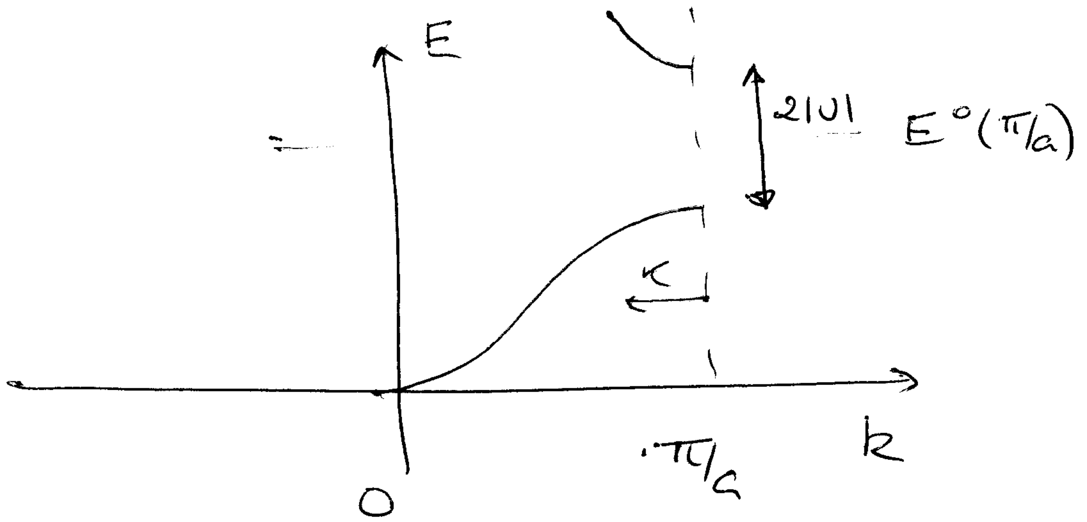
Question 28

From (6.14)

$$E^{\pm}(k) = \frac{1}{2}\left[E^0(k) + E^0\left(k-\frac{2\pi}{a}\right)\right] \pm \frac{1}{2}\sqrt{\left[\left(E^0(k)-E^0\left(k-\frac{2\pi}{a}\right)\right)^2 + 4U^2\right]}$$

where $E^0(k) = \dfrac{\hbar^2 k^2}{2m}$



Write $k = \pi/a + \kappa$, $\quad |\kappa| \ll \pi/a$

$$E^0(k) - E^0\left(k-\frac{2\pi}{a}\right) = \frac{\hbar^2}{2m}\left\{\left(\frac{\pi}{a}+\kappa\right)^2 - \left(\frac{\pi}{a}-\kappa\right)^2\right\}$$

$$= \frac{\hbar^2}{2m}\cdot\frac{4\pi}{a}\cdot\kappa$$

Then

$$E^{\pm} \simeq E^0\left(\frac{\pi}{a}\right) \pm |U|\sqrt{\left[1 + \left(\frac{\hbar^2}{2m}\cdot\frac{4\pi}{a}\cdot\frac{\kappa}{2U}\right)^2\right]}$$

$$\simeq E^0\left(\frac{\pi}{a}\right) \pm |U|\left[1 + \frac{1}{2}\left(\frac{\hbar^2}{2m}\cdot\frac{4\pi}{a}\cdot\frac{\kappa}{2U}\right)^2 + \cdots\right.$$

$$= E^0(\pi/a) \pm |U| \pm \frac{1}{2}\frac{\hbar^2\kappa^2}{m^*}$$

$$\therefore \frac{1}{m^*} = \frac{1}{m}\cdot\frac{\hbar^2\pi^2}{ma^2}\cdot\frac{1}{|U|}$$, or since $E_{gap} = 2U$

$$\frac{m^*}{m} = \frac{E_{gap}}{4E^0(\pi/a)}$$

	$E_{gap} / (m^*/m_e)$
InSb	15
InAs	16.5
InP.	19.45.

$$4E^0(\pi/a) \sim 4 \cdot \frac{\hbar^2 \pi^2}{2ma^2}$$

$a \sim 2.5 \times 10^{-10}$ m. (not much variation between materials)

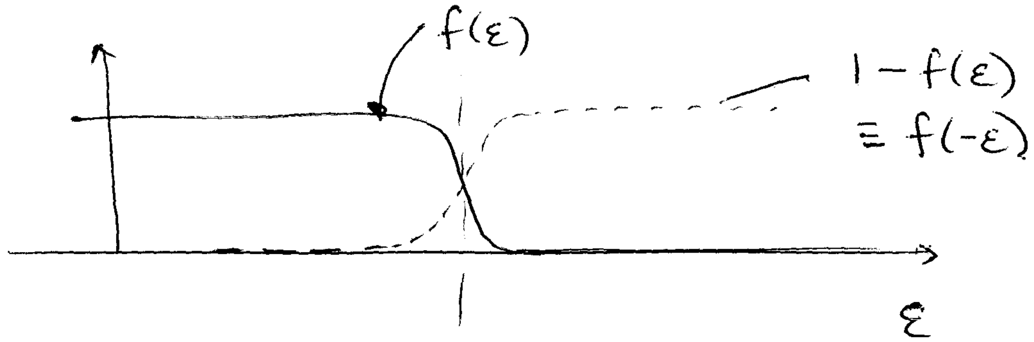
So $4E^0(\pi/a) \sim 20$ eV.

