New Structure in the Phase Diagram for Electrons in a Random Potential –The Anderson Model

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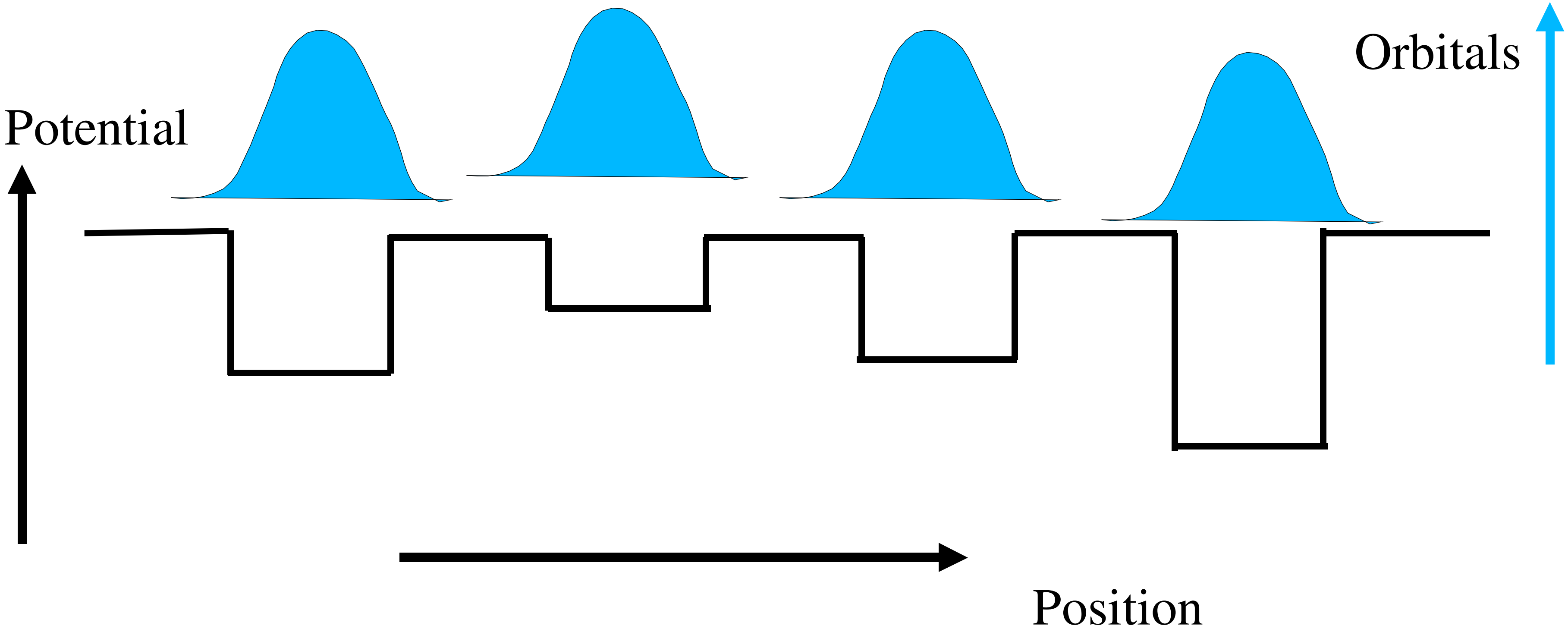
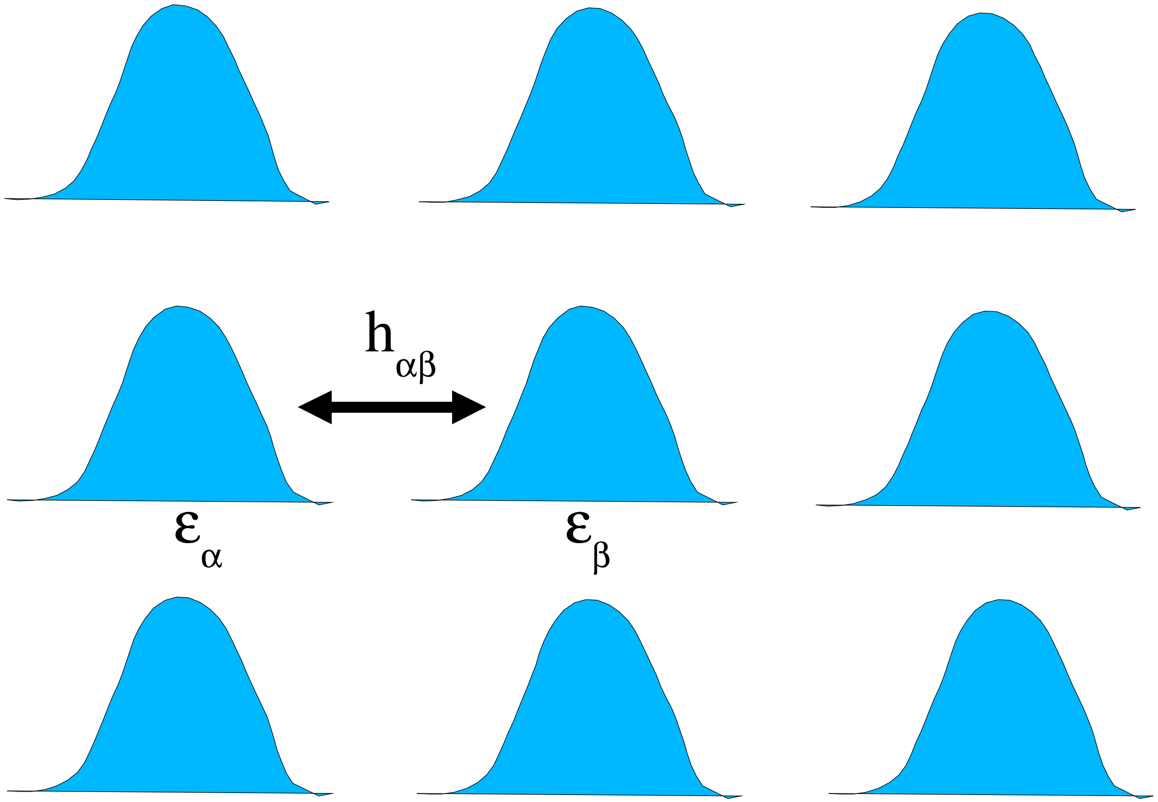
Issue: When do electrons belong to bands and when are they atom–like?



