## (2) Bloch's Theorem

Eigenstates of S.E with periodic potential satisfy

$$\psi_{nk}(r) = e^{i\underline{k}\cdot\underline{r}}\, u_{nk}(r)$$

where $u_{nk}$ is periodic

$n$ is a discrete index

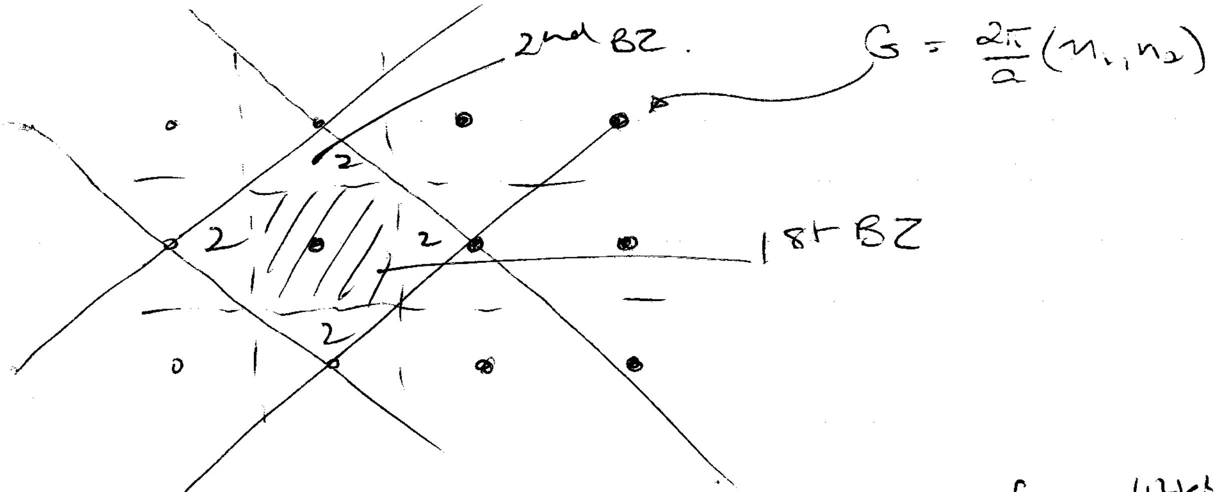
$k$ can be limited to the 1st Brillouin Zone

Eigenvalues

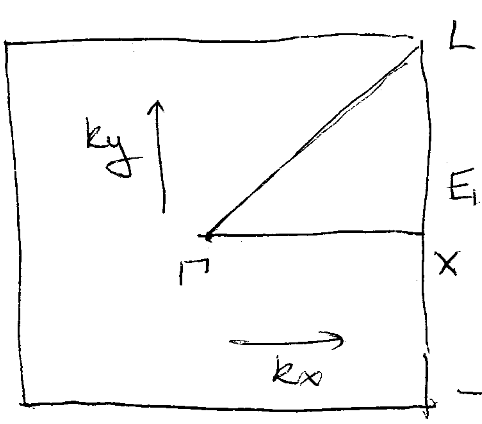
$$E_{nk} = E_{nk+G}$$

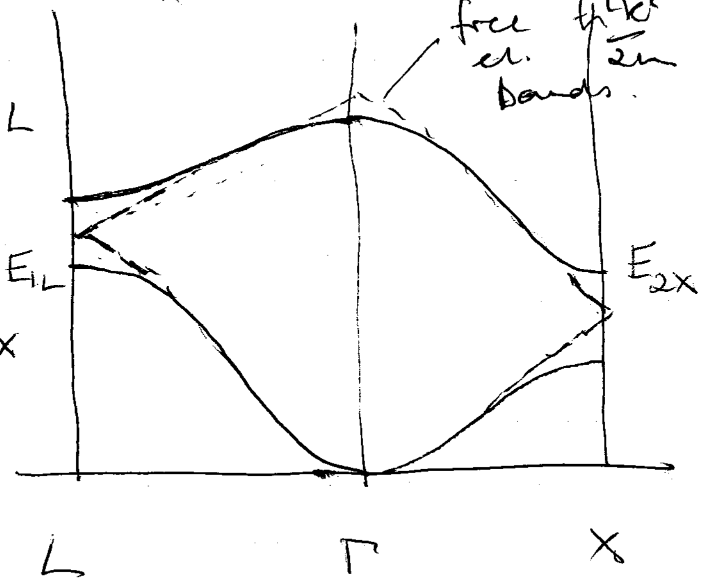
Consequence:

Discrete bands of energy (labelled by $n$) that are continuous functions of $k$.




1st B.Z



Max of lowest band at $L = \frac{\pi}{a}(1,1)$

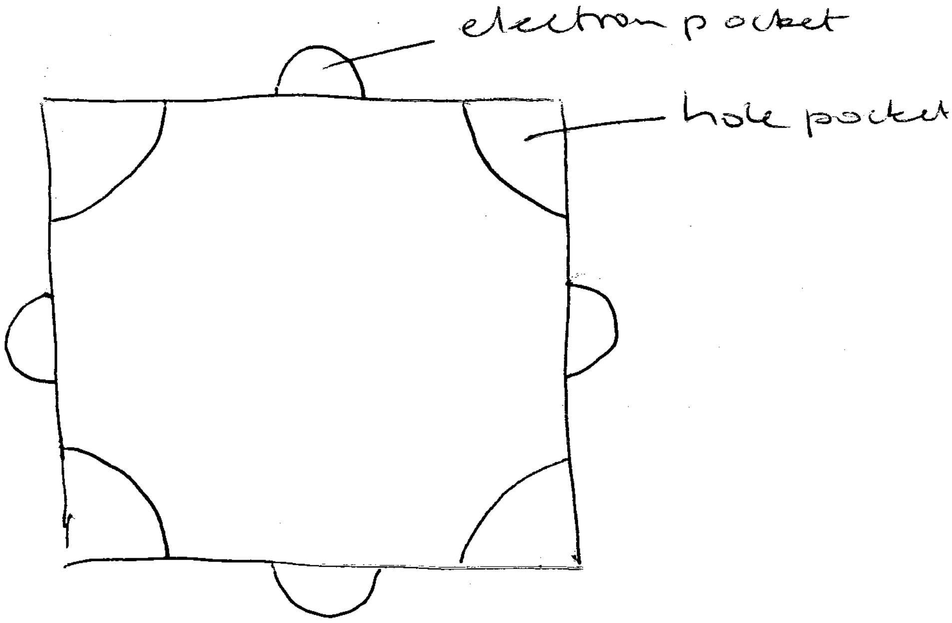
Min of second band at $X = \frac{\pi}{a}(1,0)$

$$E_{1L} \simeq \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\cdot 2 - V$$

$$E_{2X} \simeq \frac{\hbar^2}{2m}\frac{\pi^2}{a^2} + V$$

$E_{2X} > E_{1L}$ if $2V = E_g > \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}$.

(NB. factor of 2 error in original problem.




(Also OK to draw both bands in the ~~see~~ first BZ).