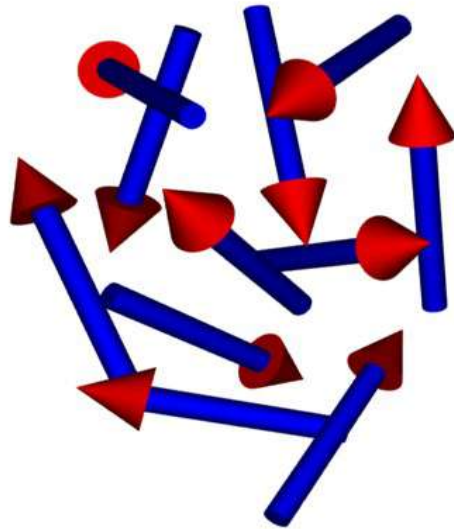
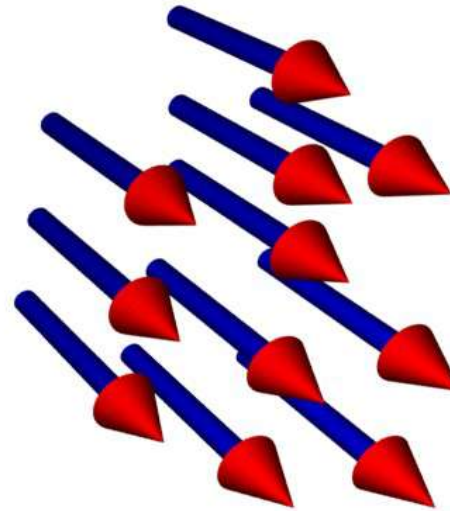


A repulsive atomic gas on the border of itinerant ferromagnetism

Weak interactions



Strong interactions



Gareth Conduit^{1, 2} & **Ben Simons**³

1. Weizmann Institute of Science, 2. Ben Gurion University, 3. University of Cambridge

G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)

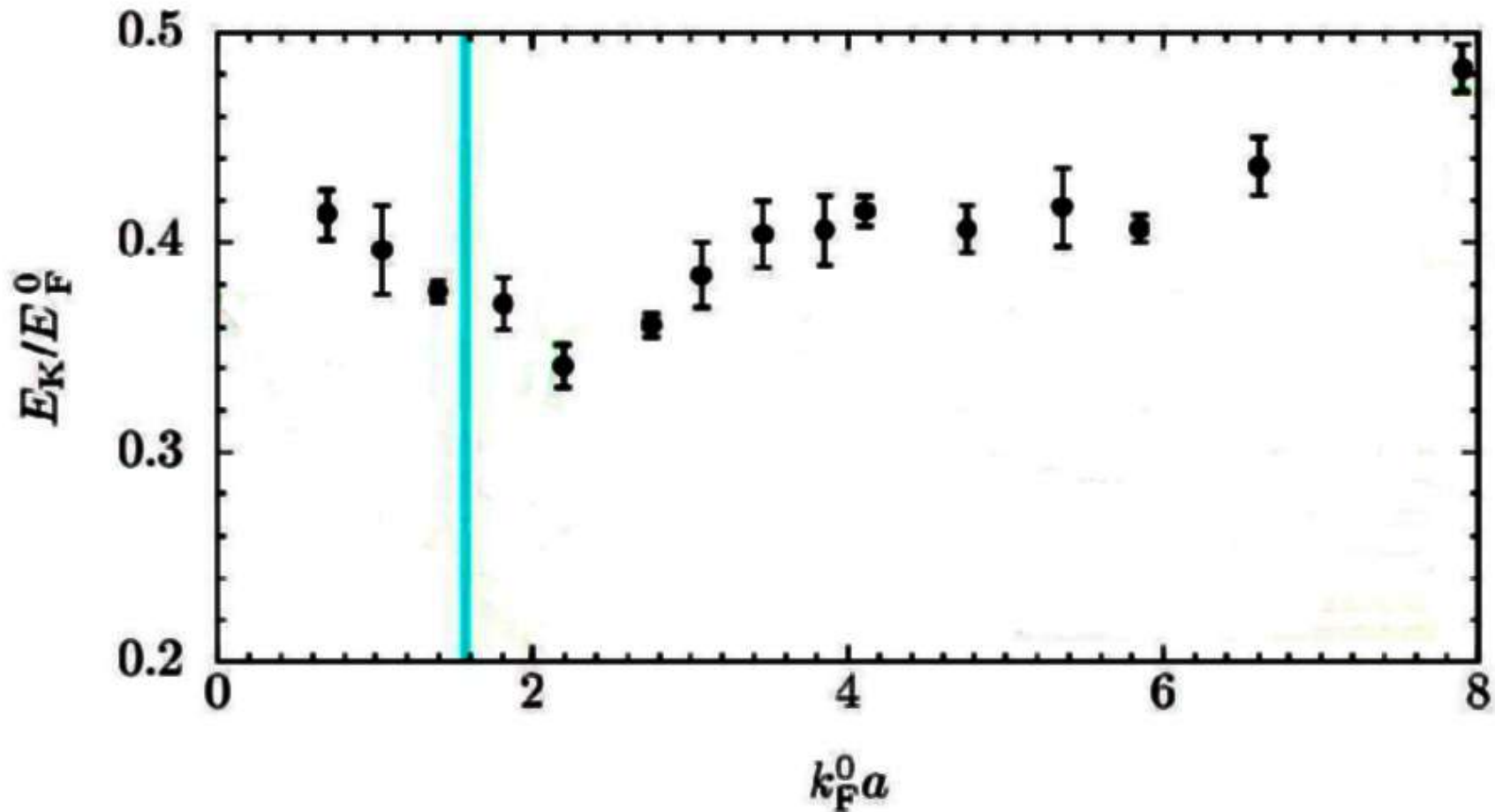
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)

