Skyrmions in Condensed Matter Systems

Nigel Cooper

Cavendish Laboratory

Skyrmion Anniversary, DAMTP

10 November, 2010

Outline

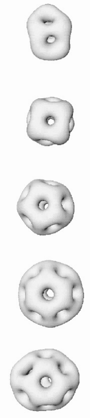
Introduction: Skyrmions

Skyrme model [T. H. R. Skyrme, Proc. Roy. Soc. **260**, 127 (1961)]

$$\vec{n}(\mathbf{r}) = (n_1, n_2, n_3, n_4) \text{ with } |\vec{n}|^2 = 1$$
$$\text{in 3D, with } \vec{n}(\mathbf{r} \to \infty) = \vec{n}_0$$

$S^3 \to S^3$ integer topological invariant

Topological solitons: Baryons



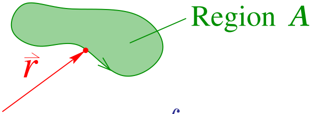
[Battye & Sutcliffe, PRL **79**, 363 (1997)]

"Baby" Skyrmions: $S^2 \to S^2$

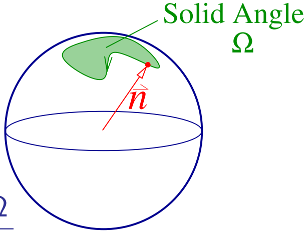
$\vec{n}(\mathbf{r}) = (n_1, n_2, n_3)$ with $|\vec{n}|^2 = 1$
in 2D, with $\vec{n}(\mathbf{r} \to \infty) = \vec{n}_0$

Topological density

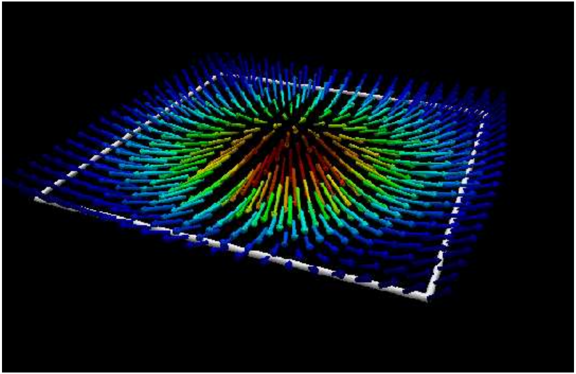
$$q(\mathbf{r}) = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu} n_i \nabla_\mu n_j \nabla_\nu n_k$$



$$\int_A q(\mathbf{r})\, d^2\mathbf{r} = \frac{\Omega}{4\pi}$$



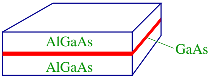
Skyrmion: $S^2 \to S^2$

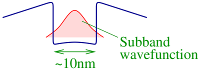


Outline

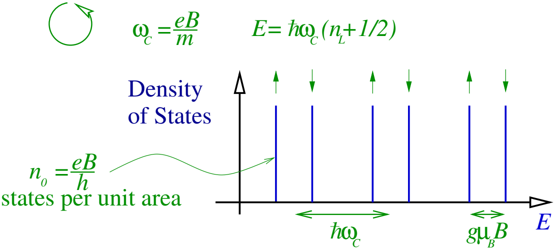
Quantum Hall Systems

Two-dimensional electron gas





$+$ strong perpendicular magnetic field B



Degeneracy

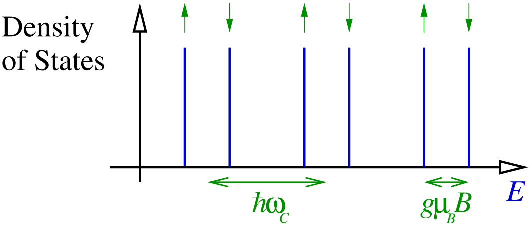
$$N_\phi = \frac{BA}{h/e}$$

Filling factor

$$\nu \equiv \frac{N_e}{N_\phi}$$

QH Ferromagnet at $\nu = 1$

$N_e = N_\phi$: one filled Landau level for one spin.

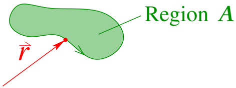


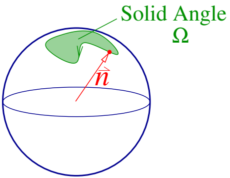
Ferromagnet $\Rightarrow$groundstate is fully polarized even for $g\mu_B B \to 0$.

