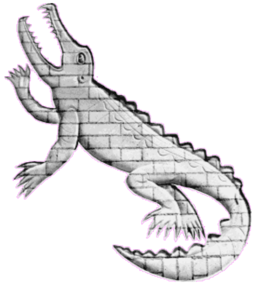

Conductivity of Granular Metals

Y. L. Loh, V. Tripathi, and M. Turlakov
Cavendish Laboratory, Cambridge, UK

UNIVERSITY OF CAMBRIDGE

Analytical and numerical study of AES model without disorder: full Kubo conductivity, no mean-field approximation

- *Cancellation between T^2 contributions leaving $e^{-E^*/T}$*
- *Numerical estimates of effective charging energies E^* and E^{**}*
- *Discussion of disorder effects*

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Outline

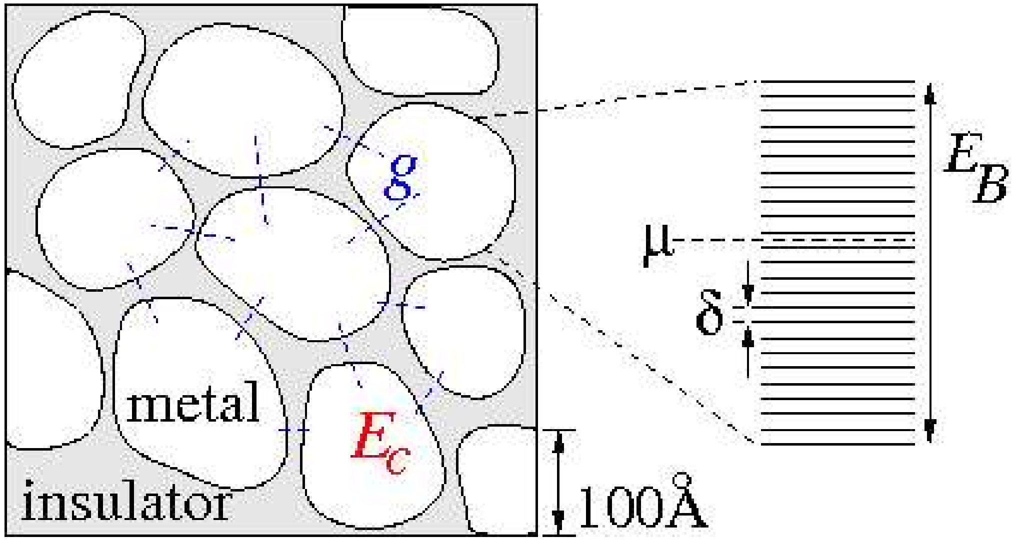
1. Granular metals
2. Experimental temperature dependence of conductivity
3. AES model
4. Regular arrays
5. Disordered arrays – hypotheses

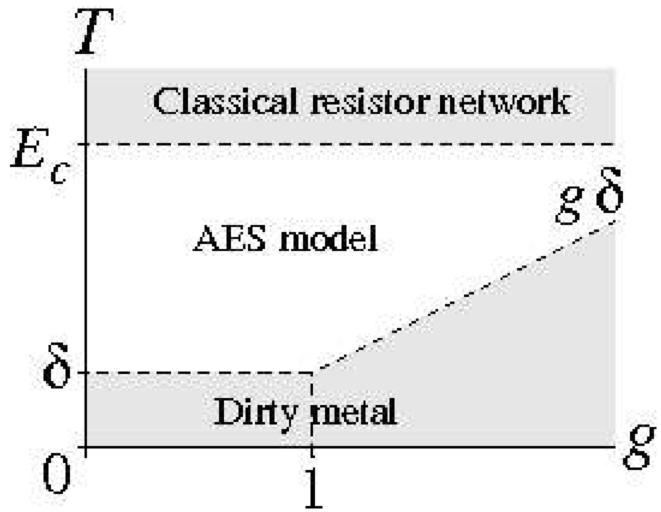
Granular metals

- Isolated metallic grains in insulating matrix
- Tunnelling vs Coulomb blockade

$$H = \underbrace{\sum_{\mathbf{x}\lambda} \xi_{\mathbf{x}\lambda} c^\dagger_{\mathbf{x}\lambda} c_{\mathbf{x}\lambda}}_{\text{electron KE}} + \underbrace{\sum_{\mathbf{xy}\lambda\mu} t_{\mathbf{xy}} c^\dagger_{\mathbf{x}\lambda} c_{\mathbf{y}\mu}}_{\text{tunnelling}} + \underbrace{\sum_{\mathbf{x}} \frac{1}{2E_c} (N_{\mathbf{x}} - Q_{0\mathbf{x}})^2}_{\text{charging}} \qquad N_{\mathbf{x}} = \sum_{\lambda} c^\dagger_{\mathbf{x}\lambda} c_{\mathbf{x}\lambda}$$

