Quantum Quenches in Chern Insulators

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M.D. Caio, NRC & M.J. Bhaseen, arXiv:1504.01910
Topological Invariants

Gaussian curvature \[ \kappa = \frac{1}{R_1 R_2} \]

\[ \frac{1}{2\pi} \int_{\text{closed surface}} \kappa \, dA = (2 - 2g) \]

Gauss-Bonnet Theorem

genus \( g = 0, 1, 2, \ldots \) for sphere, torus, 2-hole torus...

Topological invariant: \( g \) cannot change under smooth deformations
Topological Features of 2D Bands

[Thouless, Kohmoto, Nightingale & den Nijs (1982)]

Chern number

\[ \nu = \frac{1}{2\pi} \int_{\text{BZ}} d^2k \, \Omega_k \]

Berry curvature

\[ \Omega_k = -i \nabla_k \times \langle u| \nabla_k u \rangle \cdot \hat{z} \]

crystal momentum \( \mathbf{k} \), Bloch state \( |u_k\rangle \)

Topological invariant:

\( \nu \) cannot change under smooth variations of the energy band
Physical Consequences

Chern band filled with fermions ("Chern insulator"): 

- quantized Hall effect, \( \sigma_{xy} = \nu \frac{e^2}{h} \)  

[TKNN (1982)]
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Chern band filled with fermions ("Chern insulator"):

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- $\nu$ gapless chiral edge states

[TKNN (1982)]
Dynamical changes in band topology?

\[ \text{time, } t \]

\[ \nu = 0 \quad \nu = ? \quad \nu = 1 \]
Dynamical changes in band topology?

Bosons / BEC: Adiabatic

- Adiabatic formation of vortex lattice

Dynamical changes in band topology?

Bosons / BEC: Adiabatic
- Adiabatic formation of vortex lattice

Fermionic Band Insulator: Non-adiabatic
- Effects of quenching between band topologies?


[M.D. Caio, NRC & M.J. Bhaseen, arXiv:1504.01910]
Bosons: Adiabatic Formation of Vortex Lattice
  Harper-Hofstadter Model
  Adiabatic Route

Fermions: Quench of Band Topology
  Haldane Model
  Preservation of Chern Number
  Relaxation of Edge Currents
Harper-Hofstadter Model

Effective magnetic field

flux density \( n_\phi = \frac{\alpha}{2\pi} \)
Harper-Hofstadter Model

Bosons: Adiabatic Formation of Vortex Lattice
Fermions: Quench of Band Topology

Harper-Hofstadter Model
Adiabatic Route

Effective magnetic field
flux density \( n_\phi = \frac{\alpha}{2\pi} \)

Imprint phases on tunneling matrix elements
[Jaksch & Zoller '03; Mueller '04; Sørensen, Demler & Lukin '05; Gerbier & Dalibard 2010; Struck et al. (2012)...]

e.g. tilted lattice [MIT,LMU]

\[ J_{\text{eff}} \sim \frac{\Omega_1 \Omega_2^*}{\Delta'} \] inherits phase difference of the Raman beams
Flux density $n_\phi = \frac{\alpha}{2\pi}$

$n_\phi = p/q$:  
- $q$ bands with (in general) non-zero Chern numbers
- weakly interacting BEC in band minimum  
  $\Rightarrow$ vortex lattice with vortex density $n_\phi$
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Can one adiabatically create such high vortex densities?
Adiabatic Route: Essential Idea

e.g. $n_\phi = 1/3 \ (\alpha = 2\pi/3)$

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For fixed unit cell, vary phase \( \alpha = 0 \rightarrow 2\pi/3 \)

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\[ e.g. \quad n_\phi = 1/3 \quad (\alpha = 2\pi/3) \]

\[ \text{magnetic unit cell} \]

For fixed unit cell, vary phase \( \alpha = 0 \rightarrow 2\pi/3 \)

\[ e.g. \quad \text{RF + Raman} \quad \quad Je^{i\phi} = J_{\text{RF}} + J_{\text{Raman}}e^{-i\frac{2\pi}{3a}y} \]

Adiabatic Route: Essential Idea

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$\Rightarrow$ adiabatic path from uniform BEC to vortex lattice

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For fixed unit cell, vary phase \( \alpha = 0 \rightarrow 2\pi/3 \)

e.g. RF + Raman \( J e^{i\phi} = J_{RF} + J_{Raman} e^{-i\frac{2\pi}{3\alpha}} y \)

\( \Rightarrow \) adiabatic path from uniform BEC to vortex lattice
Repulsive interactions select a particular vortex lattice unit cell.

Same periodicity as

\[ J_{x,y} = J_{RF} + J_{Raman} e^{-i(x+y) \frac{2\pi}{3a}} \]

BEC is loaded into the stable vortex lattice as \( \alpha = 0 \rightarrow \frac{2\pi}{3} \)
Outline

Bosons: Adiabatic Formation of Vortex Lattice
Harper-Hofstadter Model
Adiabatic Route

Fermions: Quench of Band Topology
Haldane Model
Preservation of Chern Number
Relaxation of Edge Currents
Haldane Model

[F. D. M. Haldane, PRL 61, 2015 (1988)]

Cold atom realization:

[Jotzu et al. [ETH], Nature 515, 237 (2014)]

Honeycomb lattice

A, B sublattices

\[ \hat{H} = -t_1 \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \text{h.c.}) \]
Haldane Model

\[ \hat{H} = -t_1 \sum_{\langle i,j \rangle} \left( \hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) - t_2 \sum_{\langle\langle i,j \rangle\rangle} \left( e^{i\varphi_{ij}} \hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + M \sum_{i \in A} \hat{n}_i - M \sum_{i \in B} \hat{n}_i \]

nearest neighbour
next n.n.

