Quantum Quenches in Chern Insulators

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CUA Seminar
M.I.T., November 10th, 2015

Marcello Caio & Joe Bhaseen (KCL), Stefan Baur (Cambridge)

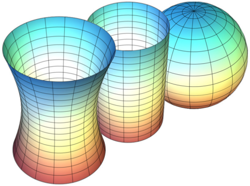
M.D. Caio, NRC & M.J. Bhaseen, arXiv:1504.01910

S. Baur & NRC, Phys. Rev. A **88**, 033603 (2013)

EPSRC
Engineering and Physical Sciences Research Council

Topological Invariants

Gaussian curvature $\kappa = \dfrac{1}{R_1 R_2}$



negative, zero and positive κ

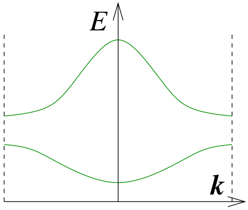
$$\frac{1}{2\pi}\int_{\text{closed surface}} \kappa \, dA = (2-2g)$$ Gauss-Bonnet Theorem

genus $g = 0, 1, 2, \ldots$ for sphere, torus, 2-hole torus...

Topological invariant: g cannot change under smooth deformations

Topological Features of 2D Bands

[Thouless, Kohmoto, Nightingale & den Nijs (1982)]



Chern number $\nu = \frac{1}{2\pi} \int_{\mathrm{BZ}} d^2\mathbf{k}\, \Omega_{\mathbf{k}}$

Berry curvature $\Omega_{\mathbf{k}} = -i \nabla_{\mathbf{k}} \times \langle u | \nabla_{\mathbf{k}} u \rangle \cdot \hat{\mathbf{z}}$

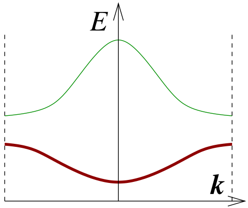
crystal momentum $\mathbf{k}$, Bloch state $|u_{\mathbf{k}}\rangle$

Topological invariant:

ν cannot change under smooth variations of the energy band

Physical Consequences

Chern band filled with fermions
("Chern insulator"):



- quantized Hall effect, $\sigma_{xy} = \nu \frac{e^2}{h}$

[TKNN (1982)]

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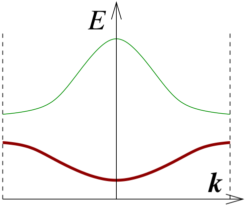
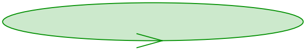
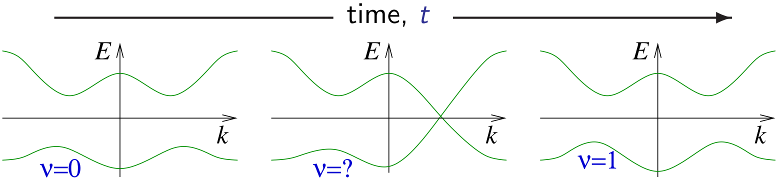


- quantized Hall effect, $\sigma_{xy} = \nu \frac{e^2}{h}$ [TKNN (1982)]

- ν gapless chiral edge states



Dynamical changes in band topology?



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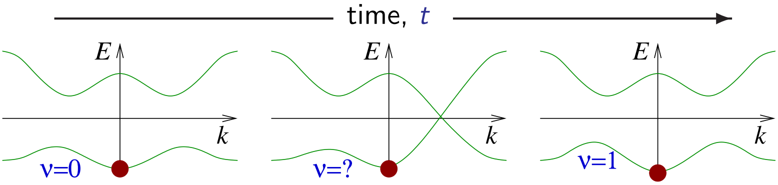

Bosons / BEC: Adiabatic

– Adiabatic formation of vortex lattice

[S. Baur & NRC, Phys. Rev. A **88**, 033603 (2013)]

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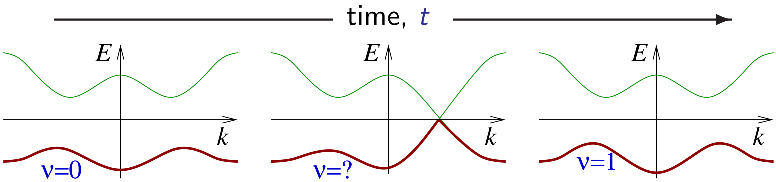

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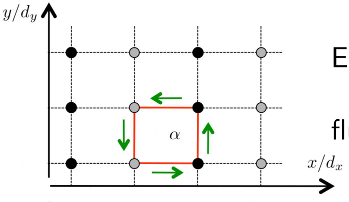
Fermionic Band Insulator: Non-adiabatic

– Effects of quenching between band topologies?

