Perspectives on itinerant ferromagnetism in an atomic Fermi gas

Gareth Conduit$^{1,2}$, Ben Simons$^3$ & Ehud Altman$^1$

1. Weizmann Institute, 2. Ben Gurion University, 3. University of Cambridge

G.J Conduit & E. Altman, arXiv: 0911.2839
Ferromagnetism in solid state

Second order in iron & nickel

First order in ZrZn$_2$

Uhlarz et al., PRL 2004
Further phase reconstruction in ZrZn$_2$

Uhlarz et al., PRL 2004
Stoner instability with repulsive interactions

- Use two $^6\text{Li}$ states to represent pseudo up and down-spin electrons

$$\hat{H} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + g \sum_{k \neq q} c_{k\uparrow}^\dagger c_{k'\uparrow}^\dagger + c_{k\downarrow} + c_{k'\downarrow} + c_{k\uparrow}$$

$$F = F_0 + \frac{1 - g \nu}{2 \nu} m^2 + um^4$$

- A Fermi surface shift increases the kinetic energy and potential energy falls

- Ferromagnetic transition occurs if $g \nu > 1$

Why study ferromagnetism with cold atoms?

• Key experimental advantages
  – Feshbach resonance
  – Clean system
  – True contact interaction

• Answer long-standing questions from the solid state
  – Is the ferromagnetic phase stable?
  – Is the transition first or second order?
  – Are there exotic phases near to the tricritical point?

• New physics
  – Two and one-dimensional ferromagnetism
  – Effects of population and mass imbalance
  – Non-equilibrium magnetism
Experimental evidence for ferromagnetism

- Minimum in kinetic energy at $k_F a \approx 2.2$

Further key experimental signatures

\[ E_K \propto n^{5/3} \]

\[ \Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow) \]

Outline

• Equilibrium analysis with mean field & fluctuation corrections
  – Fluctuation corrections lead to emergence of first order transition
• Stoner transition in the presence of atom loss
  – Condensation of topological defects
  – Renormalization of interaction strength
• Experimental protocols that circumvent three-body loss
  – Collective modes within a spin spiral
Equilibrium study of ferromagnetism

\[ Z = \int D\psi \exp\left( -\int \int d\tau d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma} (-i\hat{\omega} + \hat{\epsilon} - \mu) \psi_{\sigma} - g \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow} \right) \]

- Decouple with the average magnetisation \( m \) gives the Stoner criterion

\[ F = F_0 + \frac{1 - g\nu}{2\nu} m^2 + um^4 + vm^6 \]
Mean-field analysis & consequences of trap

- Recovers qualitative behavior\(^1\) but transition at \(k_F a = 1.8\) instead of \(k_F a = 2.2\)

Fluctuation corrections

\[ Z = \int D\psi \exp \left( -\int \int d\tau d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma} (-i\omega + \hat{\epsilon} - \mu) \psi_{\sigma} - g \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\uparrow} \psi_{\downarrow} \right) \]

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

\[ F = F_0 + \frac{1-g\nu}{2\nu} m^2 + u m^4 + v m^6 + g^2 \left( r m^2 + w m^4 \ln|m| \right) \]

\[ k_F a_{\text{crit}} = 1.05 \]

- First order transition\(^1\)

Quantum Monte Carlo verification

\[ F = F_0 + \frac{1 - g \nu}{2 \nu} m^2 + u m^4 + v m^6 + g^2 \left( r m^2 + w m^4 \ln |m| \right) \]

\[ k_F a_{\text{crit}} = 1.05 \]

- Verified by \textit{ab initio} Quantum Monte Carlo calculations\(^1\)


Fluctuation corrections encourage ferromagnetism

Outline: three-body loss

• Equilibrium analysis with mean field & fluctuation corrections
  – Fluctuation corrections lead to emergence of first order transition

• The Stoner transition in the presence of atom loss
  – Condensation of topological defects
  – Renormalization of interaction strength

• Experimental protocols that circumvent three-body loss
  – Collective modes within a spin spiral
Initial growth of domains

- Quench leads to domain growth [Babadi et al. arXiv:0908.3483], applies for \( k_F a < 1.06 k_F a_c \)

- Ferromagnetic quench *deep* beyond the spinoidal line leads to the condensation of topological defects
Condensation of topological defects

- Defects freeze out from paramagnetic state

- Defects grow as $L \sim t^{\frac{1}{2}}$
  [Bray, Adv. Phys. 43, 357 (1994)]

