① a) Characteristic properties of a metal.

(i) Fermi surface lies inside a band
— continuous distribution of excitations at low energies → 0.
→ conductor.

(ii) Screening — Bare coulomb

$$\frac{1}{r} \Rightarrow \frac{e^{-r/\lambda}}{r}$$

λ = Thomas Fermi screening length
$\propto (a_B \lambda_F)^{1/2}$ geometric mean of Bohr radius & fermi wavelength
($\lambda_F = 2\pi / k_F$)

· Added charge completely screened on length scale λ_F
· No electric fields inside a metal.
· $\varepsilon(q, \omega \sim 0) = 1 + q_{TF}^2/q^2$.

(iii) Electronic specific heat at low T
$C_{el} = \gamma T \qquad \gamma \simeq g k_B^2$
where g is density of states at E_F

(iv) Pauli susceptibility $M = \chi H$
$\chi \sim \mu_B^2 g$ where μ_B is Bohr magneton.

(v) Plasmons — collective mode of sloshing of charge — $D = \varepsilon \varepsilon_0 E$

$$\varepsilon(q \sim 0, \omega) = 1 - \frac{\omega_p^2}{\omega^2} \qquad \omega_p^2 = \frac{ne^2}{\varepsilon_0 m}.$$

Note that E is non-zero at $\omega_p = \omega$ even if applied field $D = E_{external} = 0$
Seen by energy-loss spectroscopy.

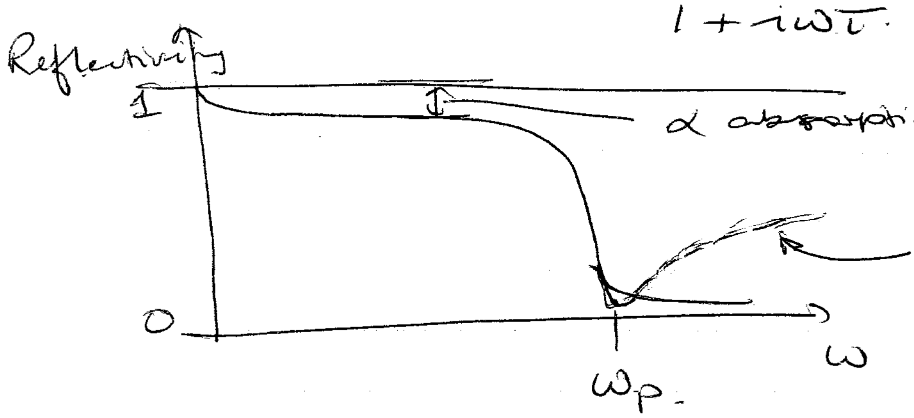
(vi) Metal is a good reflector, since

$n = \sqrt{\varepsilon}$ and $\varepsilon < 0 \Rightarrow n$ imaginary
— total reflection.

Actually, small absorption

$$\Rightarrow \quad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}$$

$$\text{or} \quad \sigma(\omega) = \frac{\varepsilon_0 \omega_p^2 \tau}{1 + i\omega\tau}.$$



$\propto$ absorption $\propto \frac{1}{\omega_p \tau} \ll 1$

NB if $\varepsilon > 1$ at high freq. get a minimum at the plasma rising to a value

$$R = \left| \frac{\sqrt{\varepsilon_\infty} - 1}{\sqrt{\varepsilon_\infty} + 1} \right|^2.$$

(b) Hartree-Fock theory of the electron gas.