PLAN

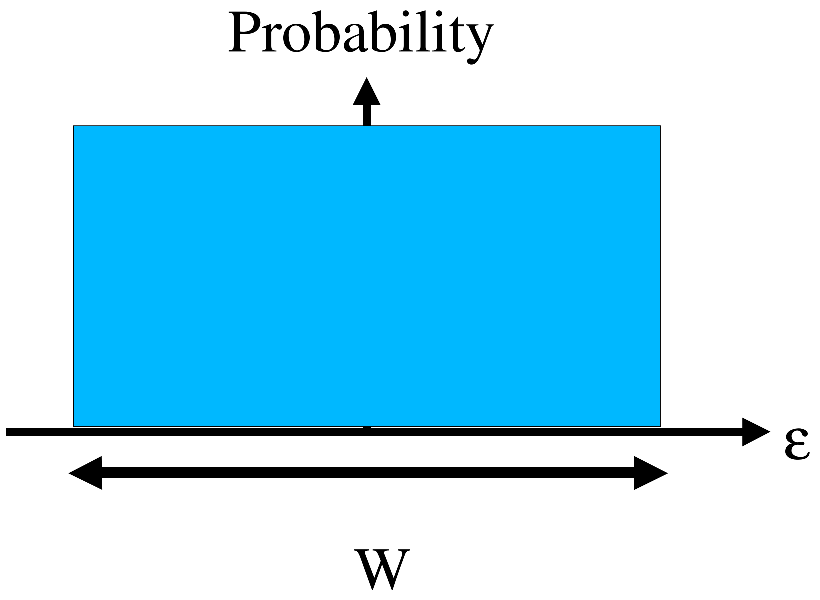
- Anderson Model
- Standard Phase Diagram
- New Phase Diagram
- Numerical Tridiagonalization (Recursion)
- Analytic Projection – Spins Interacting with an Electron
- Extended and Localized States on the Cayley Tree
- Conclusions