G.J Conduit & E. Altman, arXiv: 0911.2839

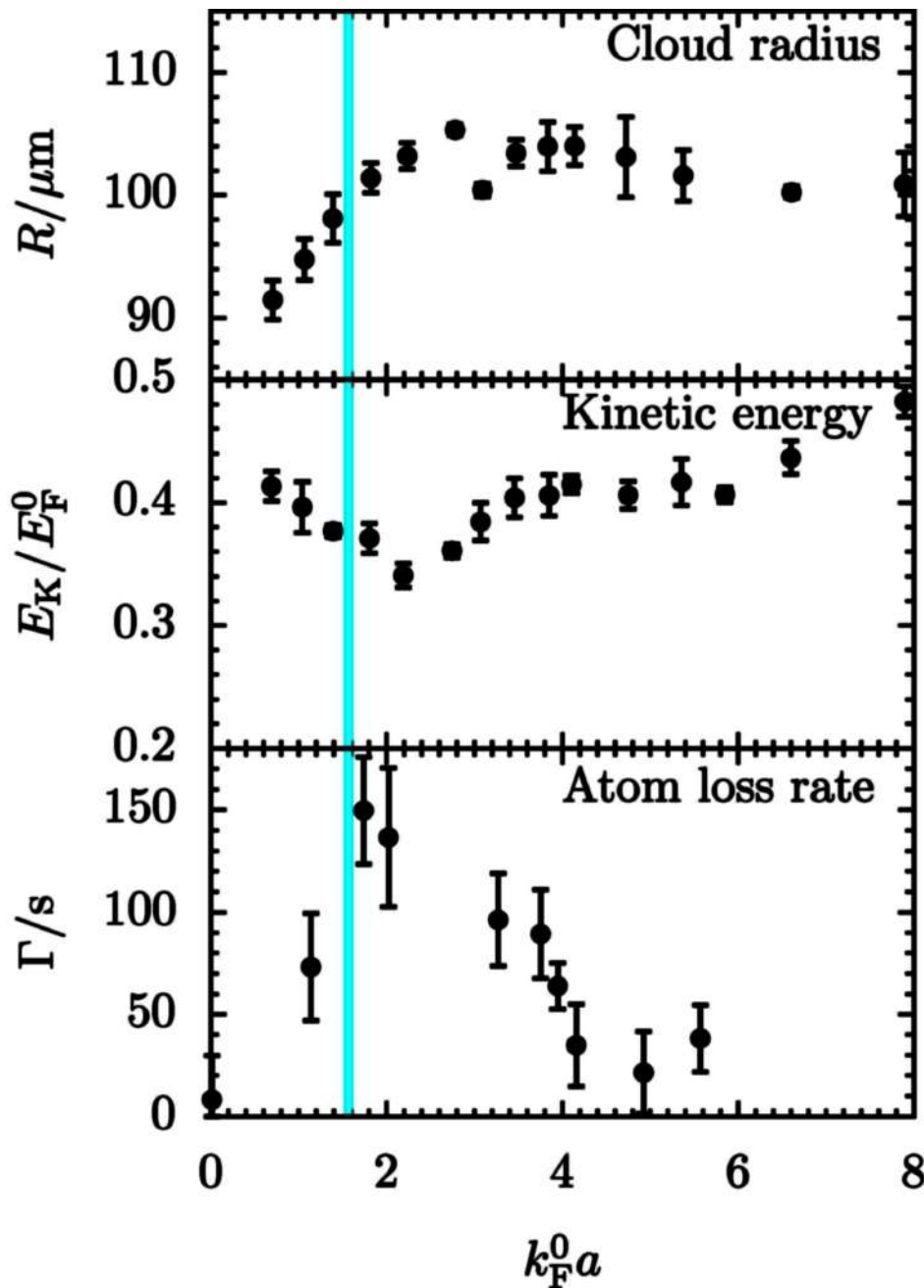
Experimental evidence for ferromagnetism

- Minimum in kinetic energy at $k_F a \approx 2.2$



Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Further key experimental signatures



$$E_K \propto n^{5/3}$$

$$\Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$$

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Equilibrium study of ferromagnetism

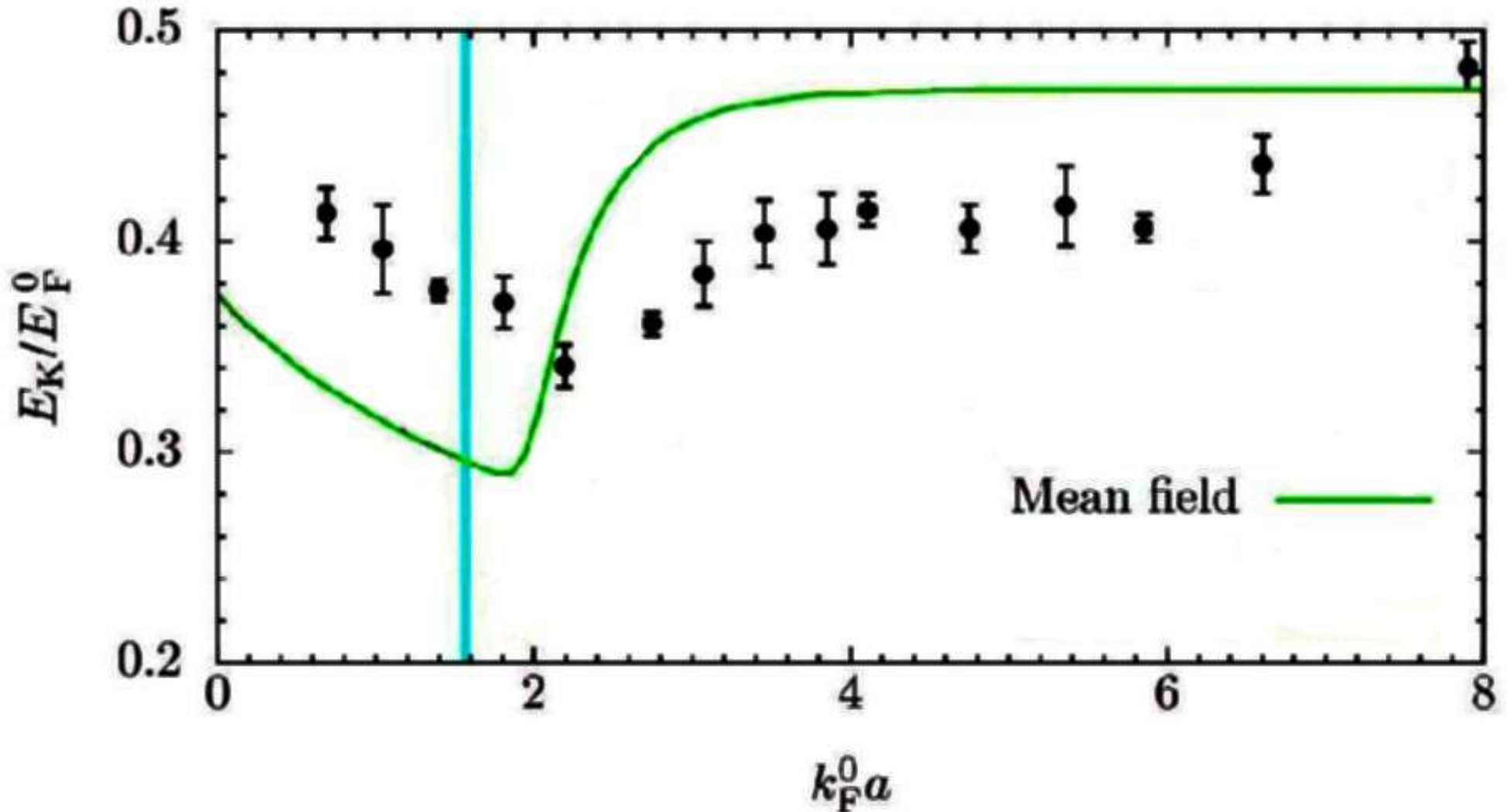
$$Z = \int D\psi \exp\left(-\iint d\tau d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma} (-i\hat{\omega} + \hat{\epsilon} - \mu) \psi_{\sigma} - g \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow}\right)$$

- Decouple with the average magnetisation m gives the Stoner criterion

$$F = F_0 + \frac{1 - g\nu}{2\nu} m^2 + um^4 + vm^6$$

Mean-field analysis & consequences of trap

- Correct qualitative behavior¹ but transition at $k_F a \approx 1.8$ instead of $k_F a \approx 2.2$



¹LeBlanc, Thywissen, Burkov & Paramakanti, Phys. Rev. A **80**, 013607 (2009) & Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Fluctuation corrections

$$Z = \int D\psi \exp\left(-\iint d\tau d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma} (-i\hat{\omega} + \hat{\epsilon} - \mu) \psi_{\sigma} - g \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow}\right)$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

$$F = F_0 + \frac{1-g\nu}{2\nu} m^2 + um^4 + vm^6 + g^2 (r m^2 + w m^4 \ln|m|) \quad k_F a_{\text{crit}} = 1.05$$