[M.D. Caio, NRC & M.J. Bhaseen, `arXiv:1504.01910`]

Outline

Harper-Hofstadter Model



Effective magnetic field

flux density $n_\phi = \dfrac{\alpha}{2\pi}$

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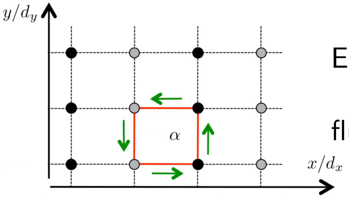

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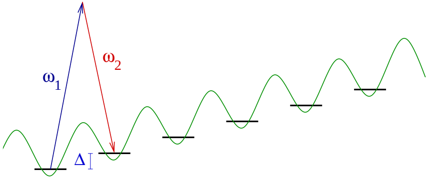
flux density $n_\phi = \dfrac{\alpha}{2\pi}$

Imprint phases on tunneling matrix elements

[Jaksch & Zoller '03; Mueller '04; Sørensen, Demler & Lukin '05; Gerbier & Dalibard 2010; Struck *et al.* (2012)...]

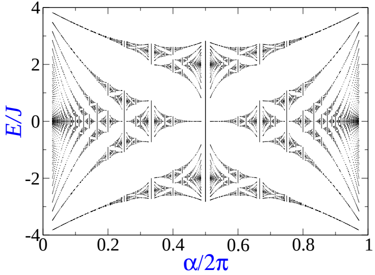
e.g. tilted lattice [MIT,LMU]



$J_{\text{eff}} \sim \dfrac{\Omega_1 \Omega_2^*}{\Delta'}$ inherits phase difference of the Raman beams

Harper-Hofstadter Spectrum

Flux density $n_\phi = \dfrac{\alpha}{2\pi}$



$n_\phi = p/q$:

– q bands with (in general) non-zero Chern numbers

– weakly interacting BEC in band minimum

$\Rightarrow$vortex lattice with vortex density n_ϕ

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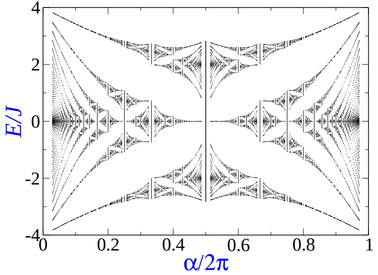

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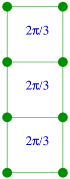
Can one adiabatically create such high vortex densities?

Adiabatic Route: Essential Idea

[S. Baur & NRC, Phys. Rev. A **88**, 033603 (2013)]

e.g. $n_\phi = 1/3$ ($\alpha = 2\pi/3$)

magnetic unit cell



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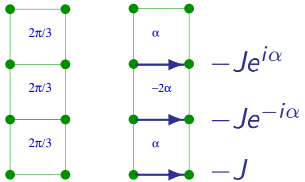

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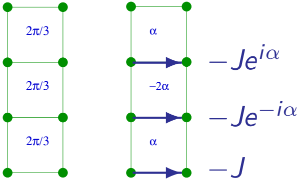

For fixed unit cell, vary phase $\alpha = 0 \to 2\pi/3$

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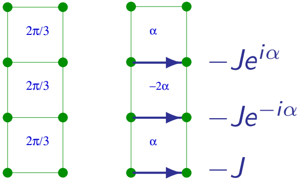

For fixed unit cell, vary phase $\alpha = 0 \rightarrow 2\pi/3$

e.g. RF + Raman $\quad Je^{i\phi} = J_{\mathrm{RF}} + J_{\mathrm{Raman}} e^{-i\frac{2\pi}{3a}y}$

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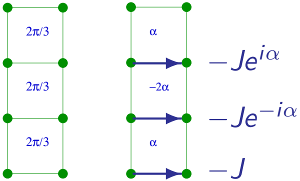

