Lecture 3

- Done repulsive interactions now
  attractive interactions with $a < 0$

- Gives BCS superconductivity

- Then looks at ongoing research
  on spin-imbalanced superconductor
Attractive interactions start from Fermi surface that is non-interacting.

Consider two electrons, opposite spin, on top of Fermi surface. Construct interaction as opposite spin attract.

COM momentum at \( g = 0 \) so construct fermion on all orbitals sharing zero COM momentum.
\[ |\psi\rangle = \sum_{k_{\text{CF}}} c_{k_{\text{CF}}}^+ c_{k_{\text{CF}}}^1 |FS\rangle \]

\[ \langle FS | c_{\text{CF}}^+ c_{\text{CF}}^1 | \hat{T} | \psi \rangle \]

\[ = 2 \alpha_{\text{p}} \left( |p_{\text{CF}}^+| |p_{\text{CF}}^-| \right) E_{\text{p}} \]

\[ \langle FS | c_{\text{CF}}^+ c_{\text{CF}}^1 | g \sum_{n_{\text{CF}}} c_{n_{\text{CF}}}^+ c_{n_{\text{CF}}}^1 c_{n_{\text{CF}}}^1 c_{n_{\text{CF}}}^1 | \psi \rangle \]

\[ = g \sum_{n_{\text{CF}}} \alpha_{n_{\text{CF}}} \]

Sum over spherical symmetry of He Fermi surface, capturing massive degeneracy.

\[ \langle FS | c_{\text{CF}}^+ c_{\text{CF}}^1 | H | \psi \rangle = \langle FS | c_{\text{CF}}^+ c_{\text{CF}}^1 | E | \psi \rangle \]

\[ 2 \left( |p_{\text{CF}}^+| |p_{\text{CF}}^-| \right) E_{\text{p}} - E \]

\[ \alpha_{\text{p}} = g \sum_{n_{\text{CF}}} \alpha_{n_{\text{CF}}} \]

\[ \sum_{p_{\text{CF}}} \frac{g \sum_{n_{\text{CF}}} \alpha_{n_{\text{CF}}}}{2 \left( |p_{\text{CF}}^+| |p_{\text{CF}}^-| \right) E_{\text{p}} - E} \]

\[ \rho_{\text{CF}} \sum_{n_{\text{CF}}} \frac{\alpha_{n_{\text{CF}}}}{2 \left( |n_{\text{CF}}^+| |n_{\text{CF}}^-| \right) E_{\text{p}} - E} \]

\[ E_{\text{p}} = 2 \hbar^2 E_{\text{p}} \exp \left( - \frac{2E_{\text{p}}}{g \gamma} \right) \]

So always hunt for fundamental properties of 3D quantum well (here 3D but Fermi surface requires a dimension.)
Cooper pair ideal building block
Once made Cooper pair can make a superconductor out of coherent state of Cooper pairs

\[ T_c = \hbar \omega \exp \left( - \frac{\epsilon}{g \nu} \right) \]

Spectrum of excitations

\[ \Delta = \sum_{n} c_n^{\dagger} c_n \epsilon_n \]
\[ \epsilon = \epsilon_0 - \frac{\epsilon}{g \nu} \]
BEC - BCS crossover

BCS: coherent state of weakly bound Cooper pairs

BEC: strongly bound pairs form bosons (why, size much smaller than average separations of pairs) the bosons condense into some g = 0 state
Imbalanced Fermi sea through different options for pairing

\[ \Delta = \Delta e^{i \varphi} \]

Pair has finite momentum

FFLO state

Could look for cold atomic gas organic SC, heavy fermion metals, thin film ferromagnet and SE.
Cooper pair in a spin-imbalanced Fermi sea
Cooper pair in a spin-imbalanced Fermi sea
Cooper pair in a spin-imbalanced Fermi sea
Multiple majority spins in the instability
Energy of the \((N_\uparrow, N_\downarrow)\)-spin instability

Binding energy of a multi-particle instability

\[
E = (N_\uparrow + N_\downarrow) \omega_D \exp \left( -\frac{(N_\uparrow + N_\downarrow) \xi'}{g N_\uparrow N_\downarrow} \frac{N_c}{\nu_c} \right)
\]

\[
E = 2 \omega_D \exp \left( -\frac{2 \xi'}{g \nu} \right)
\]

Optimal number of up and down spin electrons in the instability is

\[
\frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow}
\]
Multi-particle superconductor

Superconducting transition temperature

\[ T_c = \omega_D \exp \left( - \frac{(N_\uparrow + N_\downarrow) \xi \prime}{2 g N_\uparrow N_\downarrow} \frac{N_c}{v_c} \right) \]

Peak transition temperature is at the number ratio

\[ \frac{N_\uparrow}{N_\downarrow} = \frac{v_\uparrow}{v_\downarrow} \]
Summary of multiparticle superconductor

Number of \textbf{up to down} spin electrons is the ratio of the \textbf{density of states}

Superconducting state based on multi-particle instability in a \textbf{spin-imbalanced} system

\textbf{Analytical}, exact diagonalization, and Diffusion Monte Carlo evidence

Applications in spin-orbit coupled systems and \textbf{number fluctuations} in the BCS superconductor