Talk outline
- Cold atom systems
- The Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) instability
- Analytical approaches followed
- Results for uniform and trapped systems
- Conclusions
Fermionic alkali atoms e.g. $^6\text{Li}$ are trapped by lasers and cooled
Two contributions to spin: nucleus and valence electron
A good quantum number is total projected spin
The electron's interaction with the magnetic field dominates
In the isolated atom limit get spin-up particles
\[ |a\rangle = |m_{fa} = 1/2\rangle \approx |m_s = -1/2, m_I = 1\rangle, \text{ with a bit of } |m_s = 1/2, m_I = 0\rangle \]
And spin-down particles
\[ |b\rangle = |m_{fb} = -1/2\rangle \approx |m_s = -1/2, m_I = 0\rangle, \text{ with a bit of } |m_s = 1/2, m_I = -1\rangle \]
In the dense limit the scattering operator is not diagonal in the states $|a, b\rangle$ and $|c, d\rangle$ so scattering can occur from $|a, b\rangle$ (open channel) into $|c, d\rangle$ (closed channel)
Cold atom gases (II)

- States $|a, b\rangle$ and $|c, d\rangle$ magnetic moments differ by $\mu$ so relative energies are shifted by $\mu B$
- In a Feshbach resonance a bound state of the closed channel is brought into resonance with the open channel, affecting particle scattering
- Can have atoms with different masses
- Any population imbalance is maintained
- System is in quasiequilibrium, actual equilibrium is a solid
FFLO instability

- In strong binding limit have a Bose condensate, weak binding gives Cooper pairs, in the intermediate regime a modulated phase is possible
- BCS Cooper pairs have no total momentum (a)
- A population imbalance (or a ratio of masses) means Cooper pairs have a non-zero total momentum (b)
- This Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) instability results in a textured state
- Inferred in superconductors with external magnetic field

\[ E_{F,\uparrow} = E_{F,\downarrow} \]

- Similarities to electron-hole bilayers, where electron/holes are the normal phase and excitons the superfluid
Analytical approach

Ginzburg-Landau approach
- Expand thermodynamic potential $\Phi$ in terms of an order parameter $\Delta_q$ to quadratic order over all wave vectors $q$
  $$\Phi = \sum_q \alpha_q |\Delta_q|^2$$
- If coefficient $\alpha_q$ is negative, it is favourable for $\Delta_q \neq 0$ -- an FFLO instability
- Get analytical results for phase boundaries but it cannot pick up first order transitions

Single Fourier component approach
- Consider just a single wave vector $Q$ and minimise exact thermodynamic potential with respect to that wave vector and the order parameter $\Delta_Q$
  $$\Phi(\Delta_Q)$$
- Distinguishes between first and second order transitions but results are numerical
- The $Q=0$ state can be evaluated analytically
Equal masses $m_\uparrow = m_\downarrow$, population imbalanced system with $\mu_\uparrow = \mu + h$ and $\mu_\downarrow = \mu - h$

The superfluid (SF), partially polarised normal (PP), fully polarised normal (FP), phases and the system containing no particles (ZP) are shown.

There is a first order phase transition from the normal into the superfluid phase (dotted line).

The transitions between normal phases are second order and are straight lines (solid) with $\mu = \pm h$.

In constant $n$ system would see phase separated SF and PP normal state.
Free system: $Q \neq 0$

- The FFLO instability encroaches into the partially polarised normal state (PP) but not the superfluid.
- Second order transition from partially polarised normal state (PP) into FFLO instability.
- First order transition from FFLO instability into superfluid (SF) state.
- No FFLO instability in fully polarised state (FP) as there are no minority spin particles.
- In constraint $n$ system see FFLO rather than a phase separated region.
Free system: changing mass ratio $r$

- Generalise to allow different particle masses where $r = m_\downarrow / m_\uparrow$ and chemical potentials $\mu_\uparrow = \mu + h$ and $\mu_\downarrow = \mu - h$.
- Superfluidity is favoured if the light species is in excess.
- A trap has a varying effective chemical potential $\mu(r) = \mu_0 - V(r)$, corresponding to straight line (dotted) trajectories.
Conclusions

- Ginzburg-Landau and single Fourier component approaches were used to derive analytic expressions for phase boundaries in a 2D fermionic atomic gas.
- Superfluidity is favoured if the light species is in excess and the FFLO instability was seen.
- In a trapped system a superfluid could be bordered on one or both sides by normal phases.
- Thanks to Ben Simons and Peter Conlon.