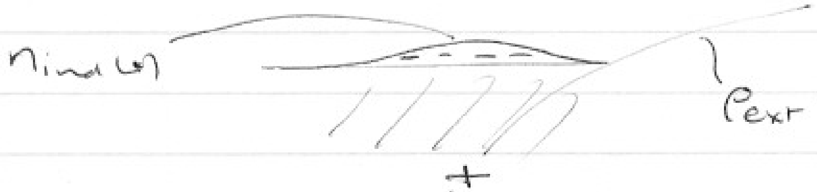
Metals

— 1.) Screening
2) Collective modes.
3) Optical prop.

Def^{ns} : Response to external charge - "free" charge



→ In vacuum generates a potential

$$\nabla^2 v_{ext} = \frac{e^2}{\varepsilon_0} n_{ext}.$$

→ makes $\vec{D}$

$$\rho_{tot} = e\left[n_{ext}(r) - n_{ind}(r)\right]$$

$$\nabla^2 v_{tot} = \frac{e^2}{\varepsilon_0}\left[n_{ext} - n_{ind}\right]$$

→ makes $\vec{E}$

$$D(r) = \varepsilon_0 \int dr'\, \varepsilon(r, r')\, E(r')$$

or equivalently $v_{ext}(r) = \int dr'\, \varepsilon(r, r')\, v_{tot}(r')$

Now $\varepsilon(r, r') \simeq \varepsilon(r - r')$

$$v_{ext}(r) = \int dr'\, \varepsilon(r - r')\, v_{tot}(r')$$

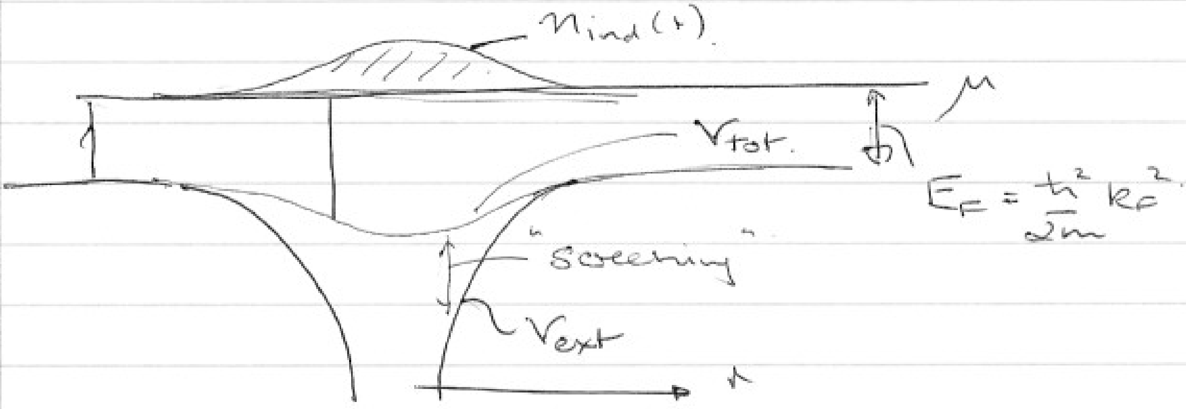
In Fourier space

$$\tilde{v}_{ext}(q) = \varepsilon(q)\, \tilde{v}_{tot}(q)$$

N.B assumes linearity. $n_{ind}(q) = \chi(q)\, v_{ext}(q)$

↑ density response

Thomas-Fermi approx. [Notes are in c.g.s.]



Slowly varying: $E(k, r) = \frac{\hbar^2 k^2}{2m} + v_{tot}(r)$

Const. μ:

$$\mu = E_F(r) + v_{tot}(r) = \mu_\infty$$

Local density approx: $k_F(r) = \left(3\pi^2 n(r)\right)^{1/3}$

Pot' $v_{tot} = v_{ext} + v_{ind} = v_{ext} + \int dr' \frac{e^2}{|r - r'|} n(r')$

$$\therefore \underbrace{\frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}}_{} + \underbrace{\int dr' \frac{e^2}{|r-r'|} n(r') + v_{ext}}_{} = \mu$$

— Thomas-Fermi.

n_0

1) Linearize $n = n_0 + \delta n$ $\quad (\delta n / n_0) \ll 1$

$$\frac{\hbar^2}{3m} \frac{(3\pi^2)^{2/3}}{n_0^{1/3}} (\delta n)$$

$E_F = \mu_\infty$

~~$\int dr' \frac{e^2}{|r-r'|}\delta n(r')$~~

(2) F.T.

$$A\, \delta n(q) + \frac{4\pi e^2}{q^2} \delta n(q) + v_{ext}(q) = 0$$

$$\delta n(q) = - \frac{v_{ext}(q)}{\frac{4\pi e^2}{q^2}\left[1 + \frac{q^2}{q_{TF}^2}\right]}$$

$q_{TF}^2 = \frac{4}{\pi} \frac{k_F}{a_B} \sim 1-2 \times 10^{-10}$ m — interparticle spacing.

