Topology of (1D) Quantum Systems Out of Equilibrium

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Controlling Quantum Matter: From Ultracold Atoms to Solids
Humboldt Kolleg, Vilnius, 30 July 2018

Max McGinley & NRC, arXiv:1804.05756

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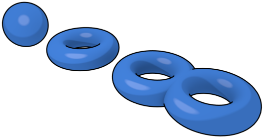
Topological Invariants

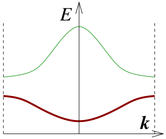


genus $g = 0, 1, 2, \ldots$

$$\frac{1}{2\pi}\int_{\text{closed surface}} \kappa \, dA \quad = \; (2-2g)$$

Gaussian curvature $\kappa = \dfrac{1}{R_1 R_2}$

2D Bloch bands [Thouless, Kohmoto, Nightingale & den Nijs (1982)]



Chern number $\nu = \frac{1}{2\pi}\int_{\text{BZ}} d^2k \; \Omega_{\boldsymbol{k}}$

Berry curvature $\Omega_{\boldsymbol{k}} = -i\nabla_{\boldsymbol{k}} \times \langle u_{\boldsymbol{k}} | \nabla_{\boldsymbol{k}} u_{\boldsymbol{k}} \rangle \cdot \hat{\boldsymbol{z}}$

- ν cannot change under smooth deformations of the energy band
- bulk insulator with ν (chiral) metallic surface states

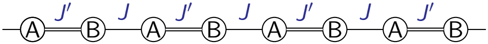
Topological Insulators

[Hasan & Kane, RMP 2010]

Many generalizations when *symmetries* included:
"symmetry-protected" topological insulators/superconductors

– Time-reversal symmetry (non-magnetic material in $B = 0$)
$\Rightarrow$ 3D bulk insulator with metallic 2D surfaces

– Su-Schrieffer-Heeger model

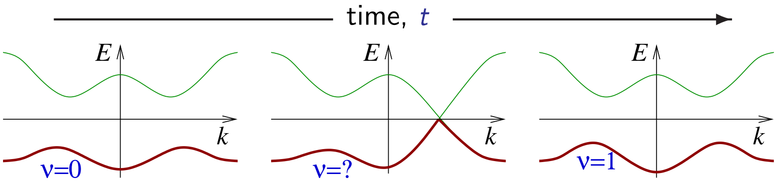


$$\mathrm{H}_k = -\begin{pmatrix} 0 & J' + J\mathrm{e}^{-\mathrm{i}ka} \\ J' + J\mathrm{e}^{\mathrm{i}ka} & 0 \end{pmatrix}$$

"chiral" symmetry
$$\sigma_z \mathrm{H}_k = -\mathrm{H}_k \sigma_z$$

$\Rightarrow$ 1D band insulator with gapless edge modes

Dynamical changes in band topology?



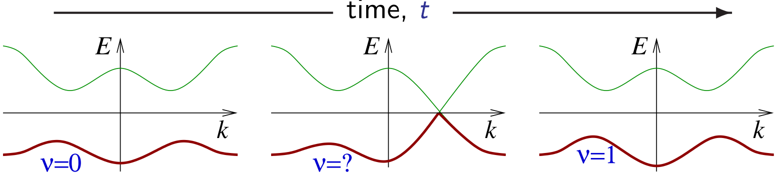
What are the consequences for the topology of the system?

– preparation of topological states?

– non-adiabatic ⇒meaning of topology out of equilibrium?

Dynamics of Chern Insulators (2D)

Quench: start in ground state of $\hat{H}^{\mathrm{i}}$, time evolve with $\hat{H}^{\mathrm{f}}$



Time-evolving Bloch state of fermion at $\boldsymbol{k}$

$$|u_{\boldsymbol{k}}(t)\rangle = \exp(-i\hat{H}^{\mathrm{f}}_{\boldsymbol{k}}t)|u_{\boldsymbol{k}}(0)\rangle$$

$$\Omega_{\boldsymbol{k}}(t) = -i\nabla_{\boldsymbol{k}} \times \langle u_{\boldsymbol{k}}(t)|\nabla_{\boldsymbol{k}} u_{\boldsymbol{k}}(t)\rangle \cdot \hat{\boldsymbol{z}}$$

$\Rightarrow$Chern number ν of the *state* is invariant

[L. D'Alessio & M. Rigol, Nat. Commun. (2015); M.D. Caio, NRC & M.J. Bhaseen, PRL (2015)]

["topological invariant" under smooth changes of the Bloch states]

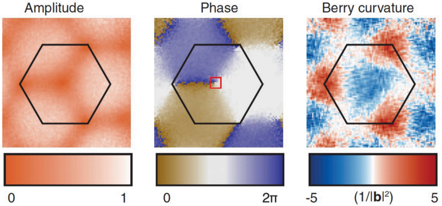
Dynamics of Chern Insulators: Physical Consequences?