Hartree-Fock is ~~the~~ a variational theory which gives the best single determinantal wavefunction

$$\begin{vmatrix} \varphi_1(r_1\sigma_1) & \cdots & \varphi_1(r_N\sigma_N) \\ \vdots & & \\ \varphi_N(r_1\sigma_1) & \cdots & \varphi_N(r_N\sigma_N) \end{vmatrix}$$

In the electron gas, translational invariance leads to $\varphi_i(r) \sim e^{i\underline{k}_i\cdot\underline{r}}$ — plane waves. All that remains is the spin occupancy. In a paramagnet we naturally will occupy states $k_i\uparrow$ and $k_i\downarrow$ equally for all $k<k_F$

Substituting in the many-particle SE, we get the total energy is

$$E_{HF} = E_K + E_{Direct} + E_x.$$

K.E. so $\sum_k \epsilon_k$
$= \frac{3}{5}E_F$
$\sim k_F^2 \sim n^{2/3}$

Coulomb interaction of electron with all the other electrons → cancels with the positive background charge

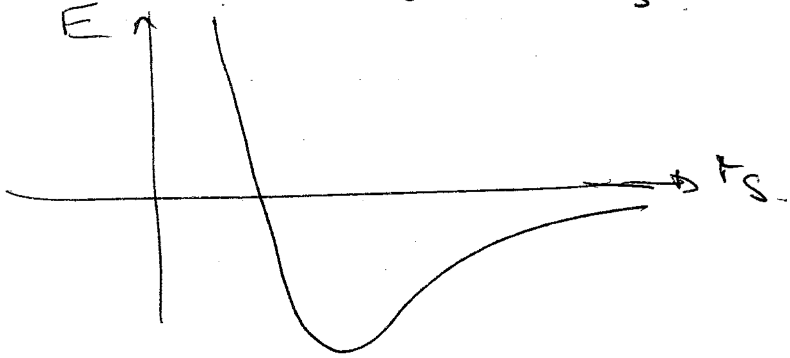
$$E_x = \langle 1\,2 | \frac{e^2}{r} | 2\,1 \rangle$$ vanishes unless states 1, 2 have the same spin.

$$\simeq \frac{e^2}{4\pi\varepsilon_0 a} \sim n^{1/3}.$$

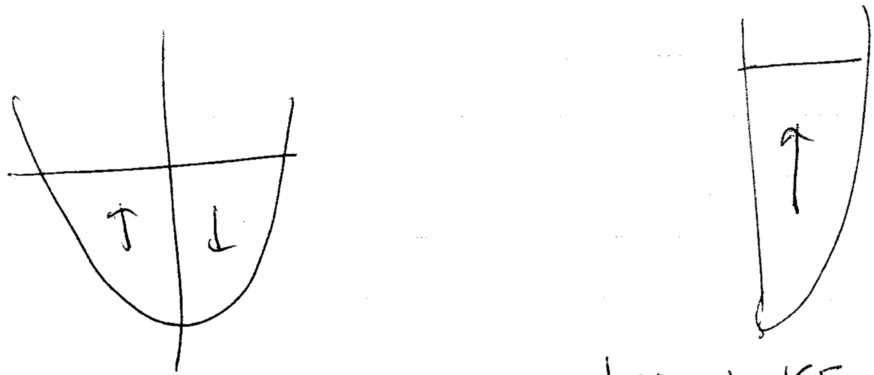
Dimensionally, measure energies in Rydbergs,
Lengths in units of Bohr radius.

$$\frac{4\pi}{3}(a_B r_s)^3 = \frac{1}{n}.$$

$$E/_{Ryd} \sim \frac{A}{r_s^2} - \frac{B}{r_s} + \text{corrections beyond HF}$$



Suppose we considered a FM arrangement of spins —



→ larger KE because $k_F \to k_F\sqrt{3}$.

→ more negative exchange because twice as many terms.

