Fluctuation Corrections to Landau Mean-Field theory

2.7 Comparison of Theory and Experiment

The validity of the mean-field approximation is assessed in the table below by comparing the results with (approximate) exponents for $d = 3$ from experiment.

Transition type	Material	α	β	γ	ν
		$C \sim \lvert t\rvert^{-\alpha}$	$\langle m\rangle \sim \lvert t\rvert^{\beta}$	$\chi \sim \lvert t\rvert^{-\gamma}$	$\xi \sim \lvert t\rvert^{-\nu}$
Ferromag. ($n = 3$)	Fe, Ni	-0.1	0.34	1.4	0.7
Superfluid ($n = 2$)	He^4	0	0.3	1.3	0.7
Liquid-gas ($n = 1$)	CO_2, Xe	0.11	0.32	1.24	0.63
Superconductors		0	1/2	1	1/2
Mean-field		0	1/2	1	1/2

Recall:

$$\beta H = \beta H[\bar{m}] + \frac{1}{2}\int d^d\underline{x}\ \varphi_l K(-\nabla^2 + \xi^{-2})\varphi_l + \text{“}\varphi_t \text{ terms”}$$

$$\left[\frac{t}{2}\bar{m}^2 + u\bar{m}^4\right]V + \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\ \varphi(q) K(q^2 + \xi^{-2})\varphi(-q)$$

Free energy

$$f \equiv \frac{\beta F}{V} = -\frac{\log Z}{V} = -\frac{1}{V}\log\left(e^{-\beta H[\bar{m}]} \det G^{\frac{1}{2}}\right)$$

$$\log \det G^{\frac{1}{2}} = -\frac{1}{2}\operatorname{tr}\log G^{-1}$$

$$\frac{1}{V}\operatorname{tr} = \int \frac{d^d q}{(2\pi)^d}$$

$$f = \frac{\beta H[\bar{m}]}{V} + \frac{1}{2}\int \frac{d^d q}{(2\pi)^d} \log\left[K\left(q^2 + \xi_L^{-2}\right)\right]$$

$$+ \frac{1}{2}(n-1)\int \frac{d^d q}{(2\pi)^d} \log\left(K\left(q^2 + \xi_t^{-2}\right)\right)$$

$$K\xi_\ell^{-2} = \begin{cases} t & t>0 \\ -2t & t<0 \end{cases} \quad ; \quad K\xi_t^{-2} = \begin{cases} t & t>0 \\ 0 & t<0 \end{cases}$$

We focus on the heat capacity

$$C_{sing} \sim -\frac{\partial^2 f}{\partial t^2} = \begin{cases} 0 + \frac{n}{2}\int \frac{d^d q}{(2\pi)^d} \frac{1}{(Kq^2+t)^2} & t>0 \\ \frac{1}{8u} + 2\int \frac{d^d q}{(2\pi)^d} \frac{1}{(Kq^2-2t)^2} & t<0 \end{cases}$$

only the longitudinal part contributes

Fluctuation corrections

$$\delta C \sim \frac{1}{K^2}\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2+\xi^{-2})^2} \sim \begin{cases} \frac{1}{K^2} a^{4-d} & d \geq 4 \\ \frac{1}{K^2} \xi^{4-d} & d<4 \end{cases}$$

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2+\xi^{-2})^2} \sim \int_0^{\frac{1}{a}} q^{d-1} dq \frac{1}{\xi^{-4}(\xi^2 q^2+1)^2}$$

$$\sim \xi^{4-d} \int_0^{\frac{1}{a}} \xi^{d-1} q^{d-1} \, d(\xi q) \frac{1}{((\xi q)^2+1)^2}$$

$$\int dx \frac{x^{d-1}}{(x^2+1)^2}$$

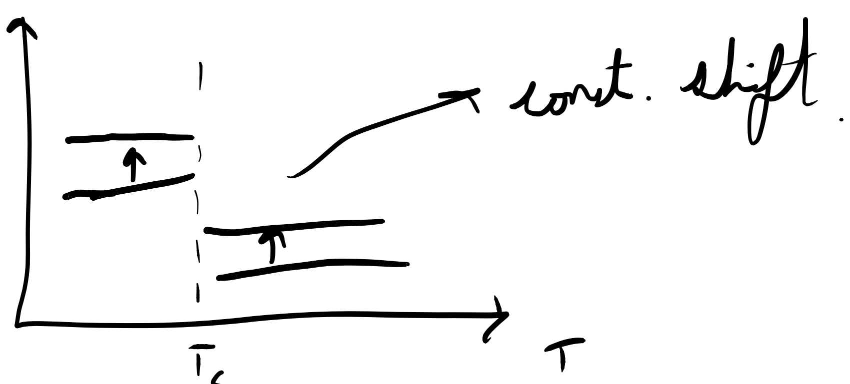
For large $x \rightarrow \frac{x^{d-1}}{x^4} = x^{d-5}$ $\quad d \geqslant 4$

