Energy of the $(N_\uparrow, N_\downarrow)$-spin instability

Binding energy of a multi-particle instability

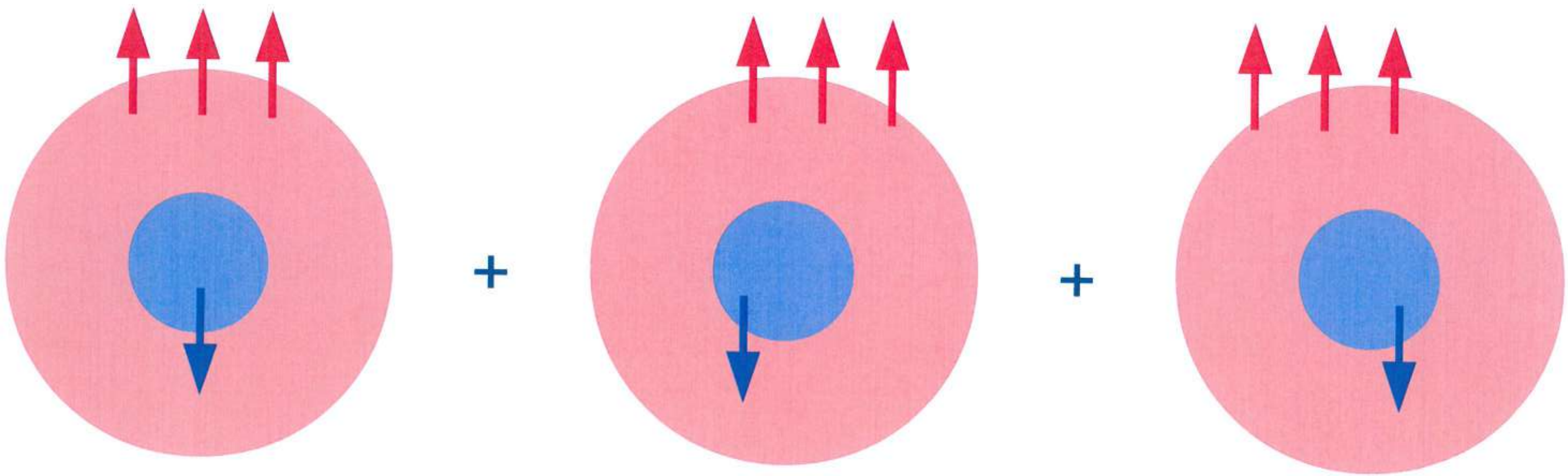
$$E = (N_\uparrow + N_\downarrow)\,\omega_D \exp\left(-\frac{(N_\uparrow + N_\downarrow)\xi'}{g N_\uparrow N_\downarrow}\frac{N_c}{\nu_c}\right) \qquad E = 2\omega_D \exp\left(-\frac{2\xi'}{g\nu}\right)$$

Optimal number of up and down spin electrons in the instability is

$$\frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow}$$



Multi-particle superconductor



Superconducting transition temperature

$$T_c = \omega_D \exp\left(-\frac{(N_\uparrow + N_\downarrow)\xi'}{2gN_\uparrow N_\downarrow} \frac{N_c}{\nu_c} \right)$$

Peak transition temperature is at the number ratio

$$\frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow}$$

Summary of multi-particle superconductor

Number of **up to down** spin electrons is the ratio of the **density of states**

Superconducting state based on multi-particle instability in a **spin-imbalanced** system

Analytical, exact diagonalization, and Diffusion Monte Carlo evidence

Applications in spin-orbit coupled systems and **number fluctuations** in the BCS superconductor