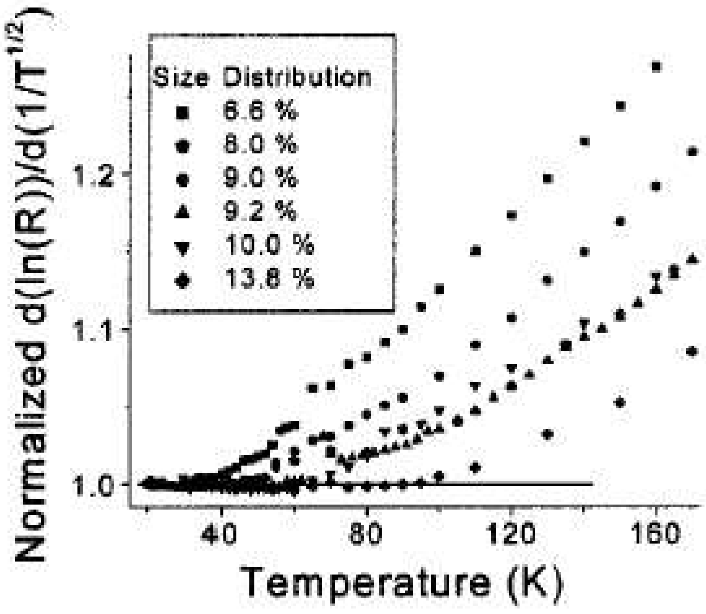
$g = 2\pi\nu^2 t^2$ = dimensionless inter-grain tunneling conductance $\qquad E_c = \dfrac{e^2}{2C}$ = on-grain charging energy



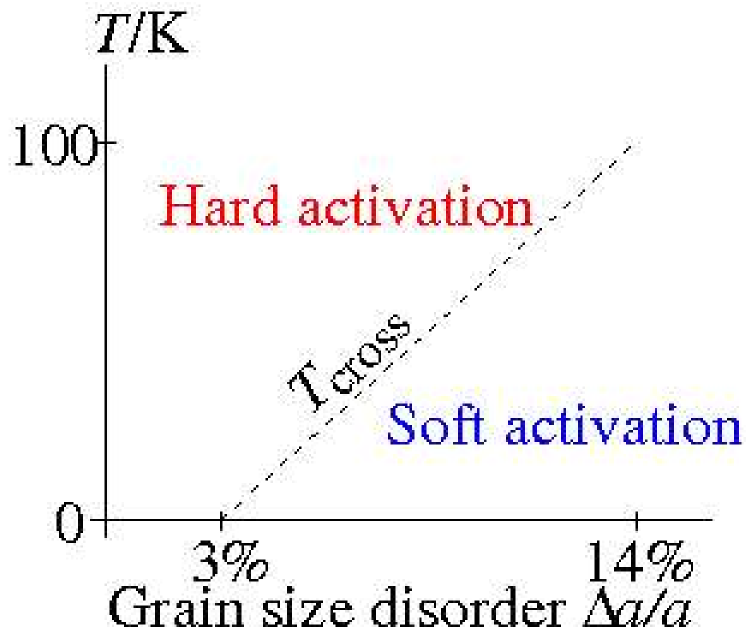


Experiments

- Ni/SiO_2 (Sheng 1973), Al/Al_2O_3 (Chui 1981), NbN/BN (Simon 1987), Al/Ge (Gerber 1997)
  - large g: logarithmic or weak power law
  - small g: soft activation $\sigma \sim \exp -(T_0/T)^{1/2}$
- Ag/dodecanethiol (Beverly 2002)
  - hard activation $\sigma \sim \exp -(E_c/T)$ in limit of zero disorder!