\( \varphi_{ij} = \pm \varphi \) breaks time-reversal symmetry
\( M \) breaks inversion symmetry

⇒ band gap

Cold atom realization:

\[ \text{Honeycomb lattice} \]
\[ A, B \text{ sublattices} \]

Haldane Model

[F. D. M. Haldane, PRL 61, 2015 (1988)]
Haldane Model: Phase Diagram

Boundaries where gaps close at two Dirac points

Non-topological ($\nu = 0$)  Topological ($\nu = \pm 1$)

[After Jotzu et al. [ETH], Nature 515, 237 (2014)]
Quenches in the Haldane Model

Change Chern number of lowest band (sign of $m_\alpha$)

\[

\begin{pmatrix}
  +m_\alpha & -c[k_x + i\alpha k_y] \\
  -c[k_x - i\alpha k_y] & -m_\alpha
\end{pmatrix}

\]
Quenches in the Haldane Model

Change Chern number of lowest band (sign of \( m_\alpha \))

- non-interacting fermions
- isolated system (unitary evolution)

\[
\begin{pmatrix}
+ m_\alpha & -c [k_x + i \alpha k_y] \\
-c [k_x - i \alpha k_y] & -m_\alpha
\end{pmatrix}
\]
(1) Momentum space: time-evolution of Chern number, $\nu$

Occupied single particle states:

$$|\psi_\alpha(k)\rangle = a_\alpha(k)e^{-iE_\alpha^l(k)t}|l_\alpha(k)\rangle + b_\alpha(k)e^{-iE_\alpha^u(k)t}|u_\alpha(k)\rangle$$

[\text{l/u denote lower/upper bands}]
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[I/u denote lower/upper bands]

Chern number of the state

$$\nu(t) = \sum_{\alpha = \pm 1} -\alpha \text{ sign } m_\alpha \left( \frac{1}{2} - |b_\alpha(0)|^2 \right)$$

$$-|b_\alpha(\infty)||a_\alpha(\infty)| \cos[(E^u_\alpha(\infty) - E^l_\alpha(\infty))t + \delta]$$

But, $b_\alpha(\infty) = 0$ always... so Chern number of state unchanged
Band Hamiltonian describes a “spin” in an effective magnetic field

\[ \hat{H}_k = -h_k \cdot \hat{\sigma} \]

band gap \( \Rightarrow |h_k| \neq 0 \)
Preservation of Chern Number

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\( \nu \) is the number of times \( \frac{\mathbf{h}_k}{|\mathbf{h}_k|} \) wraps the sphere in the BZ

e.g. BZ of the honeycomb lattice

The “spins” precess in new \( \mathbf{h}_k \), but preserve their winding number

[D’Alessio & Rigol, arXiv:1409.6319]
Preservation of Chern Number

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Out of equilibrium, the Chern number of the state is different from that of the (new) Hamiltonian
(2) Real Space: Edge States and Edge Currents

At equilibrium, the Chern insulator has $\nu$ gapless edge states.
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How do edge currents change after a quench to new Hamiltonian?
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How do edge currents change after a quench to new Hamiltonian?

**Finite-width strip** (infinite length)

$N$ rows of lattice sites
Edge States

\[
t_1 = 1, \ t_2 = 1/3, \ M = 1, \ N = 20
\]

Non-topological (\(\varphi = \pi/6\))  
Topological (\(\varphi = \pi/3\))

Topological phase has edge states
Dynamics of Edge Currents

Quench from topological to non-topological phase

\[ t_1 = 1, \quad t_2 = 1/3, \quad \varphi = \pi/3, \quad M = 1.4 \rightarrow 1.6 \quad \text{(inset, 2.2)} \]

Red Line: Ground state current of final Hamiltonian

\[ N = 30 \quad \text{(circles)} \]
\[ N = 40 \quad \text{(crosses)} \]
Dynamics of Edge Currents

Quench from topological to non-topological phase

\[ t_1 = 1, \quad t_2 = 1/3, \quad \varphi = \pi/3, \quad M = 1.4 \rightarrow 1.6 \ (\text{inset}, \ 2.2) \]

\( J_N(t) \)

Red Line: Ground state current of final Hamiltonian

⇒ fast relaxation to (close to) the ground state edge current of the new Hamiltonian

\( N = 30 \) (circles)

\( N = 40 \) (crosses)
Dynamics of Edge Currents

Quench from non-topological to topological phase

\[ J_{\text{edge}}(t) \]

Red Line: Ground state current of final Hamiltonian

⇒ fast relaxation to (close to) the ground state edge current of the new Hamiltonian
(1) Momentum space: Preservation of the Chern number $\nu$

Measure the wave function, e.g. by time-of-flight

[Zhao et al., PRA 84, 063629 (2011); Alba et al., PRL 107, 235301 (2011); Hauke et al., PRL 113, 045303 (2014)]

Amplitude  Phase  Berry curvature

[Fläschner et al. [Hamburg], arXiv:1509.05763]
Experimental Consequences

(1) Momentum space: Preservation of the Chern number $\nu$

Measure the *wave function*, e.g. by time-of-flight

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(2) Real space: Relaxation dynamics of edge currents

Measure currents, e.g. as for two-leg ladders


[Local version (microscope) should show light-cone spreading]
Summary

- Optical lattices allow dynamical changes in the topology of energy bands.
- For a BEC, the evolution can be adiabatic:
  - allows \textit{adiabatic} preparation dense vortex lattices.
- For fermionic band insulators, the dynamics is \textit{non-adiabatic}:
  - the Chern number of the state is preserved;
  - edge currents quickly relax to (close to) the equilibrium for the new Hamiltonian.

⇒ Out of equilibrium there can be a sharp distinction between the topology of the state and local observables.