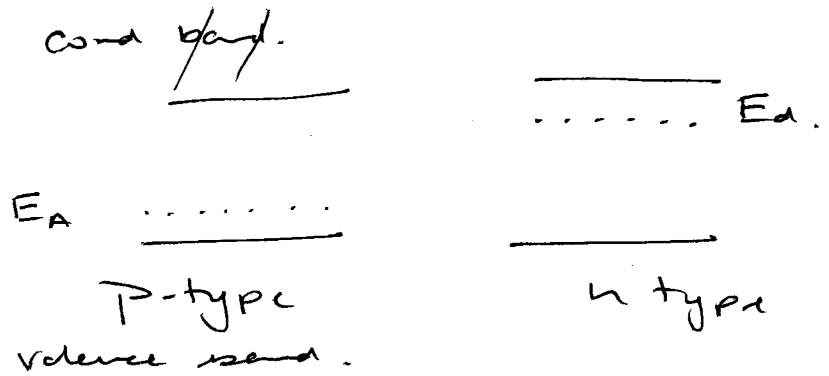
Stable for small r_s → Stable for large r_s.

Transition predicted by HF theory at around $r_s \sim 5$.

Real # is uncertain, but definitely much larger.

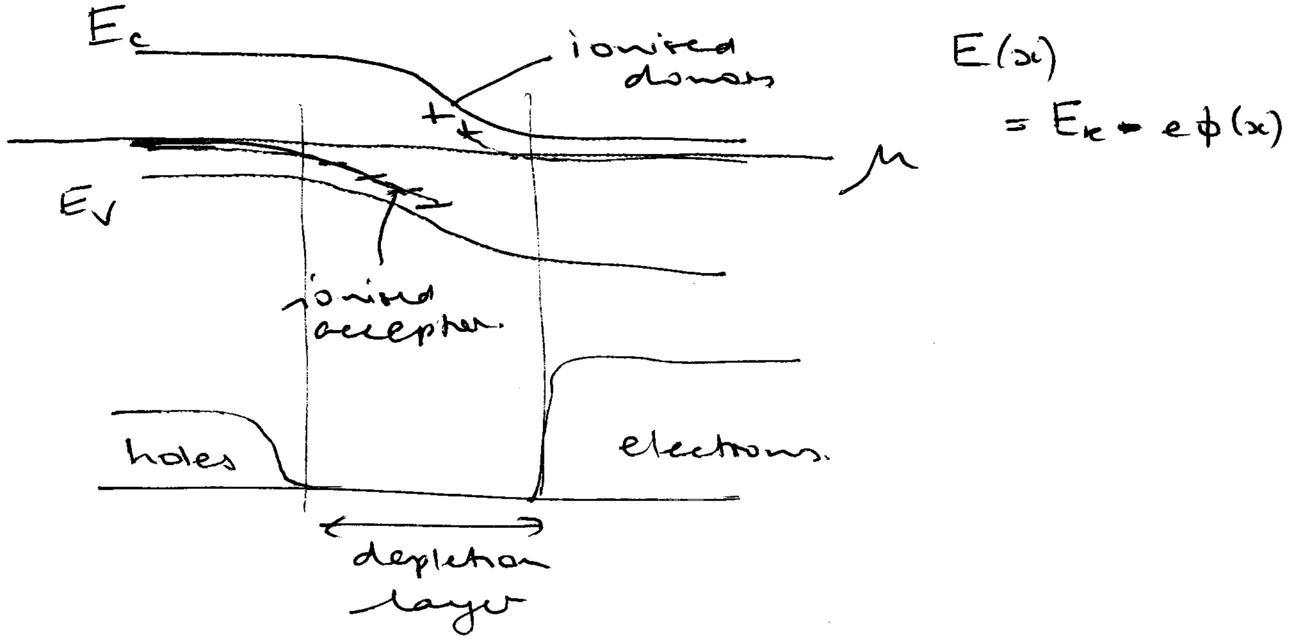
1(c) The Operation of a p-n junction.