"Zero slope indicates soft activation"

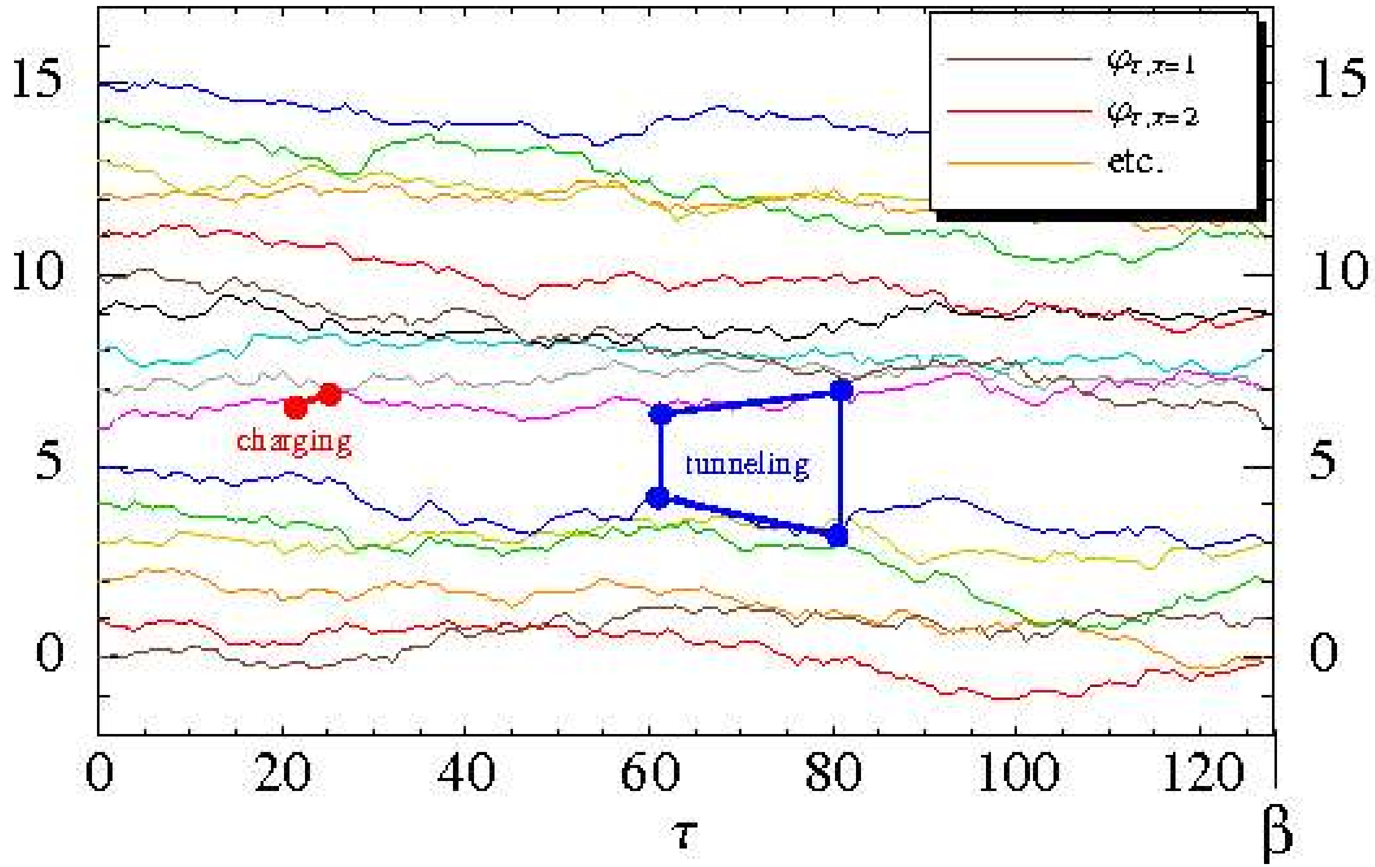


AES phase functional

Ambegaokar, Eckern, Schön (1982)

$$S[\varphi] = \underbrace{-\sum_{\tau\mathbf{x}} \frac{1}{4E_c}(\partial_\tau \varphi_{\tau\mathbf{x}})^2}_{\text{charging}} + \underbrace{\sum_{\tau\tau'\mathbf{x}\mathbf{a}} \frac{\pi g}{2}\alpha_{\tau-\tau'} \cos\left(\varphi_{\tau\mathbf{x}} - \varphi_{\tau'\mathbf{x}} - \varphi_{\tau,\mathbf{x}+\mathbf{a}} + \varphi_{\tau',\mathbf{x}+\mathbf{a}}\right)}_{\text{tunnelling}}, \quad \alpha_\tau = \frac{T^2}{\sin^2 \pi T \tau}$$