Low-energy excitations described by local magnetic orientation $\vec{n}(\mathbf{r})$

$$\boxed{\text{Topological density} \iff \text{electric charge density}}$$

Geometric Phases for Itinerant Electrons





Aharonov-Bohm phase

$$\phi_{\mathrm{AB}} = 2\pi \frac{e}{h} \int_A B \, d^2\mathbf{r}$$

Berry phase

$$\phi_{\mathrm{Berry}} = \frac{\Omega}{2} = 2\pi \int_A q(\mathbf{r}) \, d^2\mathbf{r}$$

$$B^* = B + \tfrac{h}{e} q(\mathbf{r})$$

Skyrmion: $\int (B^* - B) \, d^2\mathbf{r} = \frac{h}{e} \times 1 \Rightarrow$ extra charge $-e$

Effective Action

[Sondhi *et al.*, Phys. Rev. B **47**, 16419 (1993)]

$$E \simeq \frac{1}{2}\rho_s \int (\nabla_\mu n_i)^2 d^2\mathbf{r} + g\mu_B B n_0 \frac{1}{2}\int (1 - n_3)\, d^2\mathbf{r} + \frac{e^2}{4\pi\epsilon_0}\int\int \frac{q(\mathbf{r})q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^2\mathbf{r} d^2\mathbf{r}'$$

Scaling $E \sim \rho_s \mathcal{L}^0 + g\mu_B B n_0 \mathcal{L}^2 + \dfrac{e^2}{4\pi\epsilon_0 \mathcal{L}}$

$\mathcal{L} \sim \left(\dfrac{e^2}{\epsilon g \mu_B B n_0}\right)^{1/3}$ $\Rightarrow$ Skyrmion size diverges for $g\mu_B B \to 0$

Charged excitations carry a large spin

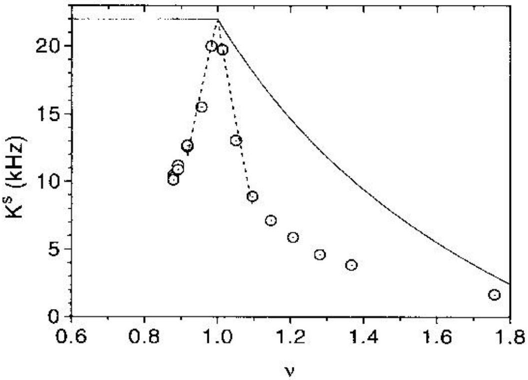
Predicted Experimental Consequences

- Formation of Skyrmions in the groundstate

$$\nu \equiv \frac{N_e}{N_\phi} = 1 \pm \epsilon$$

- Rapid loss of spin polarization
- Enhanced nuclear-spin relaxation rate
- Skyrmion crystal
- Dissipation by Skyrmion/antiSkyrmion pairs

Experimental Measurements: Spin Polarization



[Barrett *et al.*, PRL **74**, 5112 (1995)]

(anti-)Skyrmions with average spin 3.6

Fractional Quantum Hall State at $\nu = 5/2$

$$\frac{5}{2} = \underbrace{2}_{\text{filled } n_L = 0 \text{ LL for both spins}} + \underbrace{\frac{1}{2}}_{\text{half-filled } n_L = 1 \text{ LL for one spin}}$$

The "Moore-Read" state: [Moore & Read, Nucl. Phys. B360, 362 (1991)]

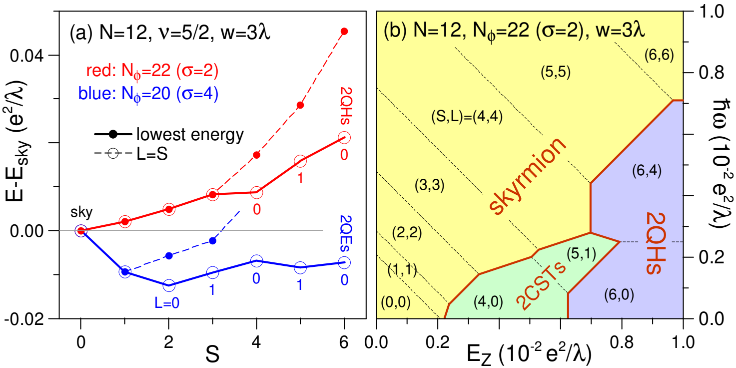
$e/4$ excitations should obey *non-abelian* exchange statistics.

$e/4$ excitations bind into $e/2$ Skyrmions at small Zeeman energies.