- First order transition¹

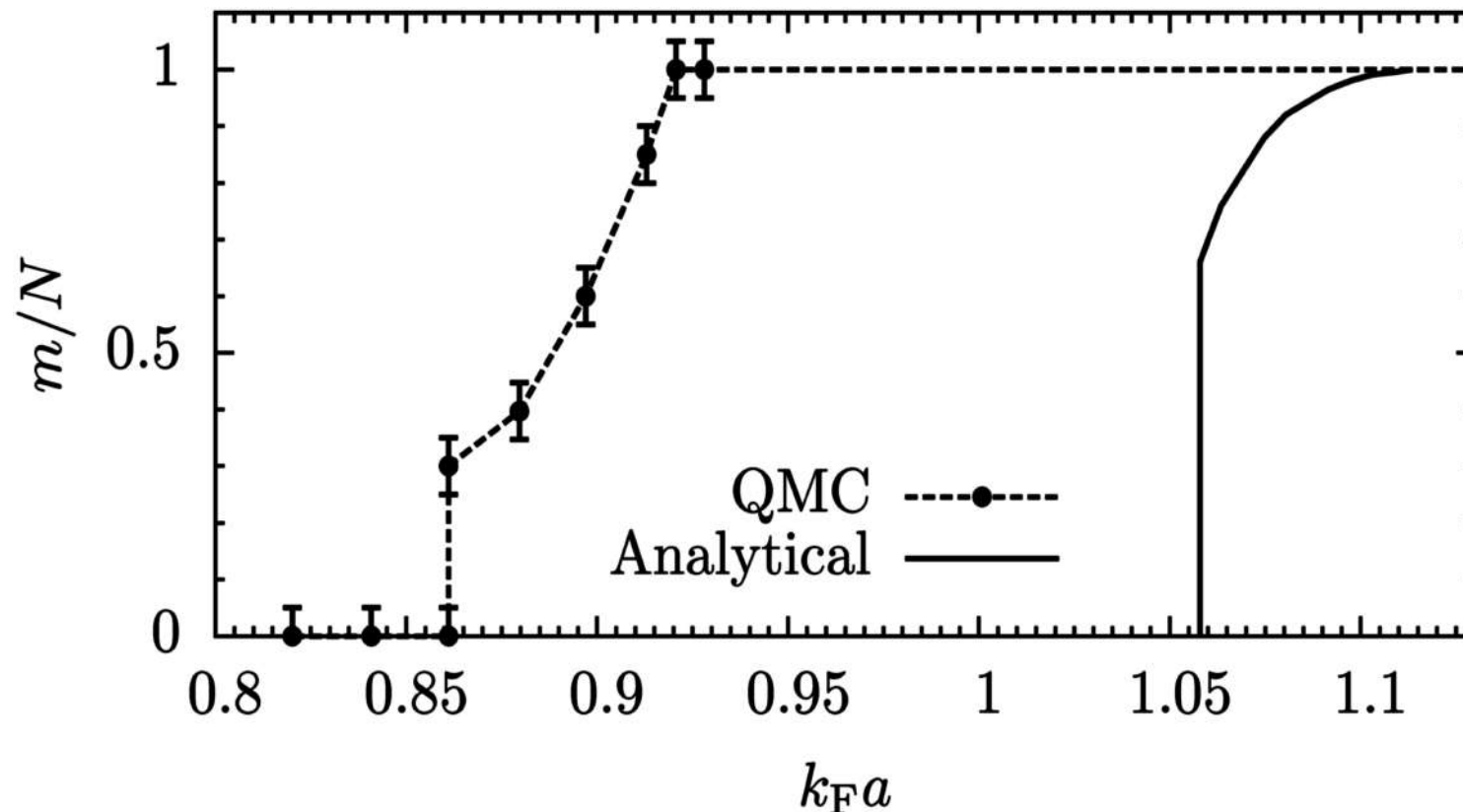
¹Abrikosov (1958), Duine & MacDonald (2005), Belitz *et al.* (1997) & Conduit & Simons (2009)

Quantum Monte Carlo

$$F = F_0 + \frac{1-gv}{2v} m^2 + um^4 + vm^6 + g^2 (r m^2 + w m^4 \ln|m|)$$

$$k_F a_{\text{crit}} = 1.05$$

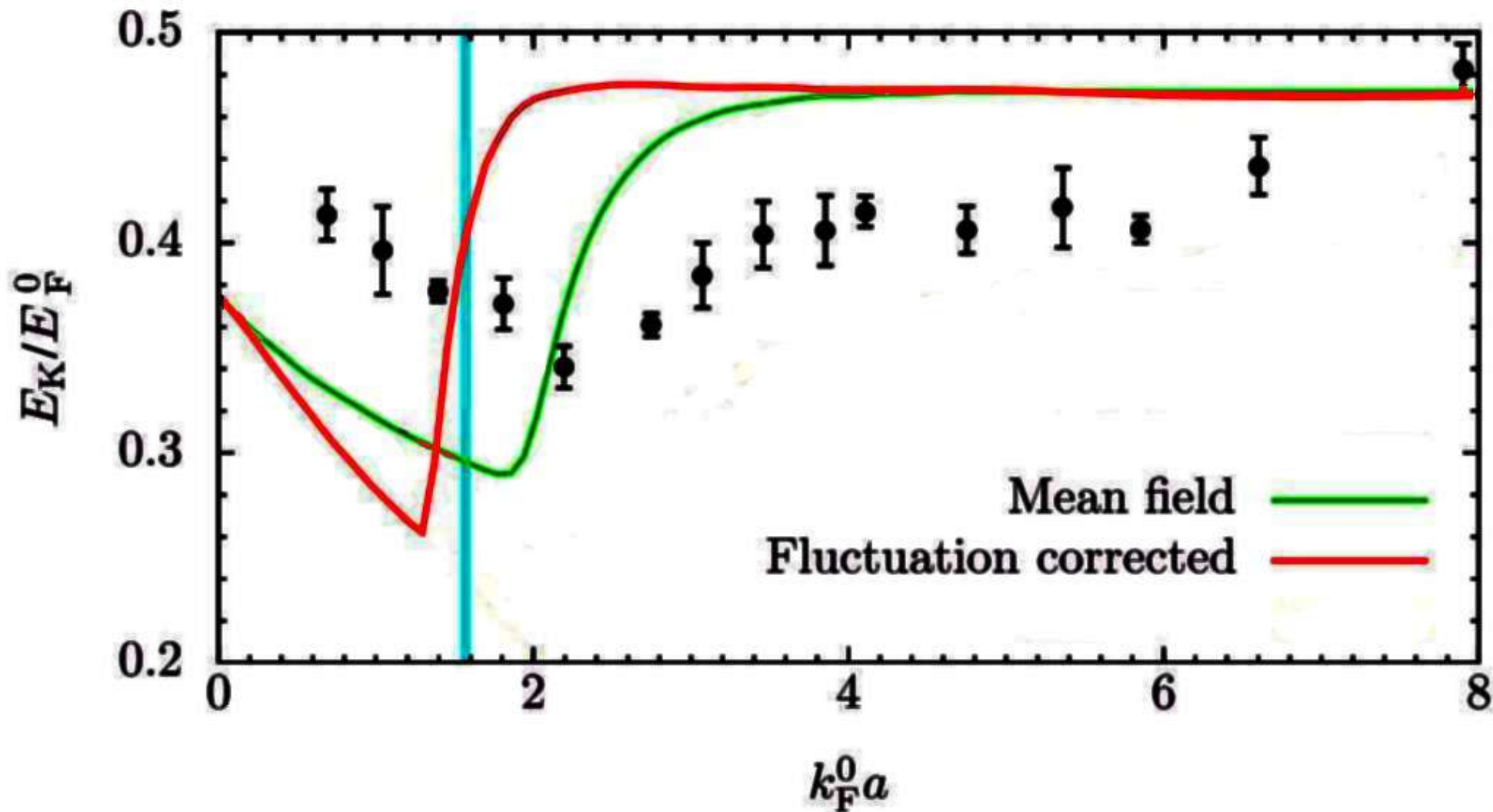
- First order transition verified with *ab initio* Quantum Monte Carlo²