Ramp up interactions

Mutual annihilation of defects
Consequences of defect annihilation

- Defect annihilation raises required interaction strength

Two versus three-body loss

Two-body mechanism

- Feshbach molecules can be formed by a two body process [Pekker, unpublished]
- Requires $k_F^2/m < 1/2ma^2$, $k_Fa < 1/\sqrt{2}$

Three-body mechanism

- A third-body can remove the excess energy
- Rate $\lambda'[n_\uparrow(r) + n_\downarrow(r)]n_\uparrow(r)n_\downarrow(r)$ [Petrov 2003]
- In boson systems, three-body scattering drives the formation of a Tonks-Girardeau gas [Syassen et al., Science 320, 1329 (2009)]
Damping of fluctuations by atom loss

- Atom loss rate, \( \lambda'[n_\uparrow(r) + n_\downarrow(r)]n_\uparrow(r)n_\downarrow(r) \), is
  \[
  \lambda'\chi(r-r')[c_\uparrow(r')c_\uparrow(r') + c_\downarrow(r')c_\downarrow(r')]c_\uparrow(r)c_\downarrow(r)c_\downarrow(r)\]

- A mean-field approximation, \( \bar{N} = n_\uparrow(r') + n_\downarrow(r') \) places interactions on same footing as interactions
  \[
  S_{\text{int}} = (g + i\lambda\bar{N})c_\uparrow(r)c_\downarrow(r)c_\downarrow(r)c_\uparrow(r)\]

- Also include atom source \(-i\gamma c_\sigma^\dagger c_\sigma\) to ensure gas remains at equilibrium

- Loss damps fluctuations so inhibits the transition
  \[
  F = F_0 + \frac{1-g\nu}{2\nu}m^2 + um^4 + vm^6 + \left( g^2 - \lambda^2 \bar{N}^2 \right)\left( r m^2 + \nu m^4 \ln|m| \right)
  \]
Phase boundary with atom loss

- Atom loss raises the interaction strength required for ferromagnetism

Conduit & Altman, arXiv: 0911.2839
Interaction renormalization with atom loss

Outline: evolution of a spin spiral

- Equilibrium analysis with mean field & fluctuation corrections
  - Fluctuation corrections lead to emergence of first order transition
- The Stoner transition in the presence of atom loss
  - Condensation of topological defects
  - Renormalization of interaction strength
- New experimental protocol to circumvent three-body loss
  - Collective modes within a spin spiral
Alternative strategy: spin spiral

(a) Fully polarized state

(b) Applied magnetic field forms spin spiral

(c) Interactions cant the spiral
Spin spiral collective modes

- Exponentially growing collective modes if \( p < Q \)

\[
\Omega(p) = \pm \left( \frac{1}{2} - \frac{2^{2/3}}{3} \frac{5k_F a}{p} \right) p \sqrt{p^2 - Q^2}
\]

- Maximum growth at \( p = Q / \sqrt{2} \)

- Critical slowing
Phase-contrast imaging displays signatures of domain growth.
Domain size fixed across the sample.

Atomic gas
Projected spin imbalance

Maximum growth
Initial spin spiral pitch

$|u|^2$ vs $\sqrt{2q/Q}$
Summary

• Equilibrium theory provides a reasonable qualitative description of the transition

• Discrepancy in the interaction strength could be accounted for by:
  1) Non-equilibrium formation of the ferromagnetic phase
  2) Renormalization of interaction strength due to atom loss

• Circumvent three-body loss by studying the evolution of a spin spiral
Feshbach resonance

- Note instability to BEC molecular state on repulsive side of resonance
Three-body losses

- In boson systems, three-body scattering drives the formation of a Tonks-Girardeau gas [Syassen et al., Science 320, 1329 (2009)]
- The experiment is an opportunity to study not only ferromagnetism but also three-body loss and dynamics
Condensation of topological defects

- Defects freeze out from disordered state

- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength

- Defect radius $L \sim t^{1/2}$ [Bray, Adv. Phys. 43, 357 (1994)]
Phase boundary with atom loss

- Atom loss raises the interaction strength required for ferromagnetism

Conduit & Altman, arXiv: 0911.2839
Interaction renormalization with atom loss

Condensation of topological defects

- Condensation of defects inhibits the transition

First order phase transition and Quantum Monte Carlo verification

- First order transition into uniform phase with TCP

- QMC also sees first order transition
Summary of equilibrium results
Momentum distribution
New approach to fluctuation corrections

\[
Z = \int D\psi \exp \left( -\int \sum_\sigma \bar{\psi}_\sigma (-i\omega + \epsilon - \mu) \psi_\sigma - g \int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right)
\]

