Electrons in a spin

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This cylinder is placed on a track as shown in the figure below. The track contains a downhill section which joins smoothly to a vertical circular loop of radius $R$, finishing with a horizontal section.

The cylinder is released from rest from a point at which its centre is at a height $h + a$ above the lowest point of the track. It rolls along the track without sliding, and
The double pendulum, as shown below, rotates at a fixed angular velocity $\Omega$. Write down the Lagrangian and hence the dynamical equations for this system.

Use this Lagrangian to show, for small $\theta$ and $\phi$, that the angular frequencies, $\omega_1$ and $\omega_2$, of the normal modes are given by:

$$\omega_{1,2}^2 = (2 \pm \sqrt{2}) \omega_0^2 - \Omega^2,$$

where $\omega_0 = \sqrt{g/l}$.

Find and sketch the normal modes. How will the double pendulum behave if $\Omega$ lies between $\omega_1$ and $\omega_2$?
Pt III physics

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Stars all interact and obey classical laws of motion
Dwarf galaxies suggest dark matter theory may be wrong

By Leila Battison
Science reporter, Bradford

Dwarf galaxies around the Milky Way are less dense than they should be if they held cold dark matter
Classical vs. quantum orbitals

- $10^{11}$ bodies all interact
- Follow circular orbits

- $10^{24}$ electrons in solids all interact and are quantum degenerate
- Follow complicated orbits
10^{24}-body physics

- Why is gold shiny, does not tarnish, gold-colored, a good conductor, and so malleable?

- Why does a superconductor expel magnetic fields and conduct electricity perfectly?
10^{24}-body physics

- How is the magnetic field generated?
- How does a hard disk drive store so much data?
Elastic bands consist of long polymer chains that interact strongly, but obey classical laws of motion.

**Natural state**

polymer chains in a state of high entropy

**Stretched**

polymer chains in a state of low entropy
Energy stored

- Potential energy stored in elastic band

\[ E = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{1}{2} \times 10 \times 0.1 = 0.5 \text{ J} \]
Energy stored

- Potential energy stored in elastic band

\[ E = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{1}{2} 10 \times 0.1 = 0.5 \text{ J} \]

- Kinetic energy in handgun bullet

\[ E = \frac{1}{2} mv^2 = \frac{1}{2} 0.005 \times 400^2 = 400 \text{ J} \]
Energy stored

- Potential energy stored in elastic band

\[ E = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{1}{2} 10 \times 0.1 = 0.5 \text{ J} \]

- Kinetic energy in handgun bullet

\[ E = \frac{1}{2} mv^2 = \frac{1}{2} 0.005 \times 400^2 = 400 \text{ J} \]

- Potential energy in enormous elastic band

\[ E = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{1}{2} 100 \times 10 = 500 \text{ J} \]
Effect of material thickness

Thin band: <10ms$^{-1}$

Cylindrical: >100ms$^{-1}$
Wave velocity

\[ c = \frac{\sqrt{gh}}{2} \left( \sqrt{\frac{\tanh kh}{kh}} + \sqrt{\frac{kh}{\tanh kh}} \right) \]

\[ k = \frac{2\pi}{\lambda} \]
How does an aircraft generate lift?

- Bernoulli's principle:
How does an aircraft generate lift?

- Bernoulli's principle:

  - Air is not incompressible
  - Violation of Newton's 3rd law
Coandă effect
Coandă effect
Coandă effect
Coandă effect
How does an aircraft generate lift?

Expels mass downwards so Newton's Law pushed plane upwards.
Upper surface of a wing generates lift
Quantum mechanics: electrons in solids

Atoms:
Quantum mechanics: electrons in solids

Atoms:

Orbitals:
Quantum mechanics: electrons in solids

Atoms:

Orbitals:

Ferro:
Quantum mechanics: electrons in solids

Atoms:

Orbitals:

Ferro:

Antiferro:
Antiferromagnetic mechanism

Heisenberg uncertainty principle

\[ \Delta p \Delta x = \hbar \]
\[ \Delta p = \frac{\hbar}{\Delta x} \]

\[ KE = \frac{\Delta p^2}{2m} = \frac{\hbar^2}{2m \Delta x^2} \]

Spreading lowers energy

\[ \Delta x \]
Antiferromagnetic mechanism

Pauli blocked
Antiferromagnetic mechanism

Pauli blocked

Exchange lowers energy

Opposite spins repel
Antiferromagnetic mechanism

Pauli blocked

Exchange lowers energy

Opposite spins repel

Antiferro
Ferromagnetism

Ferro spins
Ferromagnetism

Ferro spins

Opposite spins repel

Antiferro spins
Ferromagnetism

Ferro spins

Opposite spins repel

Antiferro spins

Ferro
Magnetic data storage

10½-inch tape (1953) 5 MB
Magnetic data storage

10½-inch tape (1953) 5 MB

8-inch Floppy disk (1973) 250¼ kB
Magnetic data storage

Cassette (1979) 660 KB
Magnetic data storage

Cassette (1979) 660 KB

5¼-inch Floppy disk (1983) 1200 KB
Magnetic data storage

3½-inch Floppy disk (1989) 1.44 MB

Hard disk drive (2010) 2 TB
Storage over the years

Full History Disk Areal Density Trend

Gb/sq. in

Year of Production

Giant-magnetoresistance (1988)

Areal Density Trend

Albert Fert  Peter Grünberg

Gb/sq. in

Year of Production

Magnetic data storage
Carrier velocity due to fields

\[ v = M (E + v \times B) \]

\[ v = \frac{M}{1 + (M B)^2} (E + E \times B) \]

which is reduced by 5% in typical metals so need large magnetic domains → low data density
Giant & colossal magnetoresistance

Change in resistance is up to 100000% so can have very small magnetic domains and high data storage density

Diagram:
- **1**:
  - The magnetic domains are aligned in the same direction.
  - The resistance is lower.
- **0**:
  - The magnetic domains are aligned in opposite directions.
  - The resistance is higher.
Summary: *more is different*

- Particles obeying well understood microscopic physics display important collective motion – *more is different*

- Many-body interactions coupled with quantum mechanics leads to new counterintuitive phenomena

- Real-life applications:
  - Electronics
  - Material science
  - Chemistry