¹Conduit & Simons, Phys. Rev. Lett. **103**, 207201 (2009)

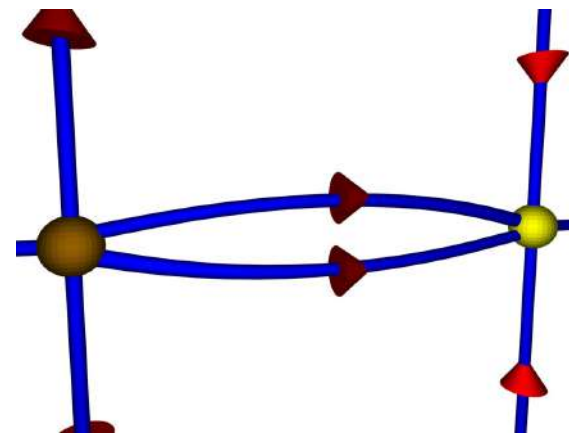
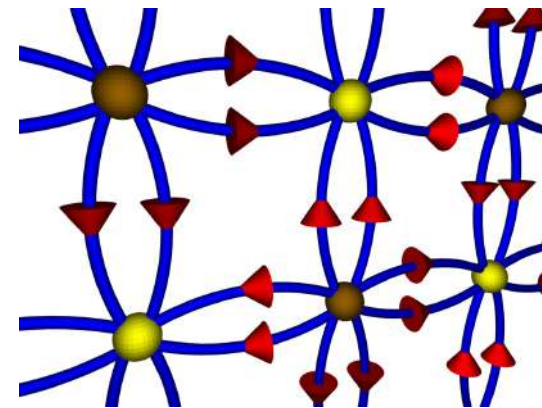
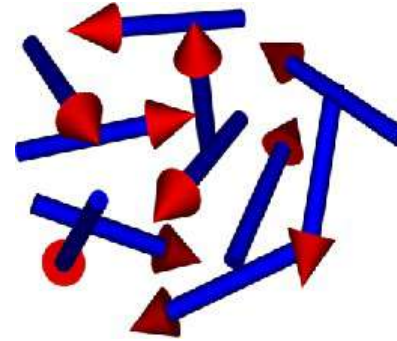
Fluctuation corrections in trap

- Extend theory through fluctuation corrections



Condensation of topological defects

Ramp up interactions



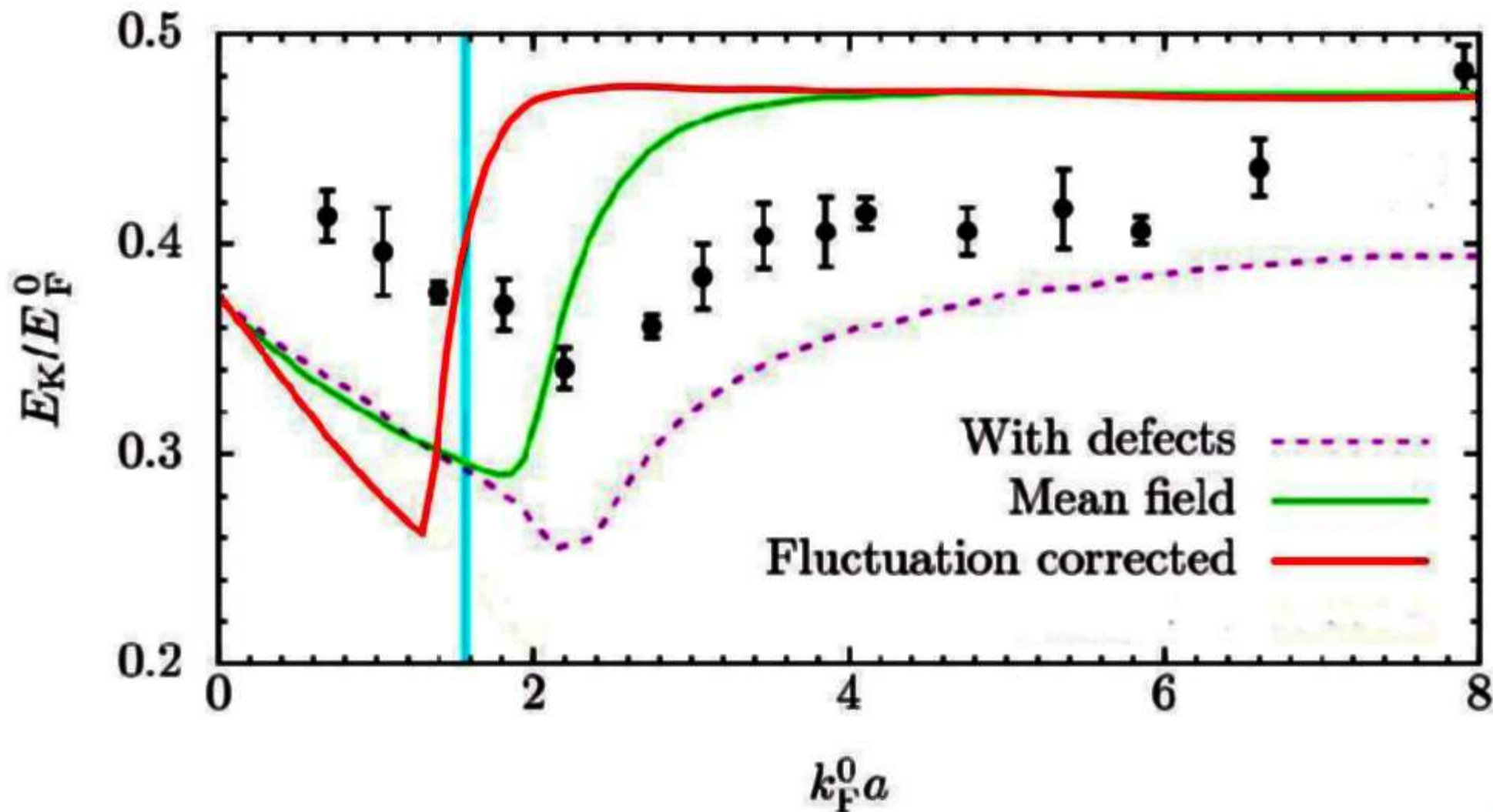
Mutual annihilation of defects

- Defects freeze out from paramagnetic state

- Defects grow as $L \sim t^{1/2}$
[Bray, Adv. Phys. **43**, 357 (1994)]

