Multi-particle instability in an imbalanced electron gas

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Cooper pair
Cooper pair
Cooper pair

\[ Q = 0 \]
$E = 2 \omega_D \exp \left( -\frac{2 \xi'}{g \nu_c} \right)$
Cooper pair on imbalanced Fermi sea
Cooper pair on imbalanced Fermi sea
Cooper pair on imbalanced Fermi sea
Orbitals that can be correlated
Orbitals that can be correlated
Few-particle instability

Binding energy of a few-particle instability

\[ E = (N_\uparrow + N_\downarrow) \omega_D \exp \left( -\frac{(N_\uparrow + N_\downarrow) \xi' N_c}{gN_\uparrow N_\downarrow \nu_c} \right) \]

Optimal number of up and down spin electrons in an instability

\[ \frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow} \]
Many-body theory

Superconducting transition temperature

\[ T_c = \omega_D \exp \left( - \frac{(N_{\uparrow} + N_{\downarrow}) \xi' N_c}{2 g N_{\uparrow} N_{\downarrow} v_c} \right) \]

Optimal number of up and down spin electrons in an instability

\[ \frac{N_{\uparrow}}{N_{\downarrow}} = \frac{\nu_{\uparrow}}{\nu_{\downarrow}} \]
Optimal number of up and down spin electrons in a Cooper particle is the ratio of the density of states.

Cooper particle is the building block for superconducting state, verified by Diffusion Monte Carlo simulations.

Energetically favorable to FFLO state.

Possibility of number fluctuations in a superconductor.
Recovering known results

Standard BCS result ($\nu_\uparrow = \nu_\downarrow$)

$$E = 2\omega_D \exp \left( -\frac{2\xi'}{g\nu} \right)$$

One-dimensional result ($\nu_\uparrow = \nu_\downarrow$)

$$E = 2\omega_D \exp \left( -\frac{2\xi'}{g\nu} \right)$$

Polaron limit ($\nu_\uparrow \gg \nu_\downarrow$)
Exact diagonalization

Spin-up orbitals

Spin-down orbitals
Exact diagonalization for 2:1 system

$S_{\uparrow}/S_{\downarrow} = 2$

2:1 instability

FFLO

Binding energy per critical species particle

$\frac{N_{\uparrow}}{N_{\downarrow}} \cdot \frac{S_{\downarrow}}{S_{\uparrow}}$
Analytical result for 2:1 system

\[ \frac{S_\uparrow}{S_\downarrow} = 2 \]

2:1 instability

FFLO

Binding energy per critical species particle

\[ \frac{N_\uparrow}{N_\downarrow} \frac{S_\downarrow}{S_\uparrow} \]
Exact diagonalization for S:1 systems
Three-particle superconducting correlations

$$\Delta_q = \left\langle c_{\uparrow k_1}^{\dagger} c_{\uparrow k_2}^{\dagger} c_{\downarrow q-k_1-k_2} \right\rangle$$

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