A p-n junction is formed by inhomogeneous doping as in the figure below



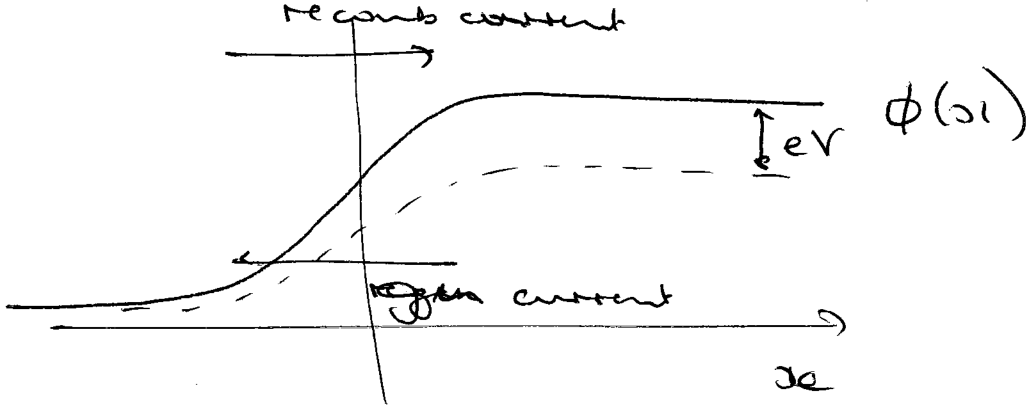
If the two sides are placed in contact, the chemical potential must be constant in equ^n.

- ⟹ Transfer of electrons from n-to p-side
- ⟹ Dipole layer of electrostatic potential formed by ionised donors / acceptors.
- ⟹ Generates electrical potential $\phi(x)$ that cancels the band offset



A p-n junction will rectify.

Suppose an external potential is applied



Overall barrier is $e(\Phi_b - V)$.

There must be a current from L to R of holes

$$J^h_{recomb} \simeq J_0 e^{-e(\Phi_0 - V)/kT}$$

balanced by current of minority holes from R → L.

$$J^h_{gen}.$$

At zero bias, net current vanishes.

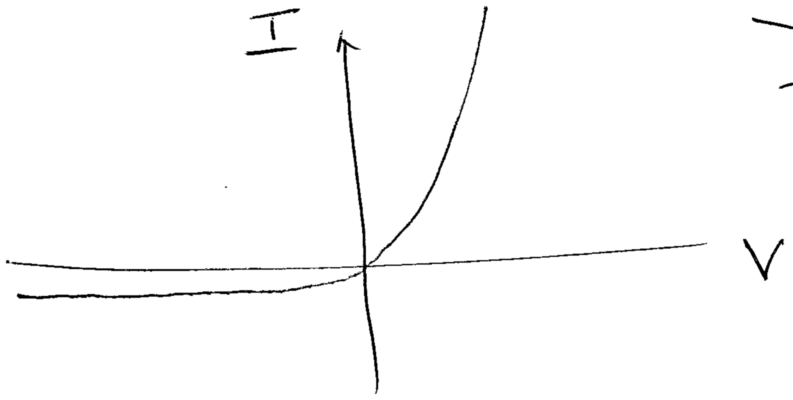
⇒

$$J^h(V) \simeq J\left(e^{eV/kT} - 1\right).$$

where J is a parameter

Same form for electrons (velocities in opp direction, electrical currents of same sign)

$$\therefore \quad I(V) = I_0\left[e^{(eV/kT)} - 1\right]$$



Diode I-V characteristic

→ a rectifier.