Consequences of defect annihilation

- Defect annihilation raises required interaction strength

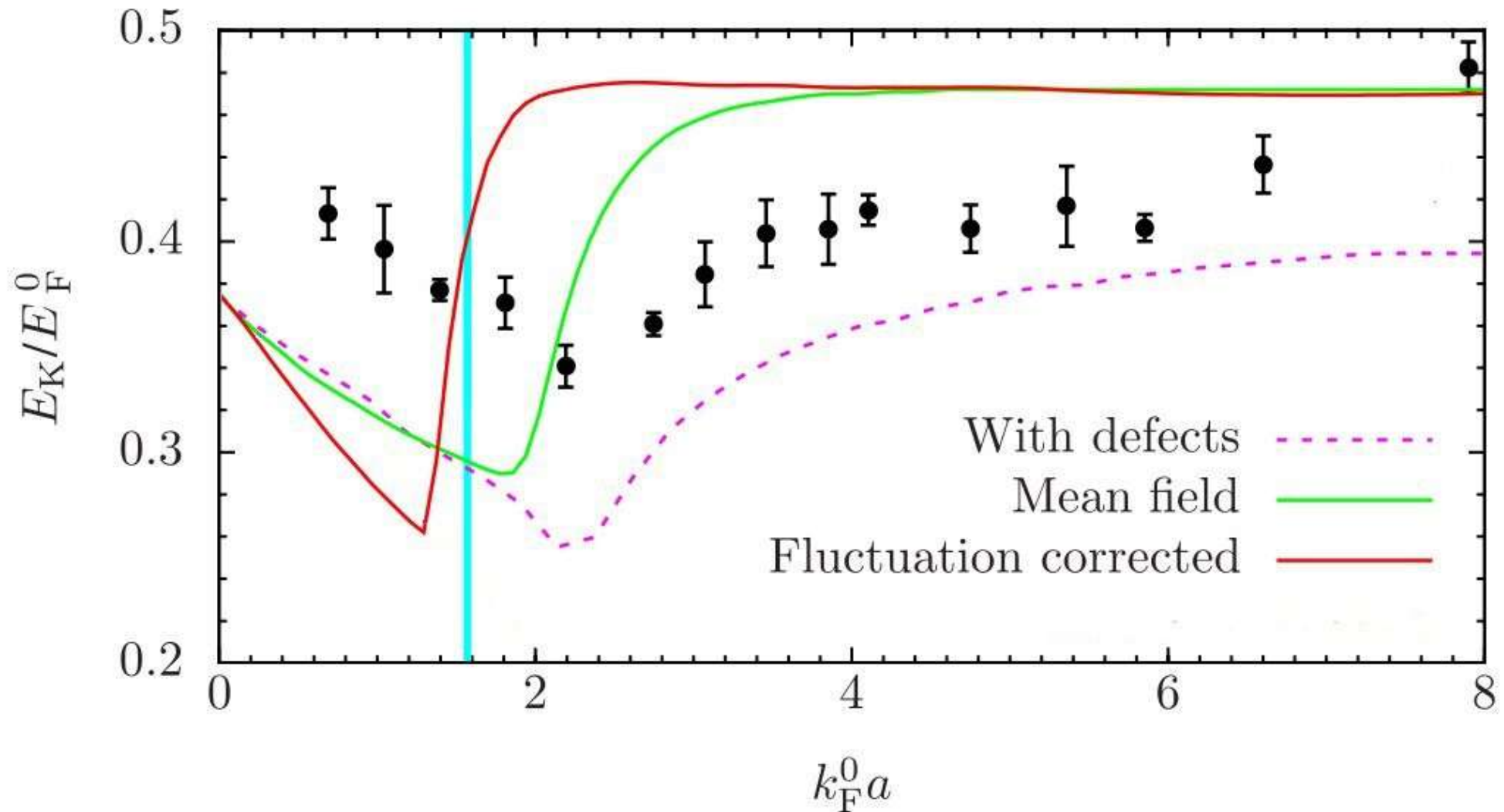


Summary

- Mean-field theory provides a reasonable qualitative description of the transition
- Fluctuation corrections drive the transition first order, backed up by Quantum Monte Carlo calculations
- Discrepancy in the interaction strength could be accounted for by the non-equilibrium formation of the ferromagnetic phase
- Atom loss can damp fluctuations and renormalize the interaction strength upwards [Y3100014]