Polymer chains with short- and long-ranged interactions ►



Kubo formula for conductivity

Efetov and Tschersich (2002)

Conductivity $\sigma = dj/dE$ related to electromagnetic response $K = dj/dA$:

$$\sigma_\omega = \frac{K_\omega}{-i\omega}, \quad K_\omega = \mathcal{K}_n\big|_{i\omega_n \to \omega + i0^+}, \quad \mathcal{K}_n = \int_0^\beta d\tau \; e^{i\omega_n \tau} K_\tau, \quad \mathcal{K}_\tau = \mathcal{K}_\tau^d + \mathcal{K}_\tau^p$$

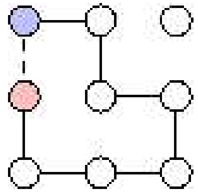
Diamagnetic and paramagnetic terms:

$$\mathcal{K}_\tau^d = \pi g \int_{\tau'} \left(\delta_{\tau-\tau'} - \delta_{\tau'}\right) \alpha_{\tau'} \Pi_{\tau'}, \qquad \Pi_\tau = \left\langle \cos\left(\varphi_{\tau\mathbf{x}} - \varphi_{\tau'\mathbf{x}} - \varphi_{\tau,\mathbf{x}+\mathbf{a}} + \varphi_{\tau',\mathbf{x}+\mathbf{a}}\right) \right\rangle$$

$$\mathcal{K}_\tau^p = \sum_{\mathbf{x}} \left\langle j_{\tau'\mathbf{x}'\mathbf{a}} j_{\tau'+\tau,\mathbf{x}'+\mathbf{x},\mathbf{a}} \right\rangle, \qquad j_{\tau'\mathbf{x}\mathbf{a}} = \pi g \int_\tau \alpha_{\tau-\tau_1} \sin\left(\varphi_{\tau\mathbf{x}\mathbf{a}} - \varphi_{\tau'\mathbf{x}\mathbf{a}}\right)$$

Regular array: Perturbation theory in g

$$S[\varphi] = S_c + S_t \qquad \langle X[\varphi]\rangle_S = \langle X\rangle_{S_c} + \langle S_t X\rangle_{S_c} - \langle S_t\rangle_{S_c}\langle X\rangle_{S_c} + \dots$$



- First order: $\sigma_d^{(1)} \sim g \exp(-E_c/T)$.
- Second order:
  - Have $K_{d\,\tau}^{(2)}/\alpha_\tau \sim g/\tau^2 \quad \therefore \quad \sigma_d^{(2)} \sim g^2 T^2$ (c.f. inelastic cotunneling)
  - but $K_{p\,\tau}^{(2)}/\alpha_\tau \sim -g/\tau^2 \quad \therefore \quad \sigma_p^{(2)} \sim -g^2 T^2$ (equal & opposite!)
- So: $K_\tau/\alpha_\tau \sim g \exp(-2E_c\tau) \quad \therefore \quad \sigma \sim g \exp(-E_c/T)$.

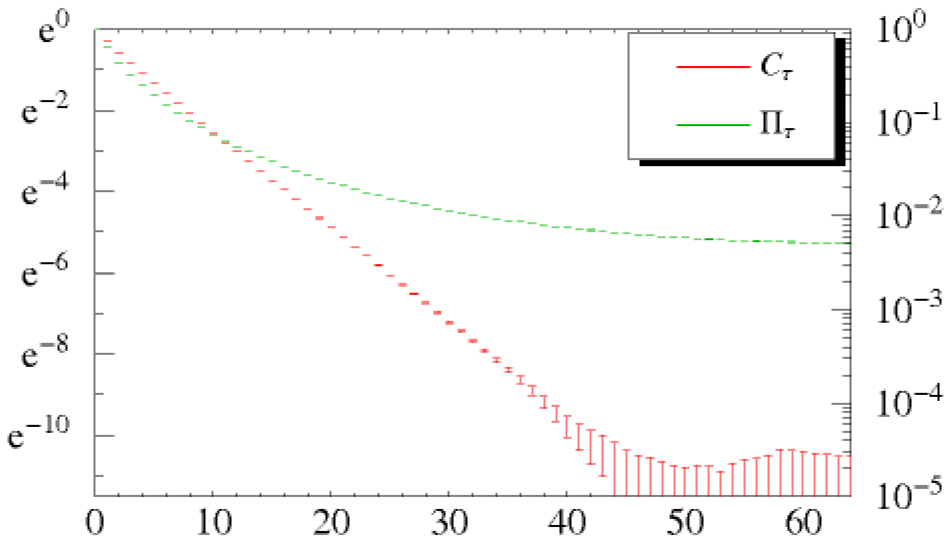
Inelastic cotunneling around closed loop does not contribute to charge transport!