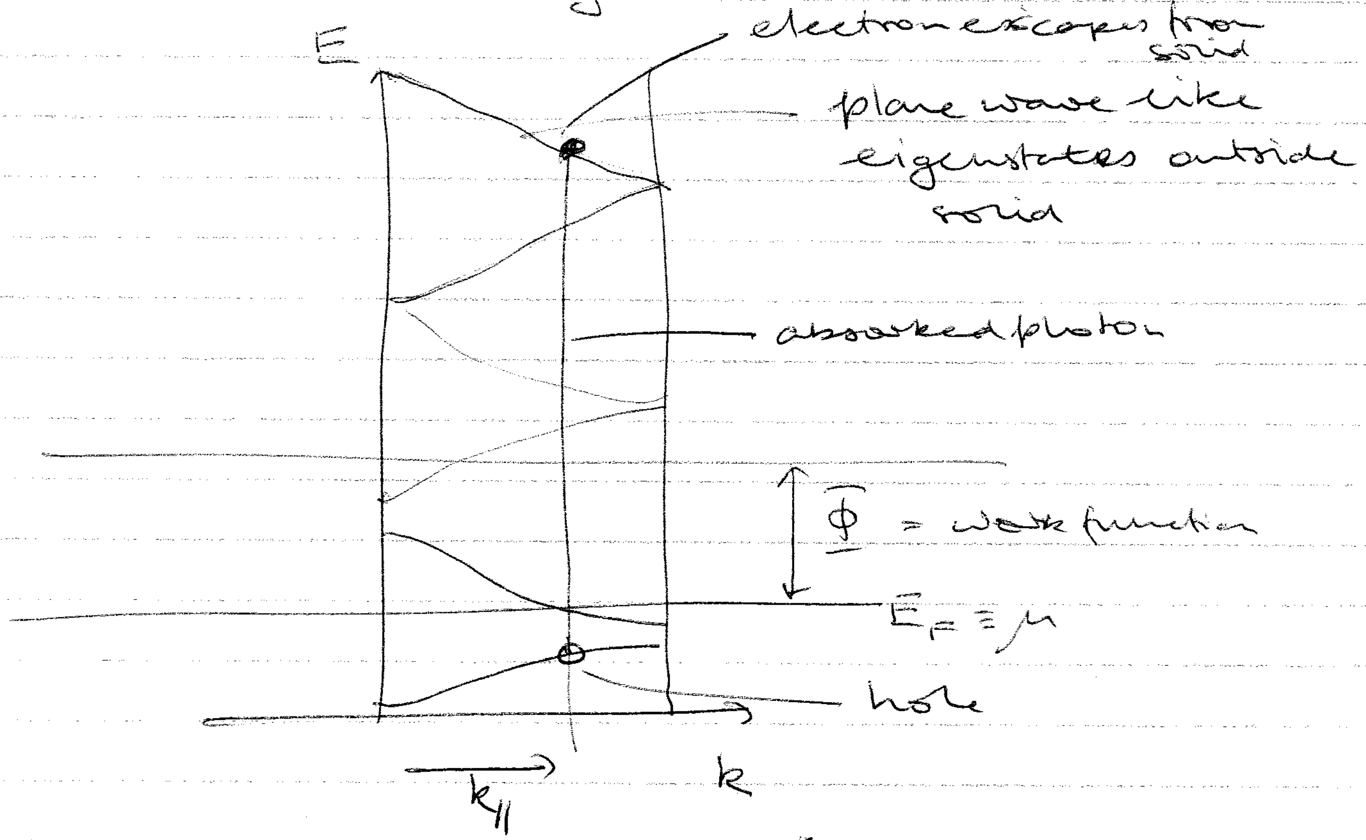
if $eV/kT \gg 1$

P-n junctions are also used as solar cells (light creates carriers that are separated in the inbuilt potential ϕ).

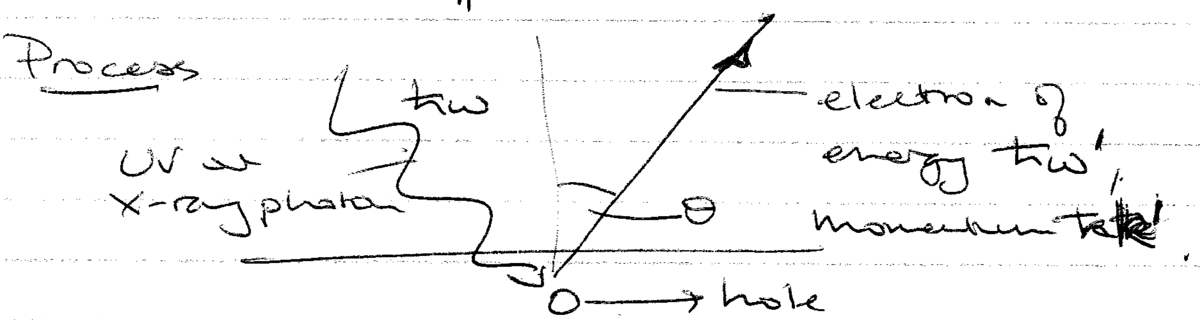
or light-emitting diodes (incident e-h pairs recombine to create light)