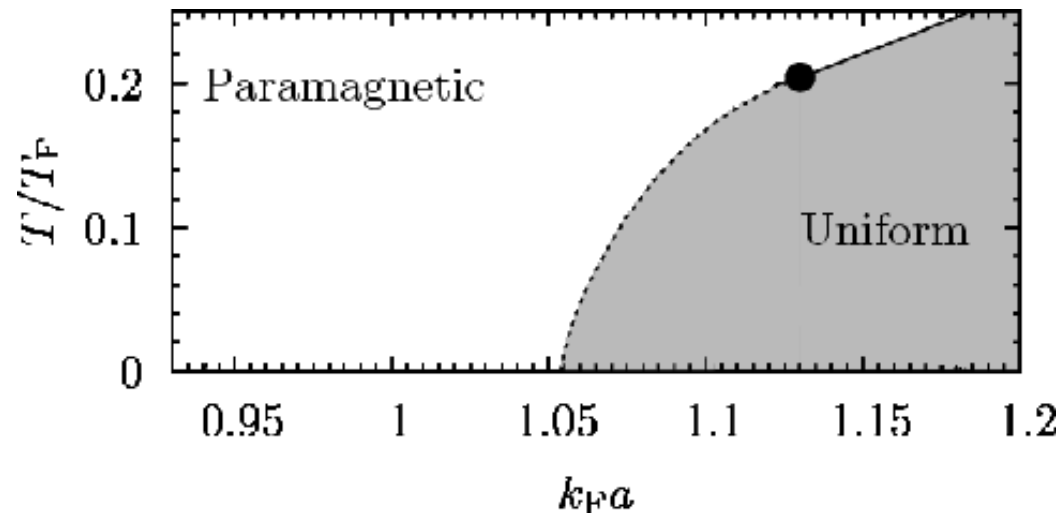
Condensation of topological defects

- Condensation of defects inhibits the transition

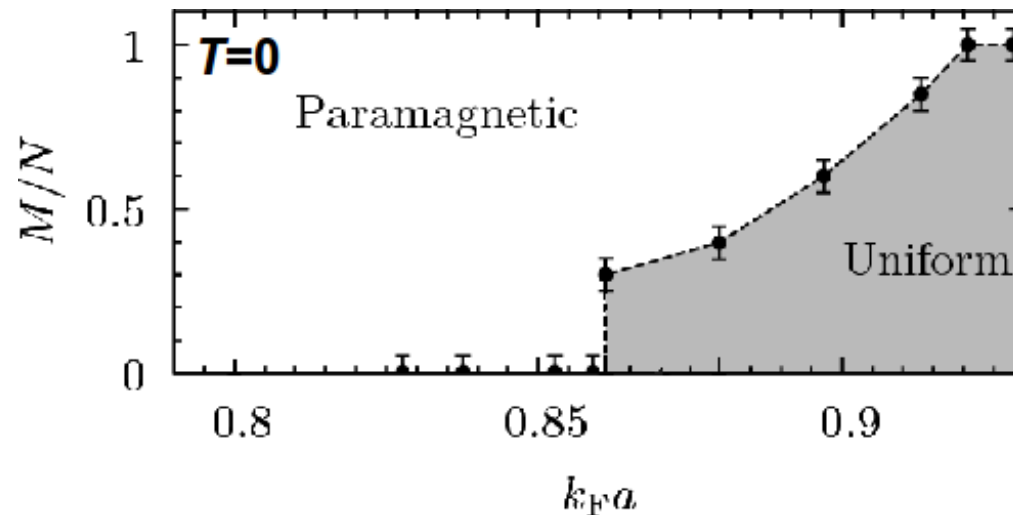


First order phase transition and Quantum Monte Carlo verification

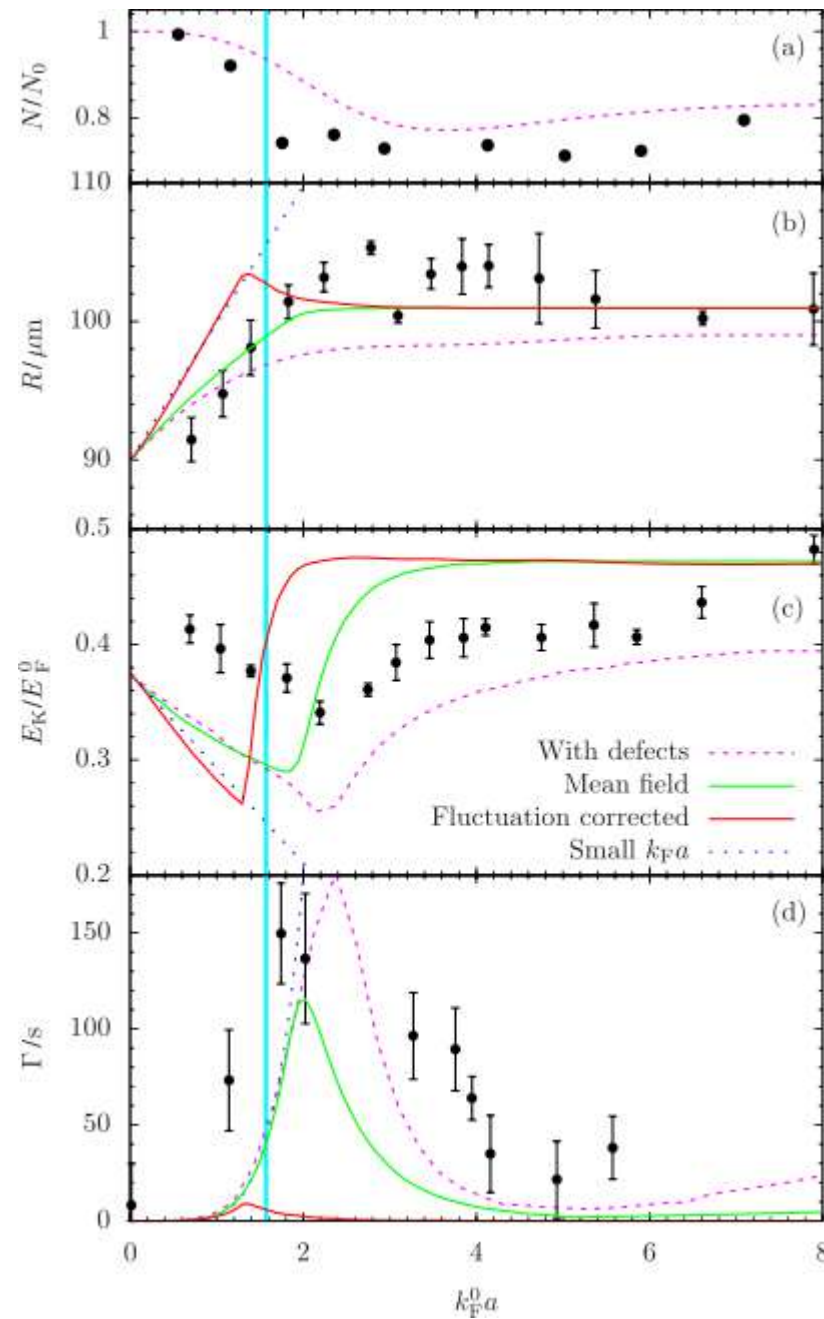
- First order transition into uniform phase with TCP



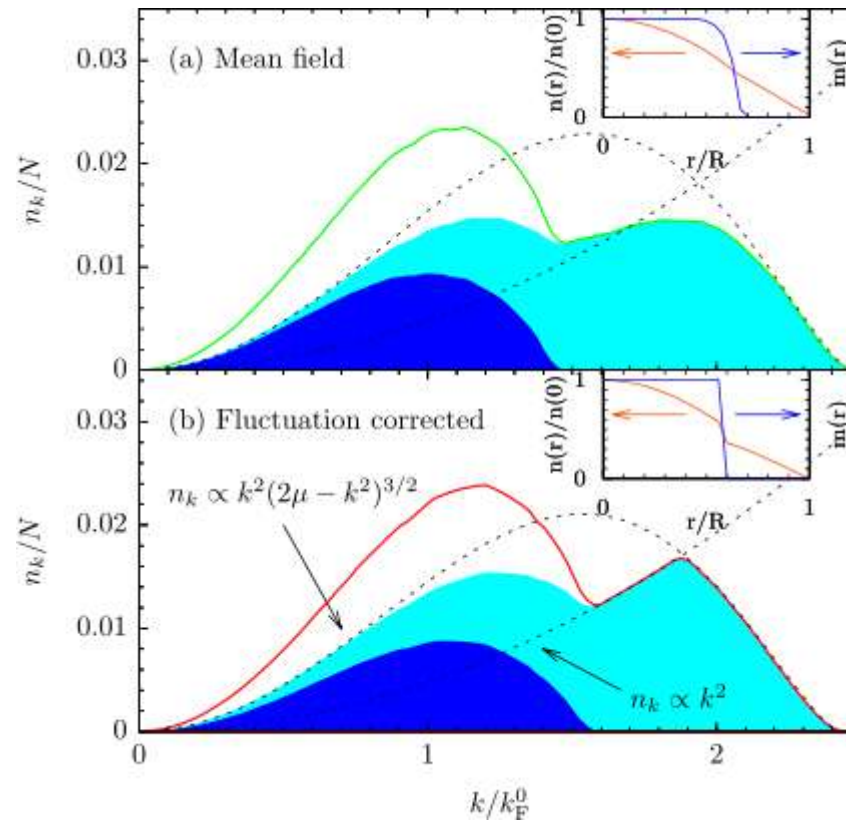
- QMC also sees first order transition



Summary of equilibrium results



Momentum distribution



New approach to fluctuation corrections

$$Z = \int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma} (-i\omega + \epsilon - \mu) \psi_{\sigma} - g \int \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow}\right)$$

- Analytic strategy:
 - 1) Decouple in both the density and spin channels (previous approaches employ only spin)
 - 2) Integrate out electrons
 - 3) Expand about uniform magnetisation
 - 4) Expand density and magnetisation fluctuations to second order
 - 5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy $F = -k_B T \ln Z$ is found via the partition function