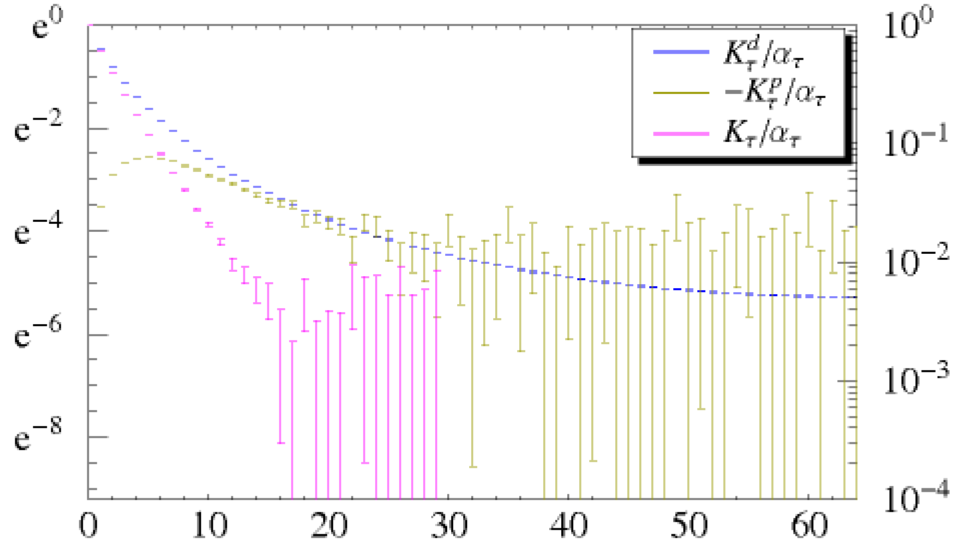
Also, single-grain correlator $C_\tau \sim \exp(-E_c\tau)$
→ tunneling DOS $\nu(E)$ is gapped.

$$C_\tau = \left\langle \cos(\varphi_{0\mathbf{x}} - \varphi_{\tau\mathbf{x}}) \right\rangle$$

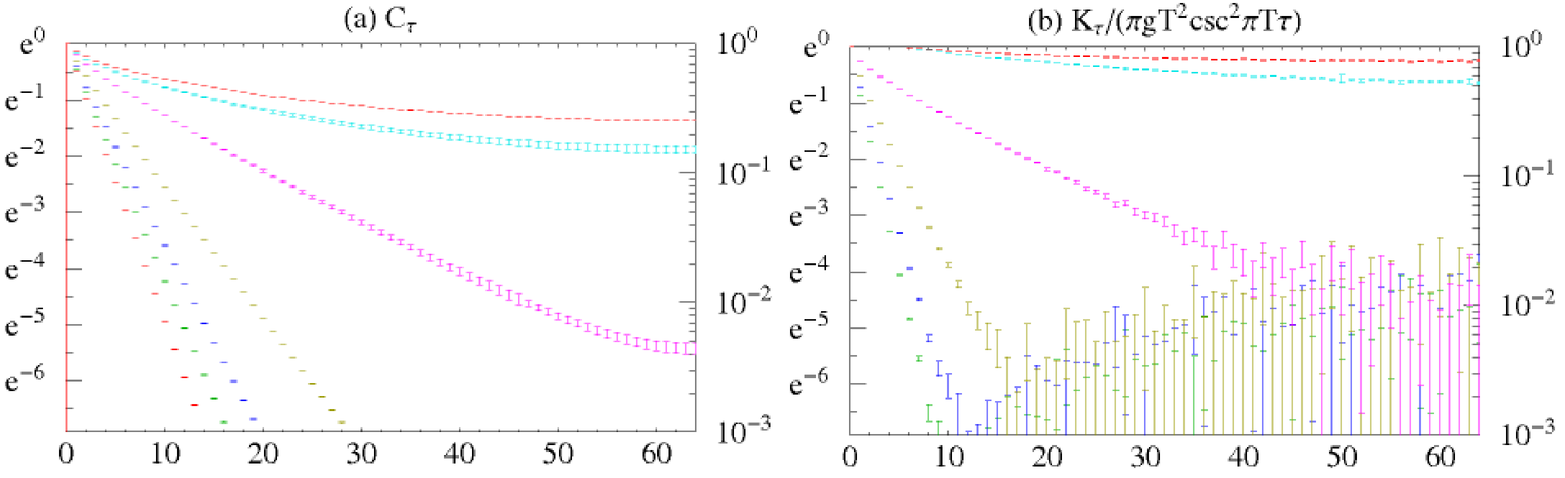
Regular array: Path integral Monte Carlo