- Analytic strategy:
  1) Decouple in both the density and spin channels (previous approaches employ only spin)
  2) Integrate out electrons
  3) Expand about uniform magnetisation
  4) Expand density and magnetisation fluctuations to second order
  5) Integrate out density and magnetisation fluctuations

- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure
**Analytical method**

- System free energy $F = -k_B T \ln Z$ is found via the partition function

$$Z = \int D\psi \exp \left( -\int \sum_\sigma \bar{\psi}_\sigma (-i \omega + \epsilon - \mu) \psi_\sigma - g \int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right)$$

- Decouple using only the average magnetisation $m = \bar{\psi}_\downarrow \psi_\uparrow - \bar{\psi}_\uparrow \psi_\downarrow$

  gives $F \propto (1 - g \nu) m^2$ i.e. the Stoner criterion


  $$F = \frac{1}{2} \left( |\omega| / \Gamma_q + r + q^2 \right) m^2 + \frac{u}{4} m^4 \ln (m^2 + T^2) + \cdots - hm$$
Quantum Monte Carlo verification

• First order transition into uniform phase with TCP

• QMC also sees first order transition
Cold atomic gases — spin

- Two fermionic atom species have a pseudo-spin:

\[ ^6\text{Li} \quad m_F=\frac{1}{2} \] maps to \( \text{spin } \frac{1}{2} \)

\[ ^6\text{Li} \quad m_F=-\frac{1}{2} \] maps to \( \text{spin } -\frac{1}{2} \)

- The up-and down spin particles \textit{cannot} interchange — population imbalance is fixed. Possible spin states are:

\[ |\uparrow\uparrow\rangle \quad S=1, \quad S_z=1 \quad \text{State not possible as } S_z \text{ has changed} \]

\[ |\downarrow\downarrow\rangle \quad S=1, \quad S_z=-1 \quad \text{State not possible as } S_z \text{ has changed} \]

\[ \left( |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)/\sqrt{2} \quad S=1, \quad S_z=0 \quad \text{Magnetic moment in plane} \]

\[ \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)/\sqrt{2} \quad S=0, \quad S_z=0 \quad \text{Non-magnetic state} \]

- Ferromagnetism, if favourable, must form in-plane
Particle-hole perspective

To second order in $g$ the free energy is

$$F = \sum_{\sigma,k} \epsilon_{k}^{\sigma} n(\epsilon_{k}^{\sigma}) + gN^{\uparrow}N^{\downarrow}$$

$$- \frac{2g^{2}}{V^{3}} \sum_{p} \int \int \frac{\rho^{\uparrow}(p, \epsilon_{\uparrow})\rho^{\downarrow}(-p, \epsilon_{\downarrow})}{\epsilon_{\uparrow} + \epsilon_{\downarrow}} d\epsilon_{\uparrow} d\epsilon_{\downarrow}$$

$$+ \frac{2g^{2}}{V^{3}} \sum_{k_{1,2,3,4}} \frac{n(\epsilon_{k_{1}}^{\uparrow})n(\epsilon_{k_{2}}^{\downarrow})}{\epsilon_{k_{1}}^{\uparrow} + \epsilon_{k_{2}}^{\downarrow} - \epsilon_{k_{3}}^{\uparrow} - \epsilon_{k_{4}}^{\uparrow}} \delta(k_{1} + k_{2} - k_{3} - k_{4})$$

with $\epsilon_{k}^{\sigma} = \epsilon_{k} + \sigma gm$ and a particle-hole density of states

$$\rho^{\sigma}(p, \epsilon) = \sum_{k} n(\epsilon_{k+p/2}^{\sigma})\left[1 - n(\epsilon_{k-p/2}^{\sigma})\right] \delta(\epsilon - \epsilon_{k+p/2}^{\sigma} + \epsilon_{k-p/2}^{\sigma})$$

- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^{4}\ln m^{2}$ at $T=0$
- Links quantum fluctuation to second order perturbation approach\(^1\)

\(^1\)Abrikosov 1958 & Duine & MacDonald 2005
Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with $q=0.2k_F$
Modified collective modes

- Collective mode dispersion

- Collective mode damping