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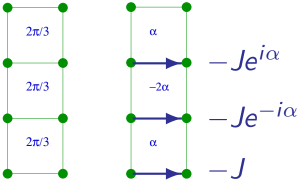

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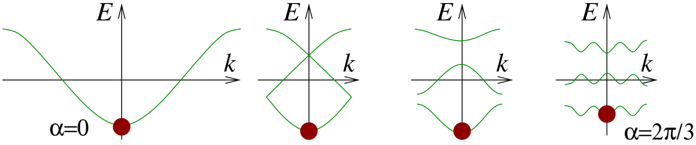
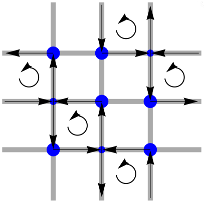


Adiabatic Route: more specifically...

Repulsive interactions select a particular vortex lattice unit cell

Same periodicity as $J_{x,y} = J_{\mathrm{RF}} + J_{\mathrm{Raman}} e^{-i(x+y)\frac{2\pi}{3a}}$ [cf. MIT, LMU]

α	α	-2α
-2α	α	α
α	-2α	α

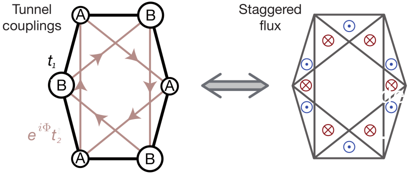


BEC is loaded into the stable vortex lattice as $\alpha = 0 \to 2\pi/3$

Outline

Haldane Model [F. D. M. Haldane, PRL **61**, 2015 (1988)]

Cold atom realization: [Jotzu *et al.* [ETH], Nature **515**, 237 (2014)]



Honeycomb lattice
A, B sublattices

$$\hat{H} = -t_1 \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right)$$

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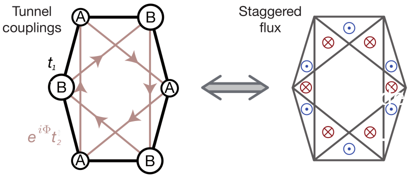


Honeycomb lattice
A, B sublattices

$$\hat{H} = -t_1 \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) - t_2 \sum_{\langle\langle i,j \rangle\rangle} \left(e^{i\varphi_{ij}} \hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right)$$

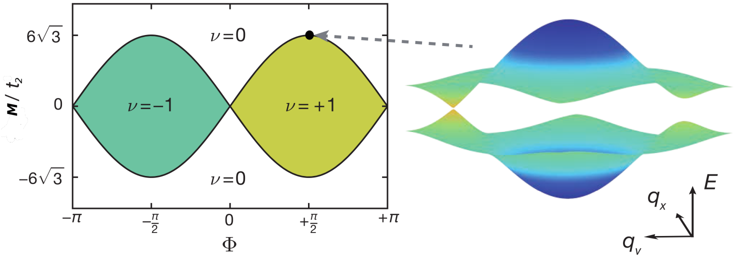
nearest neighbour next n.n.

$$+M \sum_{i \in A} \hat{n}_i - M \sum_{i \in B} \hat{n}_i$$

$\varphi_{ij} = \pm\varphi$ breaks time-reversal symmetry
M breaks inversion symmetry

$\Rightarrow$ band gap

Haldane Model: Phase Diagram



[After Jotzu *et al.* [ETH], Nature **515**, 237 (2014)]

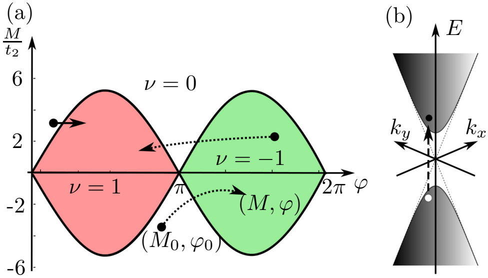
Boundaries where gaps close at two Dirac points

Non-topological ($\nu = 0$)

Topological ($\nu = \pm 1$)

Quenches in the Haldane Model

[Marcello Caio, NRC & Joe Bhaseen, arXiv:1504.01910]



Change Chern number of lowest band (sign of m_α)

$$\begin{pmatrix} +m_\alpha & -c\,[k_x + i\alpha k_y] \\ -c\,[k_x - i\alpha k_y] & -m_\alpha \end{pmatrix}$$