Conclusion: Rough consistency, most of the trend of m^* variation accounted for by change of band gap

Question 29

Simple algebra.

$$\frac{1}{e^{x}+1} + \frac{1}{e^{-x}+1} = \frac{e^{-x}+1+e^{x}+1}{1+1+e^{x}+e^{-x}} = \underline{1}$$



Question 30

Ionized donors & acceptors at RT.

$n - p = N_d - N_a \simeq n$ (p very small)

$n = 2 \times 10^{22} \, m^{-3}$

$np = n_i^2 \implies p = 2.9 \times 10^{16} \, m^{-3}$.

$\sigma = ne\mu$ $R_H = -\frac{1}{n|e|}$

Question 31

Effective Hamiltonian for a donor impurity is

$$\mathcal{H} = -\frac{\hbar^2}{2m^*}\nabla^2 + \frac{e^2}{4\pi\varepsilon\varepsilon_0 r}.$$

$\Rightarrow$ Hydrogenic, but with.

$$\text{Rydberg} = \frac{m^*}{2\hbar^2}\cdot\left(\frac{e^2}{4\pi\varepsilon\varepsilon_0}\right)^2 = \left(\frac{m^*}{m}\right)\frac{1}{\varepsilon^2} \times 13.6\,\text{eV}.$$

$$\text{Bohr radius} = \frac{\varepsilon\hbar^2 4\pi\varepsilon_0}{m^* e^2} = \frac{\varepsilon}{(m^*/m)}\, 0.5\cancel{3} \times 10^{-10}\,\text{m}.$$

For InSb. $$\text{Rydberg}^* = \frac{.015}{18^2}\, 13.6\ \text{eV}$$
$$= 6\times10^{-4}\ \text{eV}.$$

— very small (1 meV $\approx$ 10 Kelvin)

$$a^*_{Bohr} = 60\ \text{nm}.$$

Significant overlap when

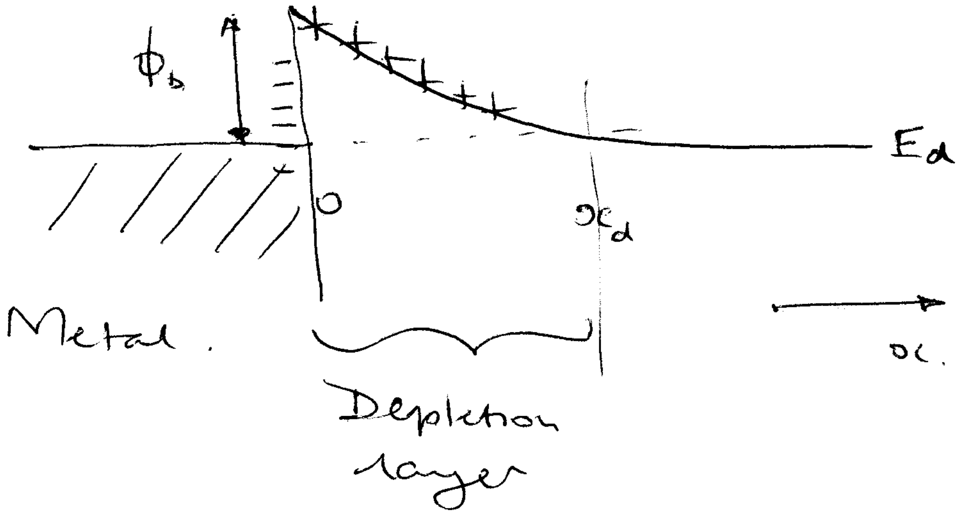
$$n(a_B^*)^3 \sim 1$$

$$\text{ie. } n \sim \left(\frac{1}{a_B^*}\right)^3 \sim \text{few}\times10^{21}\,\text{m}^{-3}$$

Question 32

Schottky barrier

See figure 13.1 (c)



In the depletion layer, charge density is $N_d e$ where N_d is donor density.