- Obstruction to preparation of state with differing Chern number

  [For slow ramps, and in finite systems, deviations can be small.]

- Chern number can be obtained by tomography of Bloch states

Two-band model: $|u_{\boldsymbol{k}}(t)\rangle = \sin(\theta_{\boldsymbol{k}}/2)|A, \boldsymbol{k}\rangle - \cos(\theta_{\boldsymbol{k}}/2)\exp(i\phi_{\boldsymbol{k}})|B, \boldsymbol{k}\rangle$



[N. Fläschner, B. S. Rem, M. Tarnowski, D. Vogel, D.-S. Lühmann, K. Sengstock & C. Weitenberg, Science (2016)]

- Topology of *final* Hamiltonian can be uncovered by tracking the time evolution of the Bloch states in $(\boldsymbol{k}, t)$ space [Next talk, Xiong-Jun Liu]

Any differences for symmetry-protected topological classes?

Topology of 1D Quantum Systems (Free Fermions)

In 1D, all topological invariants can be determined by:

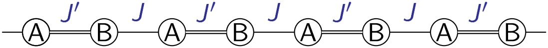
$$\mathrm{CS}_1 = \frac{i}{2\pi} \int_{\mathrm{BZ}} dk \ \langle u_k | \partial_k u_k \rangle$$

Equivalently: Berry phase around the Brillouin Zone (Zak phase)

Only quantized in the presence of symmetries

In 1D, topology must be protected by symmetry

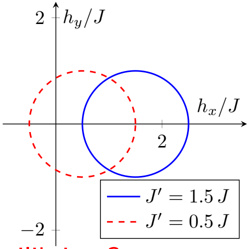
Example: Su-Schrieffer-Heeger Model



$$\mathrm{H}_k = -\begin{pmatrix} 0 & J' + J\mathrm{e}^{-\mathrm{i}ka} \\ J' + J\mathrm{e}^{\mathrm{i}ka} & 0 \end{pmatrix} = -\boldsymbol{h}(k)\cdot\boldsymbol{\sigma}$$

Chiral symmetry $\Rightarrow \boldsymbol{h} = (h_x, h_y, 0) \Rightarrow h_x + \mathrm{i}h_y \equiv |\boldsymbol{h}(k)|\mathrm{e}^{\mathrm{i}\phi(k)}$

$$\mathrm{CS}_1 = \frac{1}{2}\underbrace{\frac{1}{2\pi}\int_{\mathrm{BZ}}\frac{d\phi}{dk}dk}_{\text{integer}}$$



Is this topological invariant preserved out of equilibrium?

No... need to consider symmetries!

Symmetry-Protected Topology Out of Equilibrium

[Max McGinley & NRC, arXiv:1804.05756]

- Start in ground state of $\hat{\mathcal{H}}^{\mathrm{i}}$, then time evolve with $\hat{\mathcal{H}}^{\mathrm{f}}$
- $\hat{\mathcal{H}}^{\mathrm{f}}$ breaks symmetry $\rightarrow$ topological "invariant" can vary
  [“explicit symmetry breaking”]
- What if $\hat{\mathcal{H}}^{\mathrm{f}}$ respects symmetry?

Symmetry can still be broken after a quench

▷ Antiunitary symmetries [$\langle \hat{\mathcal{O}}\Phi, \hat{\mathcal{O}}\Psi\rangle = \langle \Phi, \Psi\rangle^*$]

$$\hat{\mathcal{O}} e^{-i\hat{\mathcal{H}}t}\hat{\mathcal{O}}^{-1} = e^{+i\hat{\mathcal{H}}t}$$

Symmetry broken in the non-equilibrium state $|\Psi(t)\rangle$
[“dynamically induced symmetry breaking”]

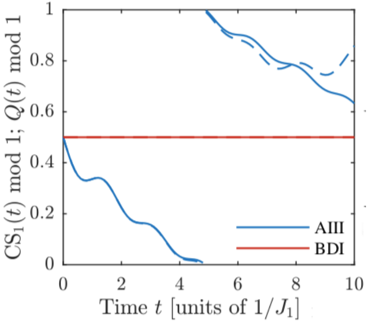
Topological "invariant" time-varying even if symmetries respected!