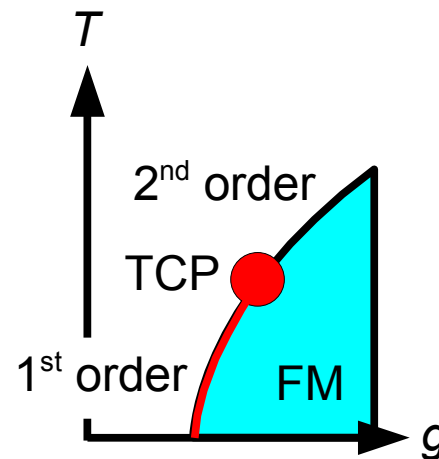
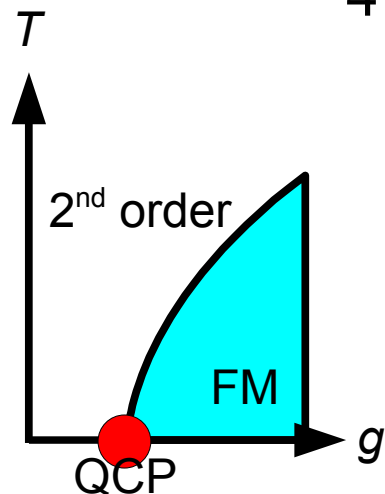
$$Z = \int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma} (-i\omega + \epsilon - \mu) \psi_{\sigma} - g \int \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow}\right)$$

- Decouple using only the average magnetisation $m = \bar{\psi}_{\downarrow} \psi_{\uparrow} - \bar{\psi}_{\uparrow} \psi_{\downarrow}$

gives $F \propto (1 - gv)m^2$ i.e. the Stoner criterion

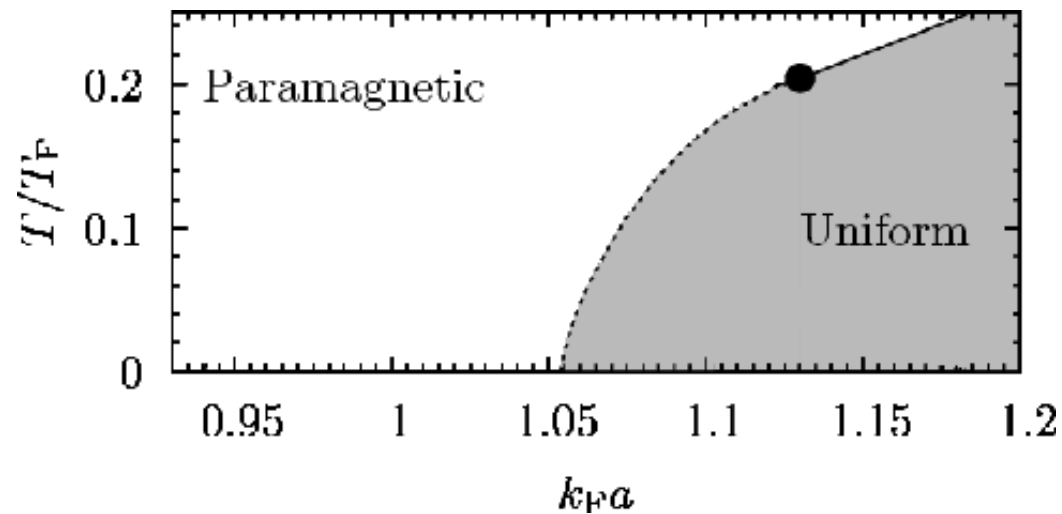
- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

$$F = \frac{1}{2} \left(|\omega| / \Gamma_q + r + q^2 \right) m^2 + \frac{u}{4} m^4 \ln(m^2 + T^2) + \dots - hm$$

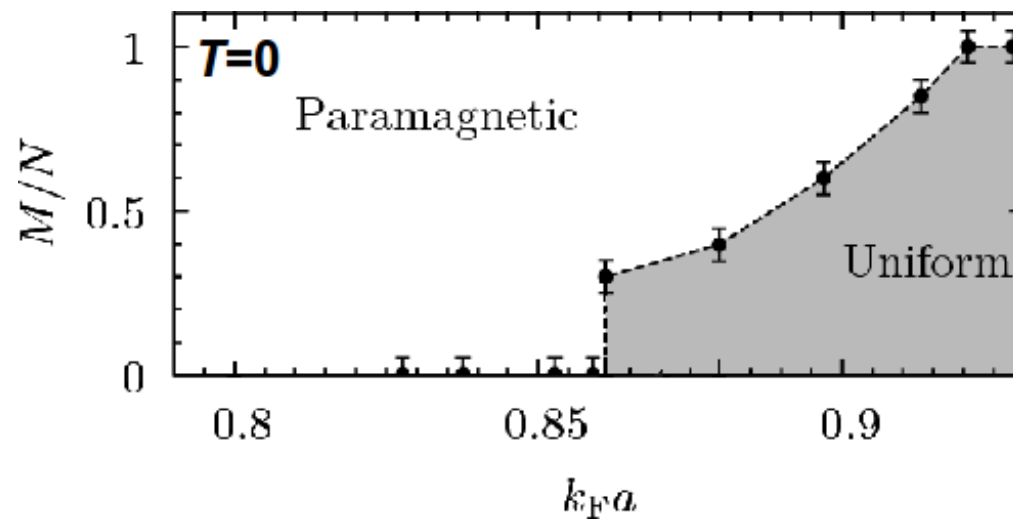


Quantum Monte Carlo verification

- First order transition into uniform phase with TCP



- QMC also sees first order transition



Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

${}^6\text{Li}$ $m_F=1/2$ maps to spin 1/2

${}^6\text{Li}$ $m_F=-1/2$ maps to spin -1/2

- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

$|\uparrow\uparrow\rangle$ $S=1, S_z=1$ State not possible as S_z has changed

$|\downarrow\downarrow\rangle$ $S=1, S_z=-1$ State not possible as S_z has changed

$(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ $S=1, S_z=0$ Magnetic moment in plane

$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $S=0, S_z=0$ Non-magnetic state

- Ferromagnetism, if favourable, must form in-plane

Particle-hole perspective

- To second order in g the free energy is

$$F = \sum_{\sigma, k} \epsilon_k^\sigma n(\epsilon_k^\sigma) + g N^\uparrow N^\downarrow$$

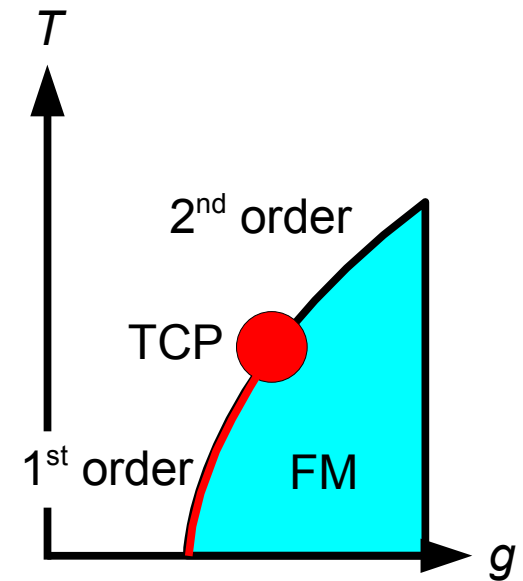
$$- \frac{2g^2}{V^3} \sum_p \int \int \frac{\rho^\uparrow(\mathbf{p}, \epsilon_\uparrow) \rho^\downarrow(-\mathbf{p}, \epsilon_\downarrow)}{\epsilon_\uparrow + \epsilon_\downarrow} d\epsilon_\uparrow d\epsilon_\downarrow$$

$$+ \frac{2g^2}{V^3} \sum_{k_{1,2,3,4}} \frac{n(\epsilon_{k_1}^\uparrow) n(\epsilon_{k_2}^\downarrow)}{\epsilon_{k_1}^\uparrow + \epsilon_{k_2}^\downarrow - \epsilon_{k_3}^\uparrow - \epsilon_{k_4}^\downarrow} \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)$$

with $\epsilon_k^\sigma = \epsilon_k + \sigma gm$ and a particle-hole density of states

$$\rho^\sigma(\mathbf{p}, \epsilon) = \sum_k n(\epsilon_{k+p/2}^\sigma) [1 - n(\epsilon_{k-p/2}^\sigma)] \delta(\epsilon - \epsilon_{k+p/2}^\sigma + \epsilon_{k-p/2}^\sigma)$$