(d) Photoemission Spectroscopy

Electrons described by bands



Process

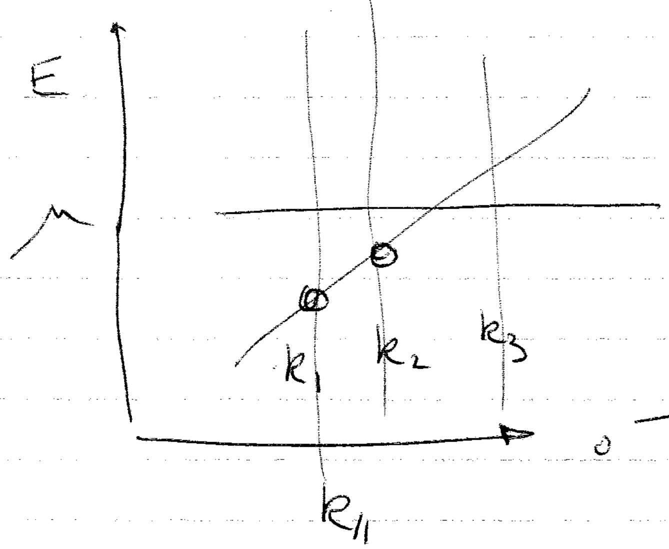


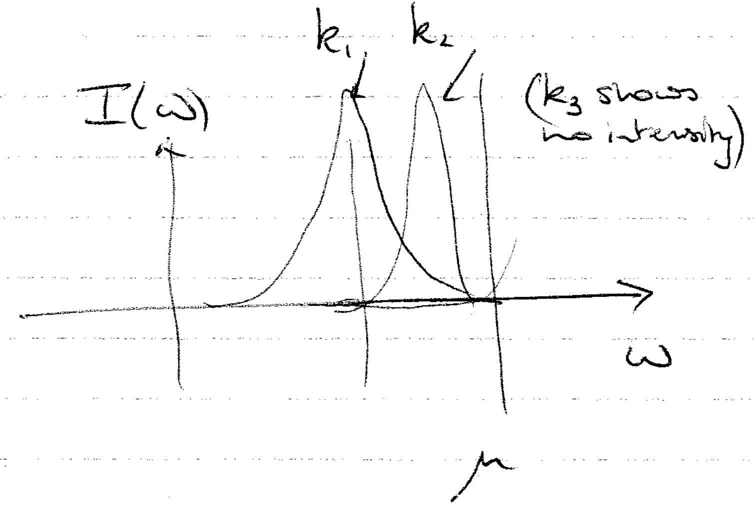
By measuring energy & momentum of emitted electron, determine momentum and energy of the "hole" left in the sample

Usually, plot spectrum as a function of energy —— integrating over angle gives the full density of states

—— at ~~a~~ particular angles θ know the momentum of the hole $k_{\parallel}$

Plotting spectrum as a function of angle





- Tracking the position of the peak gives bandstructure
- Width of the peak is the lifetime of a quasiparticle

Easiest to untangle this if one has a layered material with 2D dispersion. ($k_\perp$ not conserved).

Probe is surface sensitive — usually only a few Å penetrated into the material

Needs a clean surface — roughness destroys the momentum conservation.

Can measure only the occupied bands.