Anderson Model



- Independent Electrons
- Tight–Binding Orbitals
- Lattice of Sites

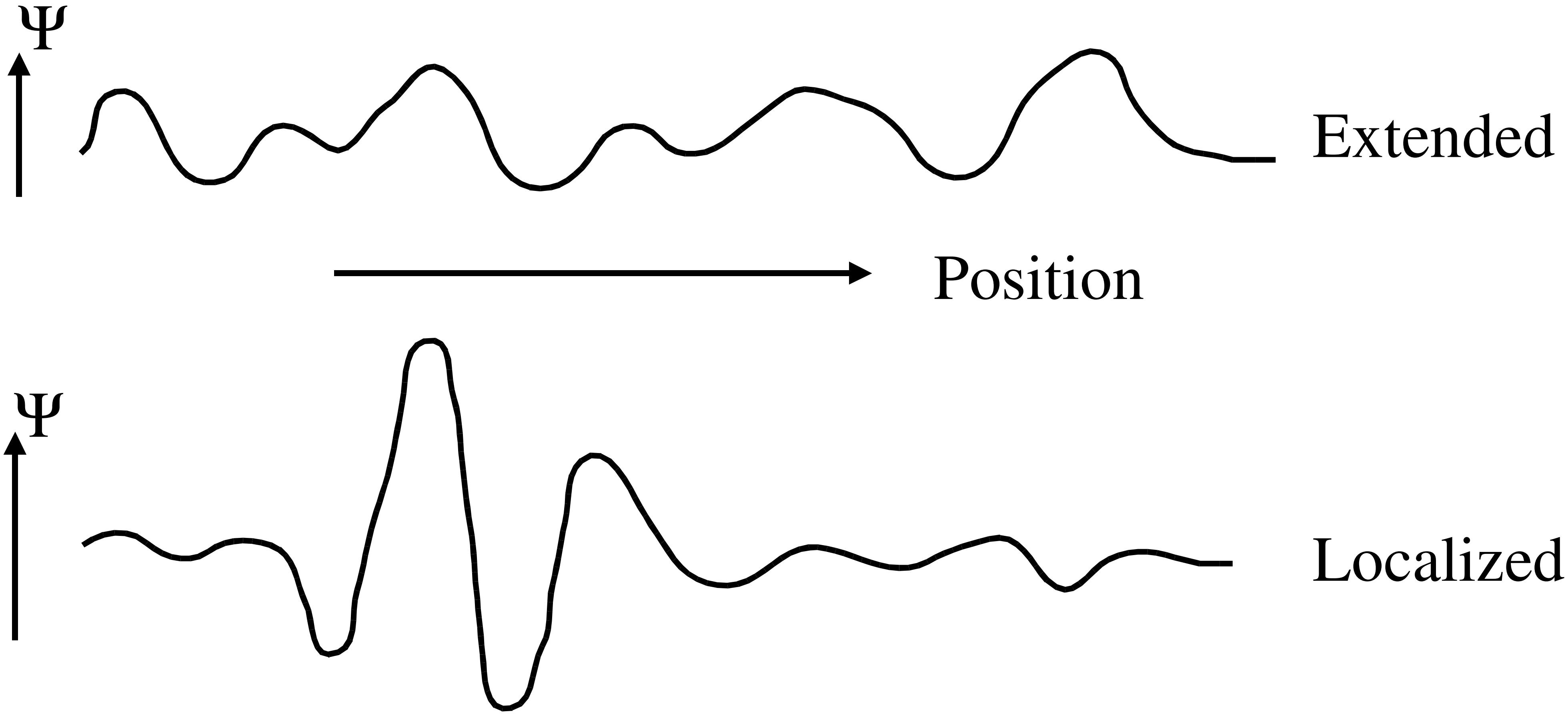