[A. Wòjs, G. Möller, S. Simon, and NRC, Phys. Rev. Lett. 104, 086801 (2010)]

Fractional Quantum Hall State at $\nu = 5/2$

[A. Wòjs, G. Möller, S. Simon, and NRC, Phys. Rev. Lett. 104, 086801 (2010)]



Outline

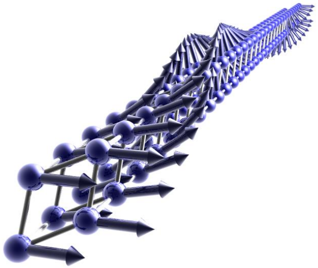
Chiral Magnets

Dzyaloshinskii-Moriya interaction

$$H = \int d^3\mathbf{r} \left[\frac{1}{2}\rho_s (\nabla_i n_j)^2 + \alpha \epsilon_{ijk} n_i \nabla_j n_k \right] + \ldots$$

Spin spiral $\vec{n} = (0, \cos qx, \sin qx)$

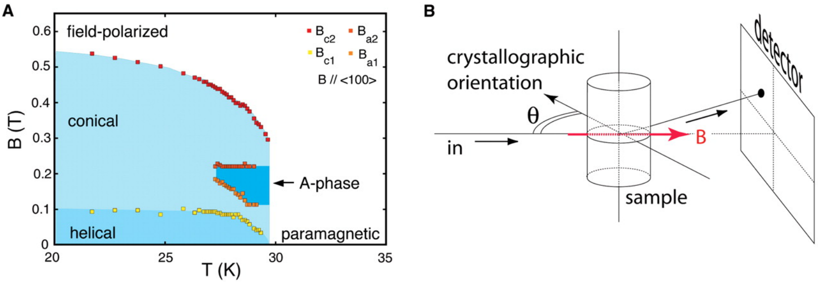
$$q = \frac{\alpha}{\rho_s}$$



MnSi: Phase Diagram

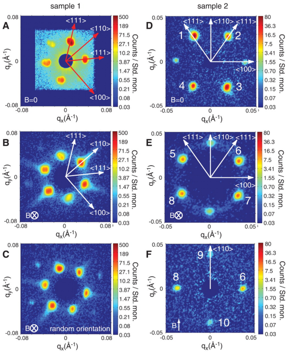
[Mühlbauer *et al.*, Science 323, 915919 (2009)]

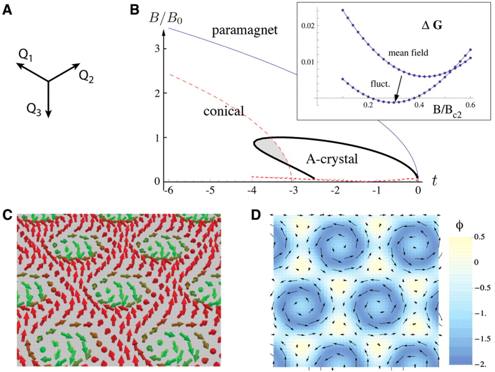
+ Zeeman field $g\mu_B B \frac{1}{2}(1-n_3)$
+ temperature T



MnSi: Neutron Scattering

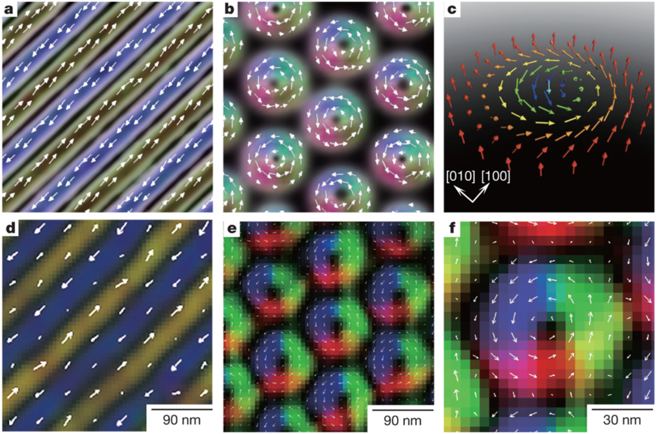
[Mühlbauer *et al.*, Science 323, 915919 (2009)]





$Fe_{1-x}Co_xSi$: Real Space Images

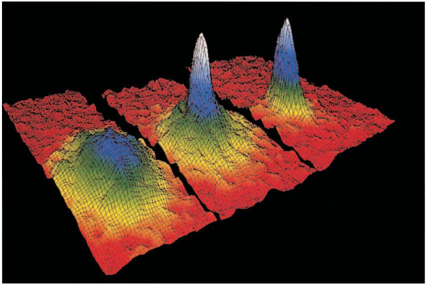
[X. Z. Yu, *et al.*, Nature **465**, 901 (2010)]



Outline

Atomic Bose-Einstein Condensates

[M. H. Anderson *et al.*, Science **269**, 198 (1995)]



$$k_B T_c \simeq \frac{\hbar^2}{m} n^{2/3} \sim 100\text{nK}$$

$$\Psi_N \propto \prod_{i=1}^{N} \psi(\mathbf{r}_i)$$

$$E = \int d^3\mathbf{r} \left[\frac{\hbar^2}{2m} |\boldsymbol{\nabla}\psi|^2 + V(\mathbf{r})|\psi|^2 + \frac{1}{2} U |\psi|^4 \right]$$

