Between two events

$$\Delta x' = \gamma \left(\Delta x - v \Delta t \right) \qquad \Delta x = \gamma \left(\Delta x' + v \Delta t' \right)$$

$$\Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right) \qquad \Delta t = \gamma \left(\Delta t' + \frac{v \Delta x'}{c^2} \right)$$

$$c^2 \Delta t^2 - \Delta x^2 = c \Delta t'^2 - \Delta x'^2 = c^2 \Delta \tau^2.$$

- Find frame in which quantity is 0 and then exploit

eg length contraction

$\Delta x = L$, $\Delta t = 0$ measure in S

$\Delta x' = \gamma L$ so stick longer in S' frame

$$E^2 - p^2 c^2 = m^2 c^4 \longrightarrow (\Sigma E)^2 - (\Sigma p)^2 c^2 = m^2 c^4$$

$$\frac{p}{E} = \frac{v}{c^2}$$

Useful due to conservation laws.

Confusing to say $m' = \gamma m$ as $m^2 c^4$ constant in all frames

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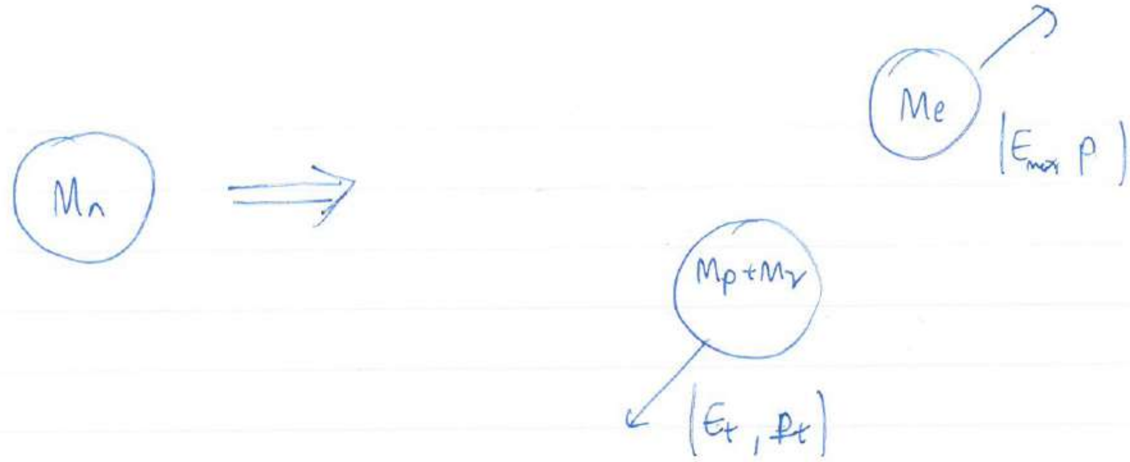
Doppler:

$$t = \frac{\lambda}{c - v} = \frac{c / f_s}{c - v} = \frac{1}{1 - \beta} \frac{1}{f_s}$$

$$t' = \frac{t}{\gamma}$$

$$\frac{1}{f'} = \sqrt{1 - \beta^2} \frac{1}{1 - \beta} \frac{1}{f_s} = \sqrt{\frac{1 + \beta}{1 - \beta}} \frac{1}{f_s} \quad \text{so} \quad f' = \sqrt{\frac{1 + \beta}{1 - \beta}} f_s$$