Time-Varying $\mathrm{CS}_1(t)$: Physical Consequences

- Could be observed in Bloch state tomography [cf. Chern number]
- Directly measure, via: $\frac{d}{dt}\mathrm{CS}_1(t) = j(t) = \dot{Q}(t)$

  [cf. in 2D, Chern number $\neq$ Hall conductance out of equilibrium]

Example: quenches in a generalized SSH model



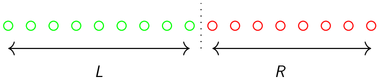
BDI: time-reversal, particle-hole & chiral

AIII: chiral symmetry only

Topological Classification Out of Equilibrium

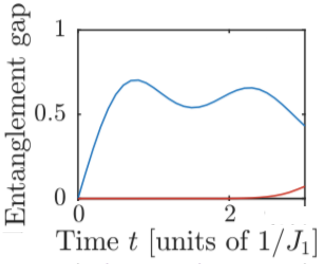
Equilibrium topological state $\Rightarrow$gapless edge state

Non-equilibrium topological state $\Rightarrow$gapless *entanglement spectrum*



$$|\Psi(t)\rangle = \sum_i e^{-\lambda_i} |\psi_L^i\rangle \otimes |\psi_R^i\rangle$$

Example: quenches in a generalized SSH model



BDI: time-reversal, particle-hole & chiral

AIII: chiral symmetry only

$\Rightarrow$Meaningful topological classification out of equilibrium

Topological Classification Out of Equilibrium

"Ten-fold way" for free fermions [Chiu, Teo, Schnyder & Ryu, RMP (2016)]
[Time-reversal, particle-hole, and chiral symmetries]

Non-equilibrium classification in 1D [Max McGinley & NRC, arXiv:1804.05756]

Class	T	C	S	$\mathrm{CS}_1(t=0)$	$\mathrm{CS}_1(t) \bmod 1$	eq. $\to$ non-eq.
AIII	0	0	1	$\mathbb{Z}/2^*$	Varies $[0,1)$	$\mathbb{Z} \to 0$
BDI	$+$	$+$	1	$\mathbb{Z}/2^*$	Const. $\{0,1/2\}$	$\mathbb{Z} \to \mathbb{Z}_2$
D	0	$+$	0	$\mathbb{Z}/2 \bmod 1$	Const. $\{0,1/2\}$	$\mathbb{Z}_2 \to \mathbb{Z}_2$
DIII	$-$	$+$	1	$\mathbb{Z} \bmod 2^*$	Const. 0	$\mathbb{Z} \to 0$
CII	$-$	$-$	1	$\mathbb{Z}^*$	Const. 0	$\mathbb{Z} \to 0$

Summary

• Out of equilibrium, topological "invariants" can vary in time:
"dynamically induced symmetry breaking"

$\Rightarrow$ no obstruction to changing the topology of the state dynamically

$\Rightarrow$ sensitivity to noise

• In 1D such time-variations appear as a measurable current

• There is a robust topological classification of non-equilibrium states, which differs from that at equilibrium:
bulk-boundary correspondence applies to entanglement spectrum

[holds also for interacting + disordered systems]

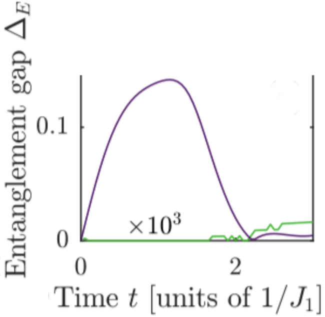
Generalization to Interacting SPT phases

[Max McGinley & NRC, arXiv:1804.05756]

Example: the Haldane phase of a $S = 1$ spin chain (e.g. AKLT model) is an SPT phase, stabilized by various symmetries:

- TRS (anti-unitary)
- $Z_2 \times Z_2$ dihedral symmetry (unitary)

Only unitary symmetries are preserved out of equilibrium



TRS only (purple)
both TRS and dihedral (green)