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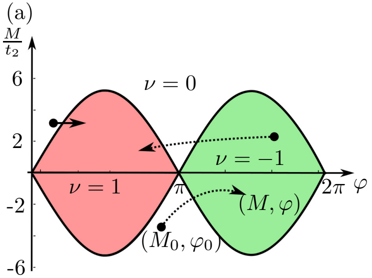

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- non-interacting fermions
- isolated system (unitary evolution)

(1) Momentum space: time-evolution of Chern number, ν

Occupied single particle states:

$$|\psi_\alpha(\mathbf{k})\rangle = a_\alpha(\mathbf{k}) e^{-\mathrm{i}E^l_\alpha(k)t} |l_\alpha(\mathbf{k})\rangle + b_\alpha(\mathbf{k}) e^{-\mathrm{i}E^u_\alpha(k)t} |u_\alpha(\mathbf{k})\rangle$$

[l/u denote lower/upper bands]

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[l/u denote lower/upper bands]

Chern number of the state

$$\nu(t) = \sum_{\alpha=\pm1} -\alpha\,\mathrm{sign}\,m_\alpha\left(\frac{1}{2} - |b_\alpha(0)|^2\right.$$
$$\left. -|b_\alpha(\infty)||a_\alpha(\infty)|\cos[(E^u_\alpha(\infty) - E^l_\alpha(\infty))t + \delta]\right)$$

But, $b_\alpha(\infty) = 0$ always... so Chern number of state unchanged

Preservation of Chern Number

Band Hamiltonian describes a “spin” in an effective magnetic field

$\hat{H}_{\mathbf{k}} = -\mathbf{h}_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}}$ band gap $\Rightarrow |\mathbf{h}_{\mathbf{k}}| \neq 0$

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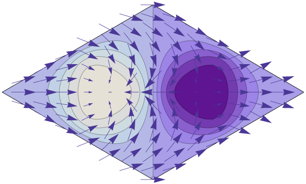
ν is the number of times $\dfrac{\mathbf{h}_{\mathbf{k}}}{|\mathbf{h}_{\mathbf{k}}|}$ wraps the sphere in the BZ

e.g. BZ of the honeycomb lattice 

The "spins" precess in new $\mathbf{h}_{\mathbf{k}}$, but preserve their winding number

[D'Alessio & Rigol, arXiv:1409.6319]

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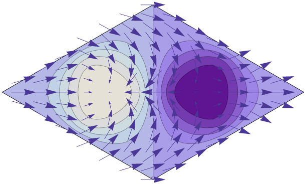

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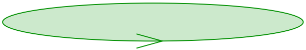
Out of equilibrium, the Chern number of the *state* is different from that of the (new) *Hamiltonian*

(2) Real Space: Edge States and Edge Currents

At equilibrium, the Chern insulator has ν gapless edge states



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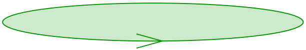

How do edge currents change after a quench to new Hamiltonian?

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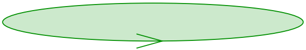
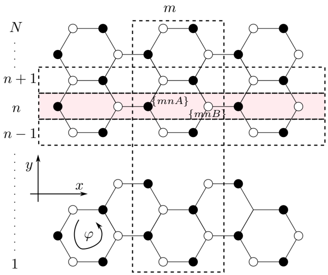


How do edge currents change after a quench to new Hamiltonian?

Finite-width strip (infinite length)

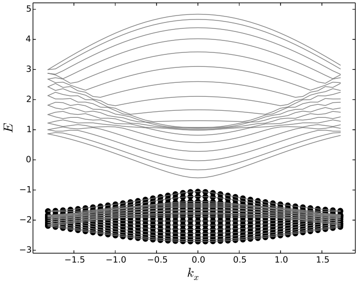
N rows of lattice sites



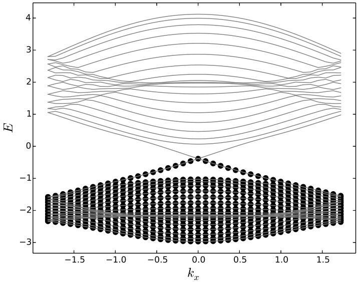
Edge States

$t_1 = 1,\ t_2 = 1/3,\ M = 1,\ N = 20$

Non-topological ($\varphi = \pi/6$)



Topological ($\varphi = \pi/3$)