Random Orbital Energies



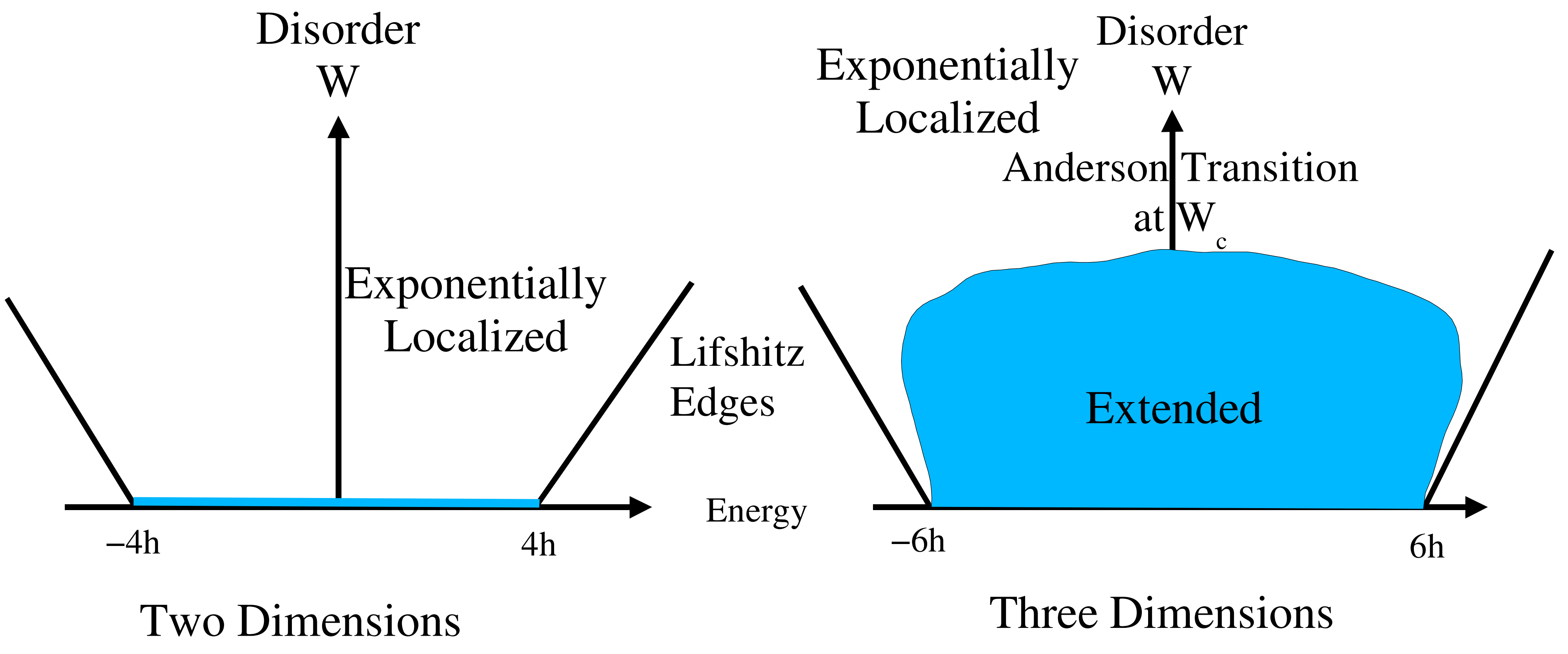
- Infinite number of inequivalent Orbitals – Infinite Dimension