Poisson $\Rightarrow$ $\nabla . D = \frac{N_d e}{\varepsilon_0}$

So the electrostatic potential ~~[illegible]~~

$$\frac{d^2\Phi}{dx^2} = -\frac{N_d e}{\varepsilon \varepsilon_0}$$

Solution is

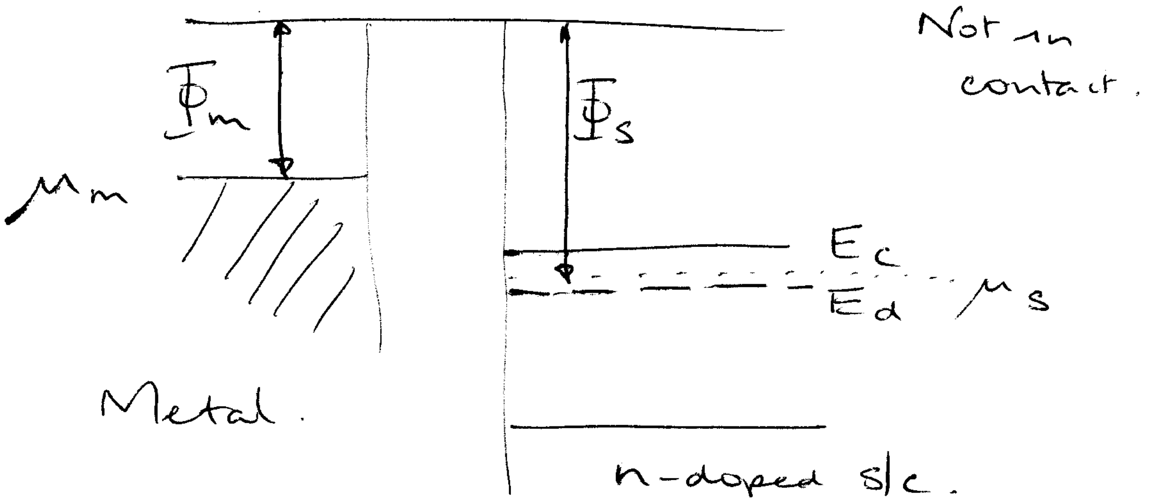
$$\Phi = \Phi_b - \frac{N_d e}{2\varepsilon_0\varepsilon} x^2$$

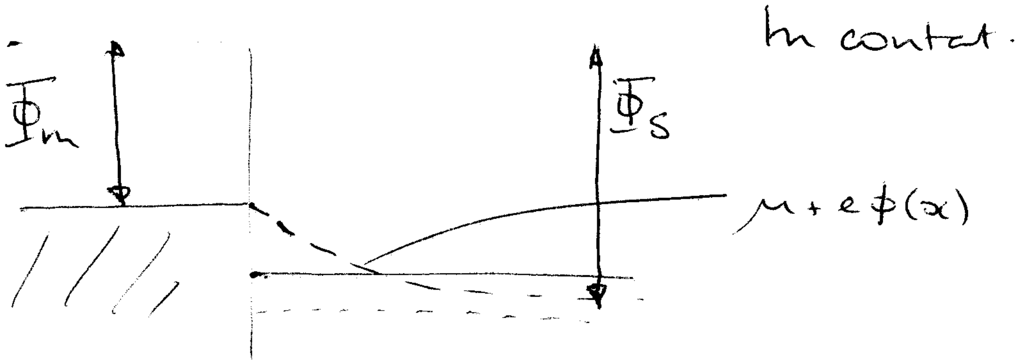
$\Rightarrow$ Width of depletion layer

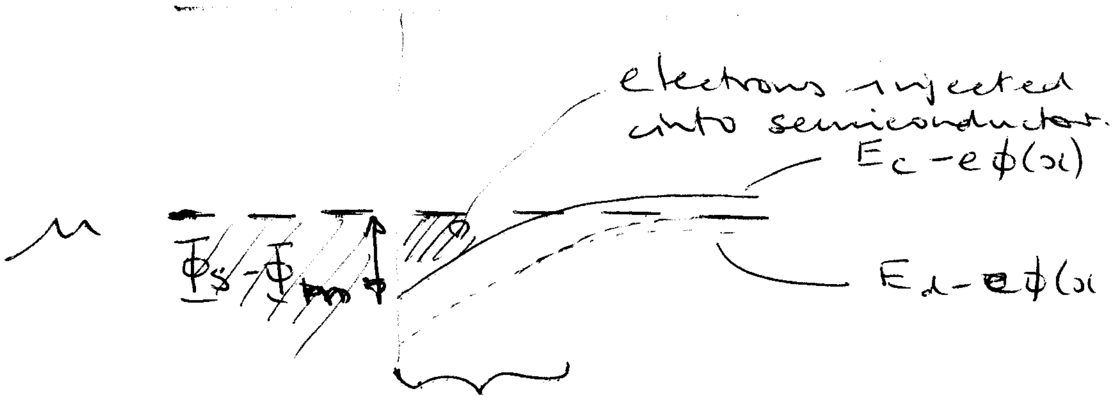
$$x_d = \left(\frac{2\varepsilon\varepsilon_0 \Phi_b}{N_d e} \right)^{1/2}$$

$N_d = 10^{22}\,m^{-3}$; $\phi_b = 0.5V$ $\varepsilon = 12$

$\Rightarrow$ $\boxed{x_d \sim 0.27\,\mu m}$







this length scale is short, because the n-doped s/c is metallized & screens ϕ well

— no barrier

— good metallic contact

— "ohmic" contact