- 1D array with periodic boundary conditions: X=64 grains, N=128 timeslices
- Hybrid MC + Fourier acceleration to improve sampling efficiency
- Indeed observe cancellation between T^2 terms ...



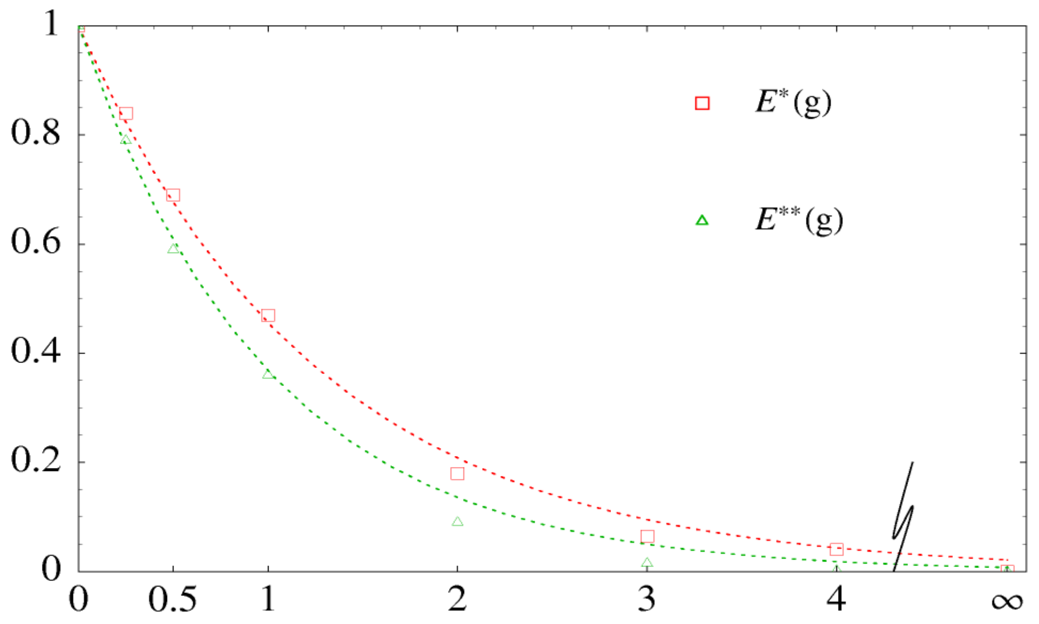


- ... but conductivity has activation energy $E^{**}(g) < E_c$ (from K_τ).
- Also, tunneling DOS has activation energy $E^{*}(g) < E_c$ (from C_τ). ($E^* \neq E^{**}$?)



◄ C_τ and K_τ for $g = 0, 0.25, 0.5, 1, 2, 3, 4$

Numerical estimates of E^* and E^{**} ►



Disordered arrays – hypotheses

- Cancellation still occurs
- Background charge disorder (random Q_0):
  - *open* chain of n inelastic cotunnelings:

    trade-off between $(gT^2)^n$ and exp $(-E_n/T)$

    gives soft activation law (c.f. ES-VRH; Feigel'man)
- Site disorder (random E_c): similar, but need large $\Delta a/a$
- Bond disorder (random g):
  - puddles with various E^*

    simulate effect of random E_c (with large $\Delta a/a$)