2003



$$M_n c^2 = E_{max} + E_t$$

$$p = p_t$$

$$E_t^2 = p_t^2 c^2 + M_t^2 c^4$$

$$M_n^2 c^4 - 2M_n c^2 E_{max} + E_{max}^2 = p^2 c^2 + M_t^2 c^4$$

$$= E_{max}^2 - M_e^2 c^4 + M_t^2 c^4$$

$$E_{max} = \frac{c^2}{2M_n}\left(M_n^2 + M_e^2 - M_t^2\right)$$

$$= \frac{M_n^2 + M_e^2 - M_p^2}{2M_n} c^2$$

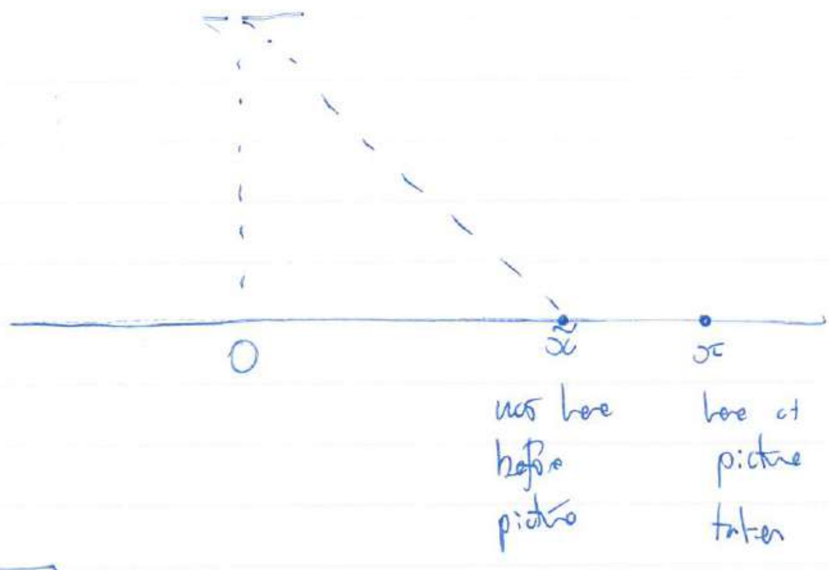
$$= 1.29 \text{ MeV}$$

$$\frac{V_m}{c} = \frac{p_t}{E_t} \; p$$

$$= \frac{\sqrt{\frac{E_{max}^2}{c^2} - M_e^2 c^2}}{M_n c^2 - E_{max}}$$

$$= 0.00127$$

2006



$$T = \frac{\sqrt{D^2 + \tilde{x}^2}}{c} \qquad \beta = \frac{v}{c}$$

2.1 $$x = \tilde{x} + \beta \cdot \sqrt{D^2 + \tilde{x}^2}$$

$$x^2 - 2x\tilde{x} + \tilde{x}^2 = \beta^2 D^2 + \beta^2 \tilde{x}^2$$

$$\tilde{x}^2 \left(1 - \beta^2\right) - 2x\tilde{x} + x^2 - \beta^2 D^2 = 0$$

$$\tilde{x} = \frac{2x \pm \sqrt{4x^2 - 4(1-\beta^2)(x^2 - \beta^2 D^2)}}{2(1-\beta^2)}$$

$$= \gamma^2 x \pm \gamma^2 \sqrt{x^2 - \left(x^2 - \beta^2 D^2 - \beta^2 x^2 + \beta^4 D^2\right)}$$

$$= \gamma^2 x \pm \beta \gamma^2 \sqrt{D^2 (1 - \beta^2) + x^2}$$

$$= \gamma^2 x \pm \beta \gamma \sqrt{D^2 + (\gamma x)^2}$$

2.2 $$= \gamma^2 x - \beta \gamma \sqrt{D^2 + (\gamma x)^2}$$

other solution is moving in opposite direction

Actual rod length is L/γ

$$x_{\pm} = x_0 \pm \frac{L}{2\gamma}$$

$$\tilde{x}_{\pm} = \gamma^2\left(x_0 \pm \frac{L}{2\gamma}\right) - \beta\gamma\sqrt{D^2 + \gamma^2\left(x_0 \pm \frac{L}{2\gamma}\right)^2}$$

$$\tilde{L}(x_0) = \tilde{x}_+ - \tilde{x}_-$$

2.3 $$\tilde{L}(x_0) = \gamma L + \beta\gamma\sqrt{D^2 + \left(\gamma x_0 - \frac{L}{2}\right)^2} - \beta\gamma\sqrt{D^2 + \left(\gamma x_0 + \frac{L}{2}\right)^2}$$

$$\frac{dx_0}{dt} = v$$

$$\frac{d\tilde{L}}{dt}(x_0) = \beta\gamma\left[\frac{2\left(\gamma x_0 - \frac{L}{2}\right)\gamma v}{2\sqrt{D^2 + \left(\gamma x_0 - \frac{L}{2}\right)^2}} - \frac{2\left(\gamma x_0 + \frac{L}{2}\right)\gamma v}{2\sqrt{D^2 + \left(\gamma x_0 + \frac{L}{2}\right)^2}}\right]$$

$$= 2\beta\gamma^2 v\left[\frac{1}{\sqrt{1 + D^2/\left(\gamma x_0 - \frac{L}{2}\right)^2}} - \frac{1}{\sqrt{1 + D^2/\left(\gamma x_0 + \frac{L}{2}\right)^2}}\right] < 0$$