Spinor BECs

Hyperfine states, α $$\Psi_N \propto \prod_{i=1}^{N} \left[\sum_{\alpha} \psi_\alpha(\mathbf{r}_i)|\alpha_i\rangle \right]$$

$$E = \int d^3\mathbf{r} \left[\sum_\alpha \frac{\hbar^2}{2m} |\boldsymbol{\nabla}\psi_\alpha|^2 + V_\alpha(\mathbf{r})|\psi_\alpha|^2 + \frac{1}{2}\sum_{\alpha,\beta} U_{\alpha\beta}|\psi_\alpha|^2|\psi_\beta|^2 \right]$$

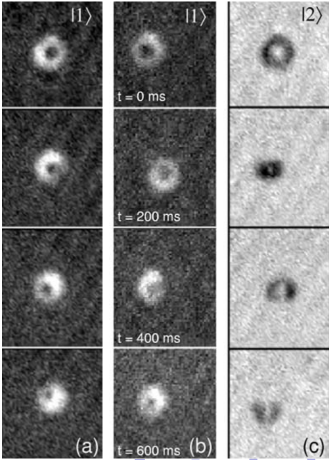
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}$$ $\Rightarrow$ topological solitons ?

Spinor BECs: 2D Skyrmions

[Matthews *et al.*, PRL **83**, 2498 (1998)]

$$\Psi(r) = \sqrt{n} \begin{pmatrix} \cos(\theta/2) \\ e^{i\varphi}\sin(\theta/2) \end{pmatrix}$$

$$\langle \vec{\sigma} \rangle = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

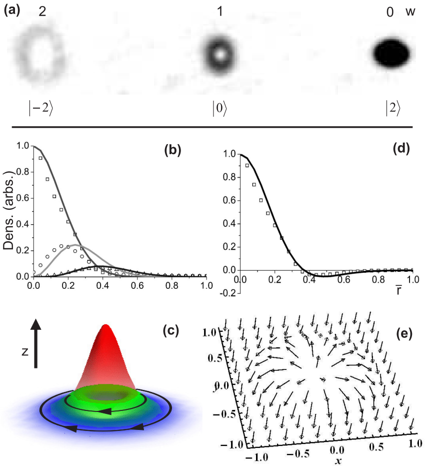


Spinor BECs: 2D Skyrmions

[Leslie *et al.*, PRL **103**, 250401 (2009)]

$$\sqrt{n}\begin{pmatrix} \cos^2(\theta/2) \\ 0 \\ \sqrt{2}e^{i\varphi}\sin(\theta/2)\cos(\theta/2) \\ 0 \\ e^{2i\varphi}\sin^2(\theta/2) \end{pmatrix}$$

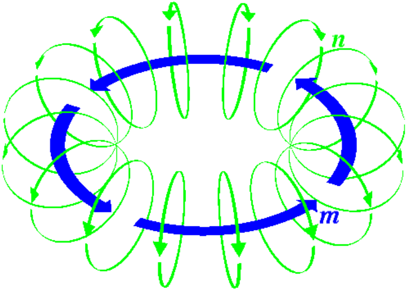
$$\vec{\ell} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$



Spinor BECs: 3D Skyrmions

Two-component BEC $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \sqrt{n} \begin{pmatrix} n_1 + in_2 \\ n_3 + in_4 \end{pmatrix}$

$$n_1^2 + n_2^2 + n_3^2 + n_4^2 = 1$$



$$Q = mn$$

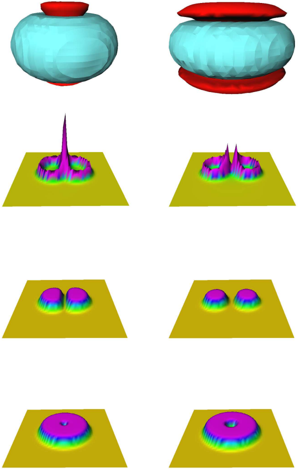
[cf. "Cosmic vortons"]

Stable 3D Skyrmions

[R. Battye, NRC & P. Sutcliffe, PRL (2002)]

$U_{12}^2 > U_{11} U_{22}$

$Q = 1$



$Q = 2$

Summary

Skyrmions appear in a variety of condensed matter settings.

- Quantum Hall Ferromagnets:
  topological density $\Leftrightarrow$ electric charge density
- Chiral Magnets: Skyrmion crystal phase
- Spinor BECs