NATURAL SCIENCES TRIPOS EXAMINATIONS
Part II: Quantum Condensed Matter Physics

29 March 2005

Mock exam questions

I believe the exam format will be a choice of two questions out of three, so you will have 45 minutes per question. Here are five "exam-style" questions.

1 Write short notes on **two** of the following : (Mathematical results may be stated without proof, and marks will be given for demonstrating a rounded understanding of the topic, and for appropriate use of examples)

(a) Five characteristic properties of a metal. [15]

(b) The Hartree-Fock approximation for the electron gas and the explanation it gives for magnetism. [15]

(c) The operation of a p-n junction. [15]

(d) The measurement of electronic bandstructure by photoemission spectroscopy. [15]

2 Explain Bloch's theorem and its consequence for the electronic band structure of periodic solids in the independent electron approximation. [6]

A two-dimensional solid has atoms arranged on a square lattice with lattice constant a. The electronic band structure is well described by the nearly-free-electron approximation with a constant band gap E_g on the first Brillouin zone boundary. There are two electrons per atom.

Draw the reciprocal lattice, and mark the first and second Brillouin zones. [4]

Sketch the lowest two energy bands within the first Brillouin zone, (a) in a direction toward the middle of the zone face, and (b) in the direction toward the zone corner. [6]

At which value of the crystal momentum do you find (a) the maximum of the lowest energy band, and (b) the minimum of the second lowest band. [4]

Show that the material will be an insulator provided that [6]

$$E_g > \frac{\hbar^2 \pi^2}{m a^2}$$

.

Sketch the fermi surface if this condition is *not* satified. [4]

3 Explain impurity doping of a semiconductor, discussing both acceptors and donors. [8]

(a) A quantum well of 10 nm thickness is made of GaAs, where the barriers may be assumed infinite. By scaling arguments based on the hydrogen atom, or otherwise, estimate the ionisation energy of an electron bound to a donor. [6]

Estimate the density of donors at which the overlap of wavefunctions is such that the carriers will form a metallic band. [4]

(b) A deep quantum well of 10 nm width is formed in $GaAs$ and has electrons introduced by modulation doping. Estimate the maximum density of carriers that can be introduced while keeping the fermi energy in the lowest sub-band. [4]

Sketch the frequency dependent conductivity $\sigma(\omega)$ for both cases (a) and (b). [8]

Electrons in GaAs have an effective mass $m^* = 0.066\ m_e$*, and the medium has a dielectric constant* $\epsilon = 13$*. The ionisation energy of the H atom is 13.6 eV and the Bohr radius is 0.05 nm.*

4 Figure 1 shows the optical absorption of Si. Comment on the features and explain their relationship to the electronic band structure of Si. [15]

Figure 2 shows the reflectivity of Al. Explain how this is related to the dielectric function $\epsilon(\omega)$ and give a qualitative explanation of the data. Use the data to estimate the electron density. [10] [5]

5 Atoms in a one dimensional chain modelled by a $\delta-$function potential, so that the Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - aV_o \sum_n \delta(x - na)$$

where $V_0 > 0$.

(a) Assuming that the nearly free electron approximation applies, determine the band gaps between the bands. [10]

(b) Assuming a tight-binding model, determine the ground state of the single-atom problem. [10]

Show that the bandwidth of the lowest band is [10]

$$W = 4V_0\kappa a e^{-\kappa a}$$

where $\kappa = aV_0m/\hbar^2$, provided $\kappa a \gg 1$.

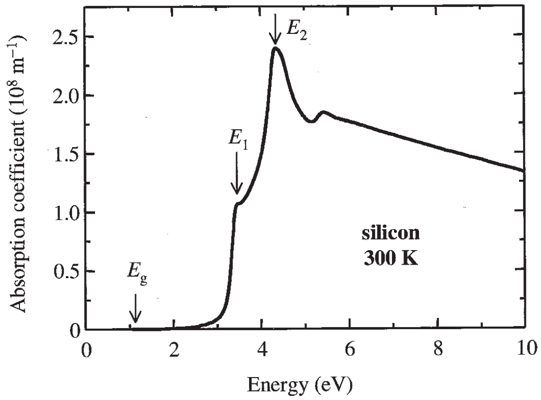

Figure 1: Optical absorption of Si.

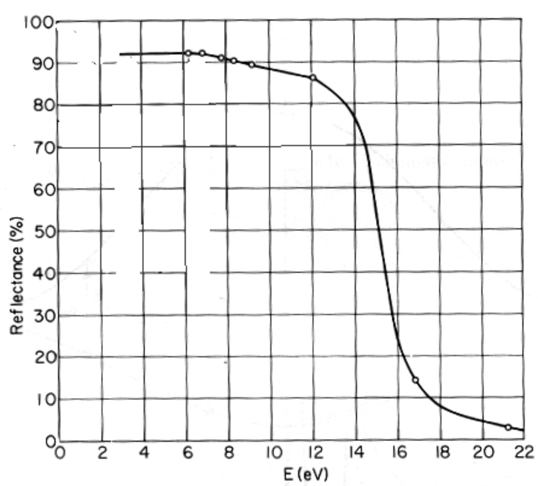

Figure 2: Reflectivity of Al, plotted as a function of $\hbar\omega = E$.