- $\Delta\varepsilon \approx h$ Strong Coupling

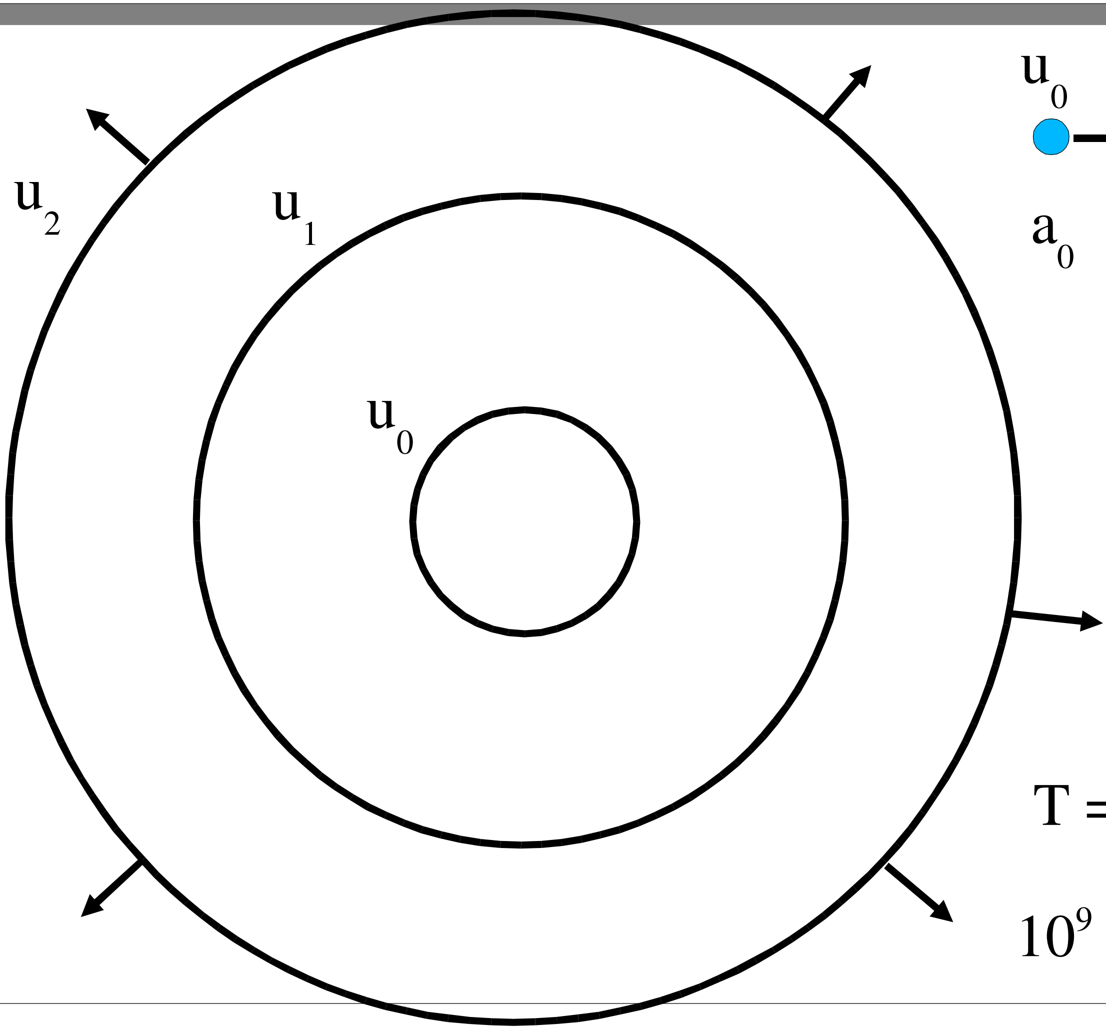
Extended (Band) and Localized (Atomic) Wavefunctions