diverge at the upper limit

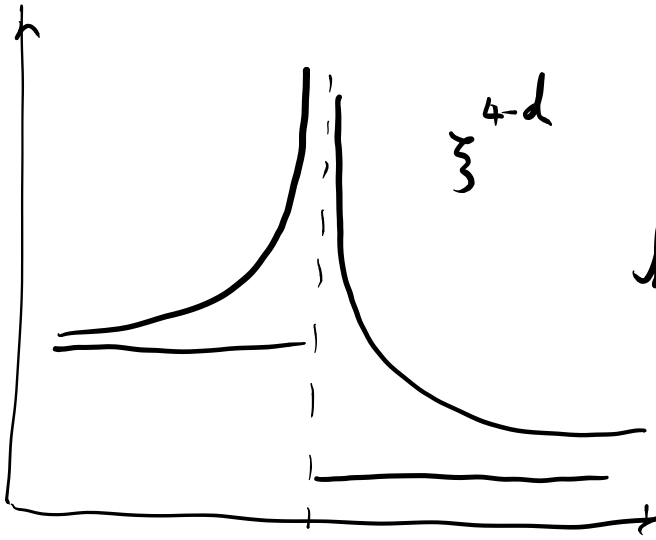
$\Rightarrow$ impose a cut-off at $\frac{1}{a}$.

For small $x \longrightarrow x^{d-1}$

$d \geqslant 4$



$d < 4$



discontinuity has been replaced by a divergence

i.e. saddle point correction is <u>not</u> small.

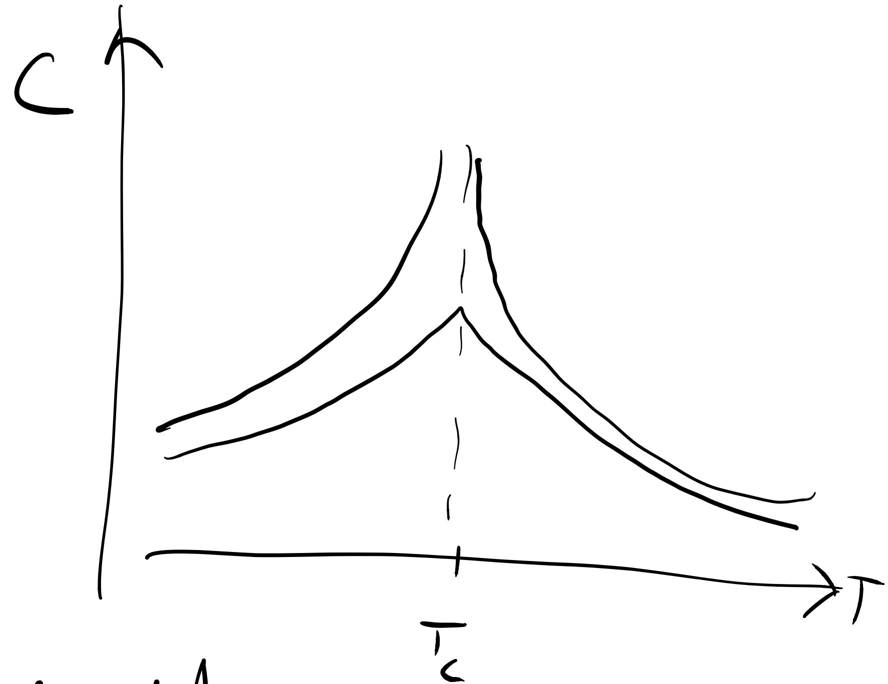
In $d < 4$, there is a breakdown of MFT (it applies to χ, m, ...). Simple perturbation theory fails.

$d = 4$ is known as the upper critical dimension

$d \geq 4$ MFT exponents are correct.

· But what about superconductors?

Ginzburg criterion



To see fluctuation dominated behavior, we must have experimental resolution around T_c.

$$\delta C > \Delta C \nearrow \text{saddle point jump}$$

$$\frac{1}{K^2}\xi^{4-d} \sim \frac{1}{K^2}\left(\sqrt{K}\, t^{-\frac{1}{2}}\right)^{4-d} \simeq \Delta C$$

$$\Rightarrow \quad \xi_0^{4-d}\, t_G^{-\frac{1}{2}(4-d)} \triangleq \Delta C\, \xi_0^{4}$$

$$t \ll t_G \sim \left(\frac{k_B a^d}{\Delta C\, \xi_0^d}\right)^{\frac{2}{4-d}}$$

Typically $\quad \Delta C \sim 1 k_B$ for particle

$\xi_0 \sim$ range of interaction

e.g. superfluid $\quad \xi_0 \sim$ thermal wavelength $\sim \lambda \sim 2-3$ Å

$\rightsquigarrow t_G \sim 10^{-1} - 10^{-2}$ accessible

superconductor $\quad \xi_0 \sim$ size of Cooper pair $\sim 10^3$ Å

$t_G \sim 10^{-18}$! — out of experimental reach.