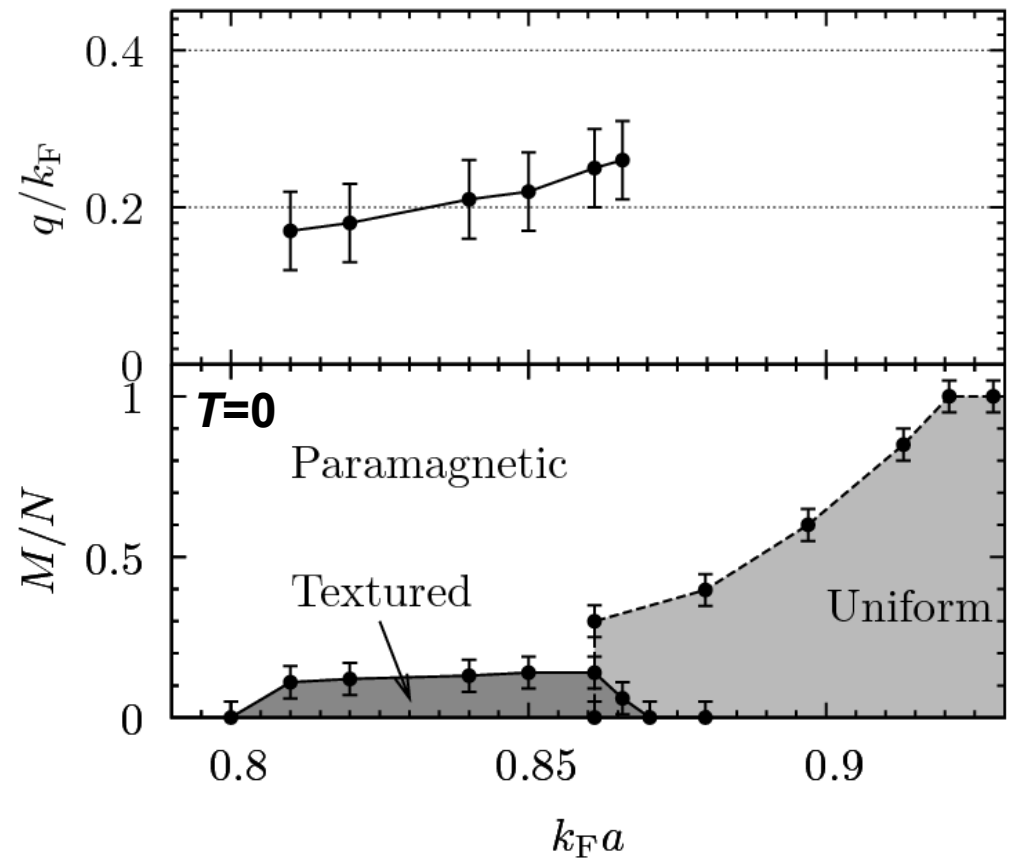
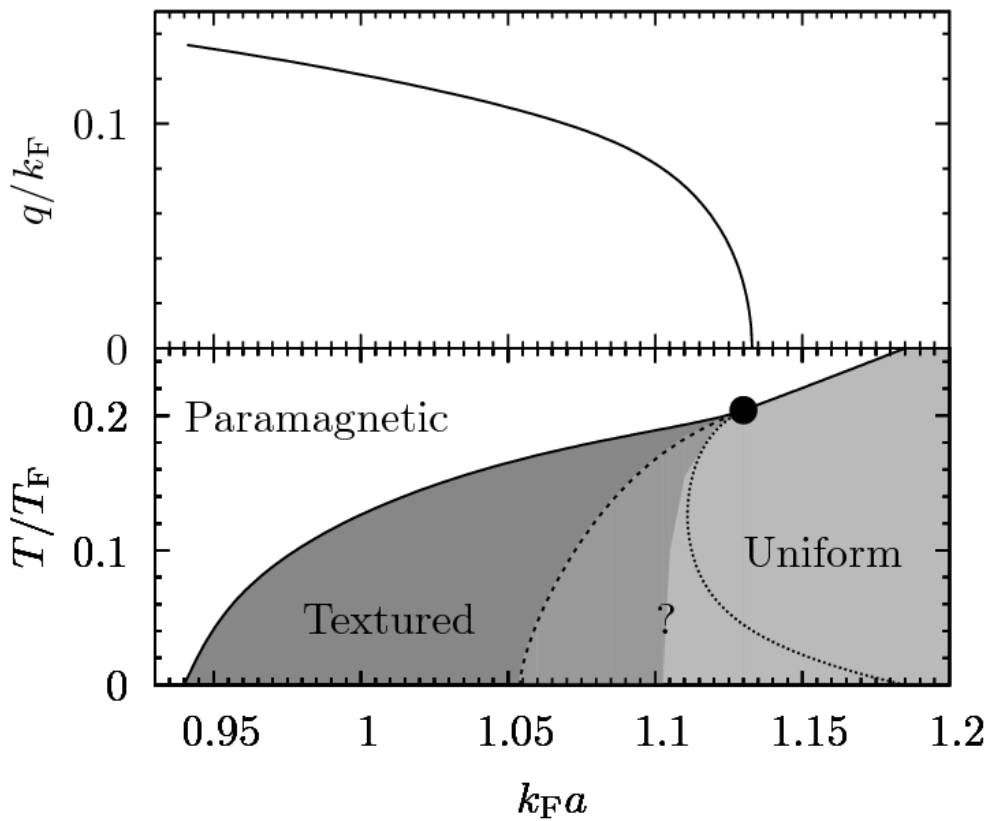
- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^4 \ln m^2$ at $T=0$
- Links quantum fluctuation to second order perturbation approach¹



¹Abrikosov 1958 & Duine & MacDonald 2005

Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with $q=0.2k_F$



Modified collective modes

- Collective mode dispersion

$$\Omega = \frac{q^2}{2} \left(1 - \frac{2^{5/3} 3}{5k_F a} \frac{1}{1 + \tilde{\lambda}^2 / (k_F a)^2} \right)$$

- Collective mode damping

$$\Gamma = \frac{q^2}{2} \frac{2^{5/3} 3 \tilde{\lambda}}{5(k_F a)^2} \frac{1}{1 + \tilde{\lambda}^2 / (k_F a)^2}$$