2.4 Apparent length always decreases

Light from two ends emitted simultaneously

2.5 $$\tilde{L} = \frac{L}{\gamma}$$

$$0 = \tilde{x}_+ + \hat{x}_-$$

$$0 = 2\gamma^2 x_0 - \beta\gamma\left[\sqrt{D^2 + \left|\gamma x_0 - \frac{L}{2}\right|^2} + \sqrt{D^2 + \left|\gamma x_0 + \frac{L}{2}\right|^2}\right]$$

also $$\frac{L}{\gamma} = \gamma L + \beta\gamma\left[\sqrt{D^2 + \left|\gamma x_0 - \frac{L}{2}\right|^2} - \sqrt{D^2 + \left|\gamma x_0 + \frac{L}{2}\right|^2}\right]$$

$$\sqrt{D^2 + \left|\gamma x_0 \pm \frac{L}{2}\right|^2} = \frac{2\gamma^2 x_0 \pm \left(\gamma L - \frac{L}{\gamma}\right)}{2\beta\gamma}$$

$$= \frac{\gamma x_0}{\beta} \pm \frac{\beta L}{2}$$

$$D^2 + \gamma^2 x_0^2 \pm \cancel{\gamma x_0 L} + \frac{L^2}{4} = \frac{\gamma^2 x_0^2}{\beta^2} \pm \cancel{\gamma x_0 L} + \frac{\beta^2 L^2}{4}$$

$$x_0^2 \gamma^2 \left(1 - \frac{1}{\beta^2}\right) = \frac{L^2}{4}\left(\beta^2 - 1\right) - D^2$$

$$x_0^2 \frac{1}{\beta^2} = D^2 + \frac{L^2}{4\gamma^2}$$

$$x_0 = \pm\beta\sqrt{D^2 + \left|\frac{L}{2\gamma}\right|^2}$$

2.6 $$x_0 = \beta\sqrt{D^2 + \left|\frac{L}{2\gamma}\right|^2}$$

$$\tilde{x}_0 = \gamma^2 x_0 - \beta\gamma\sqrt{D^2 + (\gamma x_0)^2}$$

$$= \gamma^2\beta\sqrt{D^2 + \left|\frac{L}{2\gamma}\right|^2} - \beta\gamma\sqrt{D^2 + \gamma^2\beta^2\left(D^2 + \left|\frac{L}{2\gamma}\right|^2\right)}$$

2.7 $$= \beta\gamma\left[\sqrt{|\gamma D|^2 + \left|\frac{L}{2}\right|^2} - \sqrt{|\gamma D|^2 + \left|\frac{\beta L}{2}\right|^2}\right]$$

2.8 The apparent length is 3m on the early picture and 1m on the late picture

$$\tilde{L}_{early} = \sqrt{\frac{1+\beta}{1-\beta}}\, L \qquad \left(\text{from Doppler, or } x_0 \to -\infty \right)$$

$$\tilde{L}_{late} = \sqrt{\frac{1-\beta}{1+\beta}}\, L$$

$$\frac{L_{early}}{L_{late}} = \frac{1+\beta}{1-\beta} = 3. \implies \beta = \frac{1}{2}$$

2.9

$$V = \frac{c}{2}$$

$$L = \sqrt{\tilde{L}_{early}\tilde{L}_{late}}$$

2.10

$$= 1.73\,m$$

$$\tilde{L} = \frac{L}{\gamma}$$

$$= \sqrt{3}\,.\sqrt{1-\frac{1}{2^2}}$$

2.11

$$= 1.50 \ m.$$