Substitute back: $v_{tot} = v_{ext} + v_{int}$

$$= v_{ext} + \frac{4\pi e^2}{q^2} \cdot \frac{-v_{ext}}{\frac{4\pi e^2}{q^2}\left[1 + \frac{q^2}{q_{TF}^2}\right]}$$

$$= v_{ext} \cdot \frac{q^2/q_{TF}^2}{1 + q^2/q_{TF}^2} = \frac{v_{ext}}{\varepsilon_{TF}(q)}$$

$$\varepsilon_{TF} = 1 + \frac{q_{TF}^2}{q^2}$$

<u>NB</u>. Add a charge Q at $r = 0$

$$v_{ext} = Q/r.$$

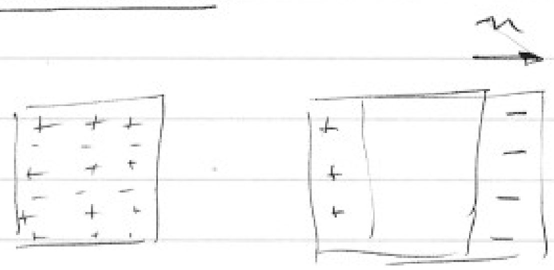
$$\tilde{v}_{ext}(q) = Q \cdot \frac{4\pi e^2}{q^2}$$

$$\tilde{v}_{tot}(q) = Q \frac{4\pi e^2}{q^2 + q_{TF}^2}$$

$$v_{Tot}(r) = \frac{Q}{r} e^{-q_{TF} r}.$$

— no potential at large r → <u>perfect</u> screening.

Plasma oscillations



Displace electron charge — sloshing modes

$$P = -nex.$$

extend. field.

$$E = \frac{D}{\varepsilon_0} - \frac{P}{\varepsilon_0}$$

Classical equ^n of motion

$$m\ddot{x} = -eE = -\frac{ne^2 x}{\varepsilon_0} - \frac{eD}{\varepsilon_0}$$

Sol^n $x \sim x_0 e^{i\omega_0 t}$

$$\left[-m\omega^2 + \frac{ne^2}{\varepsilon_0}\right] x_0 = \frac{-eD}{\varepsilon_0}$$

1) Sol^n for modes of free osc^n ($D=0$) at

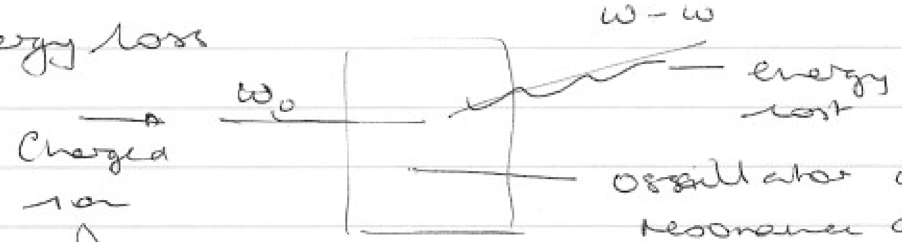
$$m\omega_p^2 = \frac{ne^2}{\varepsilon_0} \implies \text{plasmons.}$$

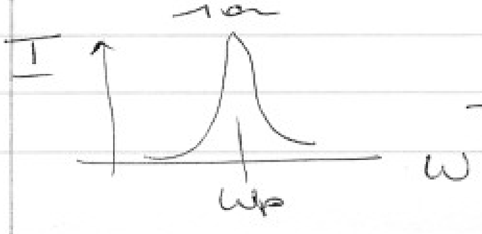
2) Dielectric f^n $\Rightarrow$ Solve for E

$$\underbrace{\varepsilon_0\left[1 - \frac{\omega_p^2}{\omega^2}\right]}_{\varepsilon(\omega,(q=0))} E = D.$$

$$\varepsilon(\omega,(q=0)) = 1 - \frac{\omega_p^2}{\omega^2}$$

3) Energy loss





$\longrightarrow$ Actually $I \propto \mathrm{Im}\left(\frac{1}{\varepsilon}\right)$

Optical conductivity — Drude model

$$m\ddot{u} + \gamma\dot{u} = -eE = -\frac{ne^2}{\varepsilon_0}u - \frac{e}{\varepsilon_0}D$$

$$\left(-m\omega^2 - i\omega\gamma + \frac{ne^2}{\varepsilon_0}\right)u = -\frac{eD}{\varepsilon_0}$$

$\Rightarrow$ Complex $\displaystyle \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}$ $\qquad \frac{1}{\tau} = \gamma/m$ relaxation rate.