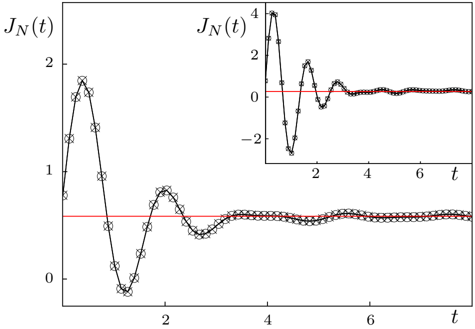


Topological phase has edge states

Dynamics of Edge Currents

Quench from topological to non-topological phase

$[t_1 = 1,\ t_2 = 1/3,\ \varphi = \pi/3,\ M = 1.4 \rightarrow 1.6\,(\text{inset}, 2.2)]$



$N = 30$ (circles)
$N = 40$ (crosses)

Red Line: **Ground state current of final Hamiltonian**

Dynamics of Edge Currents

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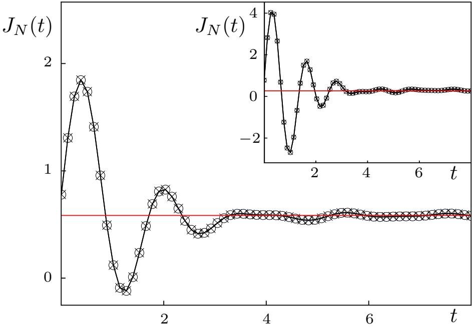

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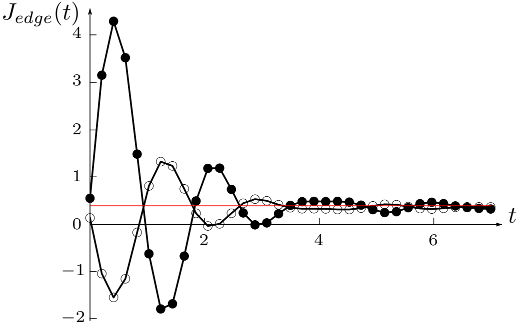
$\Rightarrow$fast relaxation to (close to) the ground state edge current of the new Hamiltonian

Dynamics of Edge Currents

Quench from non-topological to topological phase



Red Line: **Ground state current of <u>final</u> Hamiltonian**

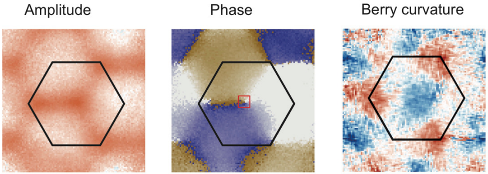
⇒fast relaxation to (close to) the ground state edge current of the new Hamiltonian

Experimental Consequences

(1) Momentum space: Preservation of the Chern number ν

Measure the *wave function*, *e.g.* by time-of-flight

[Zhao *et al.*, PRA **84**, 063629 (2011); Alba *et al.*, PRL **107**, 235301 (2011); Hauke *et al.*, PRL **113**, 045303 (2014)]



[Fläschner *et al.*[Hamburg], arXiv:1509.05763]

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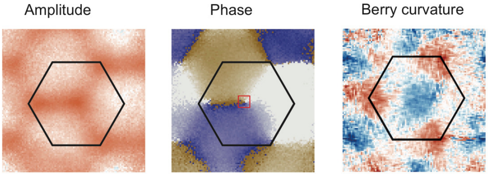

[Fläschner *et al.*[Hamburg], arXiv:1509.05763]

(2) Real space: Relaxation dynamics of edge currents

Measure currents, *e.g.* as for two-leg ladders

[M. Atala, M. Aidelsburger, M. Lohse, J. T. Barreiro, B. Paredes & I. Bloch, Nat. Phys. **10**, 588 (2014)]

[Local version (microscope) should show light-cone spreading]

Summary

- Optical lattices allow dynamical changes in the topology of energy bands.
- For a BEC, the evolution can be adiabatic:
  – allows *adiabatic* preparation dense vortex lattices.
- For fermionic band insulators, the dynamics is *non-adiabatic*:
  – the Chern number of the state is preserved;
  – edge currents quickly relax to (close to) the equilibrium for the new Hamiltonian.

$\Rightarrow$Out of equilibrium there can be a sharp distinction between the topology of the state and local observables.