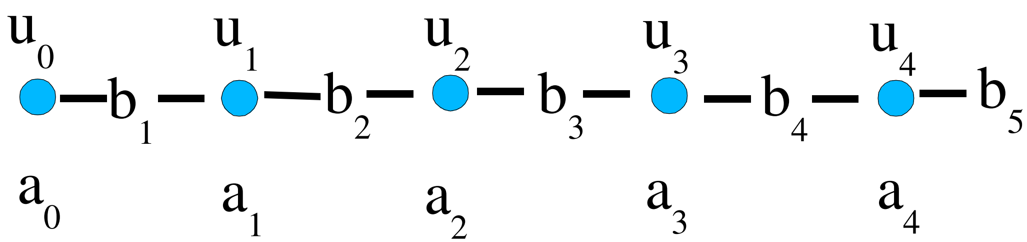


Standard Phase Diagram



Transmittance for Escape from an Initial Site





$$t_n = \begin{pmatrix} (E-a_n)/b_{n+1} & -b_n/b_{n+1} \\ 1 & 0 \end{pmatrix}$$

$$T = \ldots\, t_n\, t_{n-1}\ \ldots\, t_1\, t_0\, t^+_0\, t^+_1\, \ldots\, t^+_{n-1}\, t^+_n\, \ldots$$

10^9 compared to 10^6 sites – see plots

Analytic Projection

Transform the Basis for the Anderson Model from Localized to Extended States

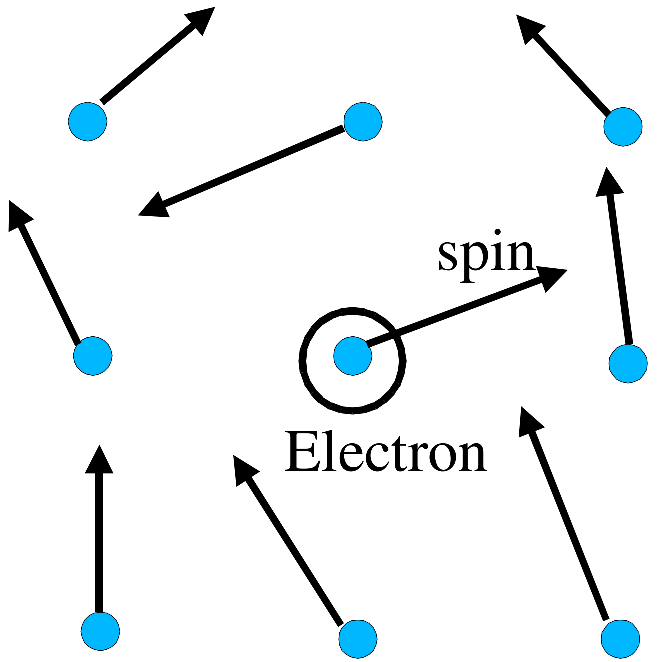
- Extended States are Waves Distorted by the Disorder
- No Disorder in the Matrix Elements because of Self–Averaging

$$\Phi_0 = \sum \phi_\alpha \qquad H\,\Phi_0 = 2\,d\,h\,\Phi_0 + \sum \varepsilon_\alpha\,\phi_\alpha = 2\,d\,h\,\Phi_0 + \Phi_1$$

$$\Phi_S = \sum_\alpha \left[\prod_\beta p_{S(\beta)}(\varepsilon_{\alpha-\beta})\right] \phi_\alpha$$

$\{p_n(\varepsilon)\}$ are orthogonal polynomials over the distribution of site energies

Electron Interacting with Spins – Augmented Space



- The Electron hops between sites with matrix element h
- The Electron changes the spin on the same site with matrix element $\propto$W
- The Electron hops between sites with matrix element h

- The configurations and interactions of this system form a structure like a Cayley Tree – Each configuration coupled to only a finite number of others, but of infinite dimensionality.

Localized and Extended States in Augmented Space

- Same or Alternating signed Configurations are Exact Eigenstates with Energies $\pm(2dh+W/2)$ – Localized states at the Lifshitz Edges.

- Band of States dominated by single extended configurations between $\pm\sim[4d^2h^2+W^2/4]^{1/2}$ – Mobility Edges and Band of Extended States.

- Anderson Transitions where Extended State Decouple from single sites .

Conclusions

- Agreement between Analytic and Numerical Mobility Edges indicates problems with the Standard Phase Diagram.

- Problems with the Standard Phase Diagram suggest there is more to be learned from the Anderson Model.

- The Challenges are to improve numerical accuracy and develop an Analytic theory for the Anderson Transitions.