Measuring response — not ~~charge~~ density but current

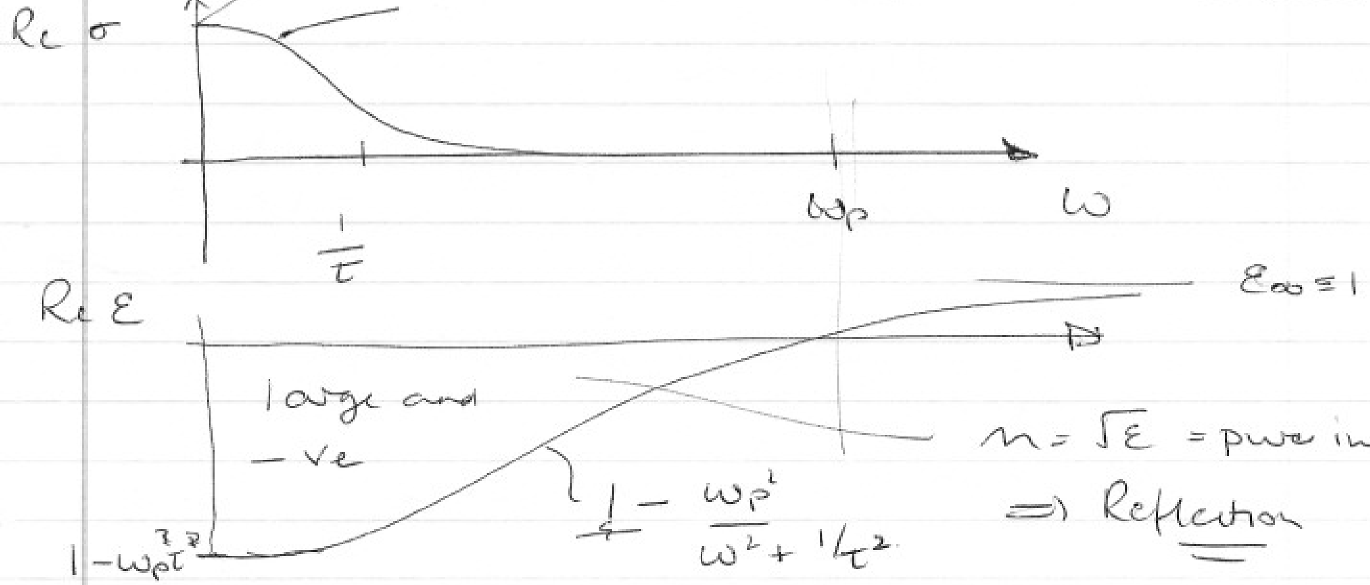
$$j(\omega) = -ne\dot{u} = i\omega n e u$$

Calculate $\displaystyle j = \underbrace{\frac{\varepsilon_0\omega_p^2}{\left(1/\tau - i\omega\right)}}_{\sigma(\omega)} E(\omega)$

σ, ω ~~related~~: $\displaystyle \varepsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0\omega}$

or, in general — ~~[illegible]~~ gas in semiconductor $(\varepsilon_\infty > 1)$

$$\frac{ne^2\tau}{m}\frac{1}{(1+\omega^2\tau^2)}$$



$$1 - \frac{\omega_p^2}{\omega^2 + 1/\tau^2}$$

$n = \sqrt{\varepsilon}$ = pure im

$\Rightarrow$ Reflection

Refl. $R = \left| \frac{\sqrt{\varepsilon} - 1}{\sqrt{\varepsilon} + 1} \right|^2$

may not be 1. (higher bands)

~~$R \to$~~ $\varepsilon = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}$

"Good metal" $\omega_p \tau \gg 1$, so for $\omega < \omega_p$.
ε nearly pure imag., $n = i|\sqrt{|\varepsilon|}| \to R = 1$.
$R \to 0$ at the plasma freq. $\omega = \frac{\omega_p}{\sqrt{\varepsilon_\infty}}$