Zero Resistance States from Surface Acoustic Waves

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Overview

- Surface Acoustic Waves
- Motivation(s)
- Classical Effects

– Weiss Oscillations (spatial periodicity)

– "Weiss-like" oscillations (spatial + temporal periodicity)

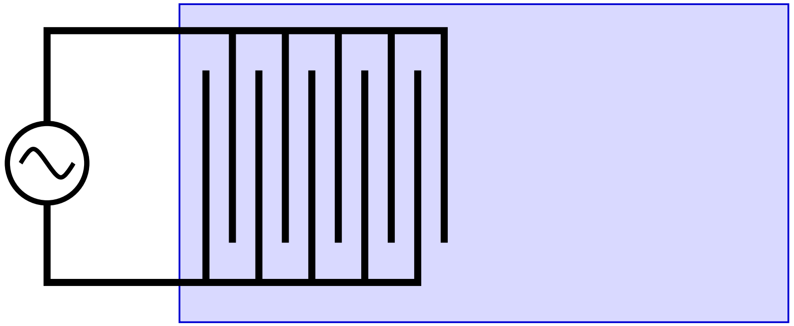
- Quantum Effects

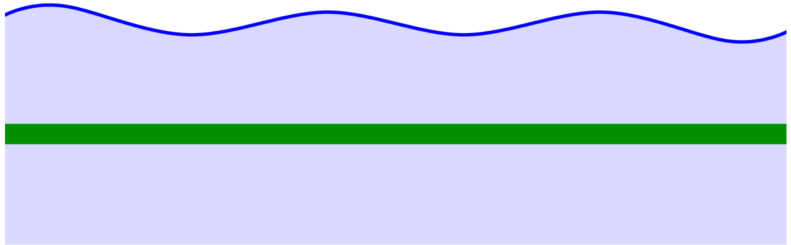
– Theories for ZRSs

– Quantum Effects of SAWs

- Summary

Surface Acoustic Waves





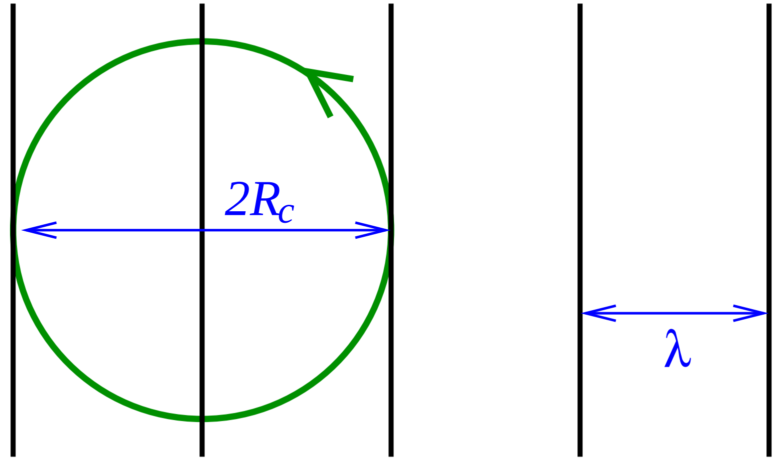
$\omega = sq \qquad s \simeq 3000$ m/s

$$\phi(x,t) = \phi_0 \sin(qx - \omega t) \quad \Rightarrow \quad \vec{E} = E_0 \cos(qx - \omega t)\hat{\vec{x}}$$

$\sigma_{xx}(\omega, q) \Rightarrow$ acoustic attenuation & velocity shift.

Acoustic attenuation in B-field [Pippard, 1957]

$\omega \simeq 0 \Rightarrow$ Geometrical commensurability



$$2R_c \simeq n\lambda \quad \Rightarrow \quad 2\frac{mv_F}{eB} \simeq n\lambda$$

Measurement of cyclotron radius of composite fermions close to $\nu = 1/2$ [Willett *et al.*, 1993]. $(\omega \sim 2\pi \times 6\text{GHz},\ \lambda \sim 0.5\mu\text{m})$

Motivation(s)

- Zero resistance states in microwave-irradiated GaAs devices. [Mani *et al.*, Zudov *et al.*]

- High-ω SAW-generation in microwave-irradiated GaAs devices. [I.V. Kukushkin *et al.*]

- SAWs + MWs $\Rightarrow$probing collective excitations. [V.I. Talyanskii, I.V. Kukushkin]

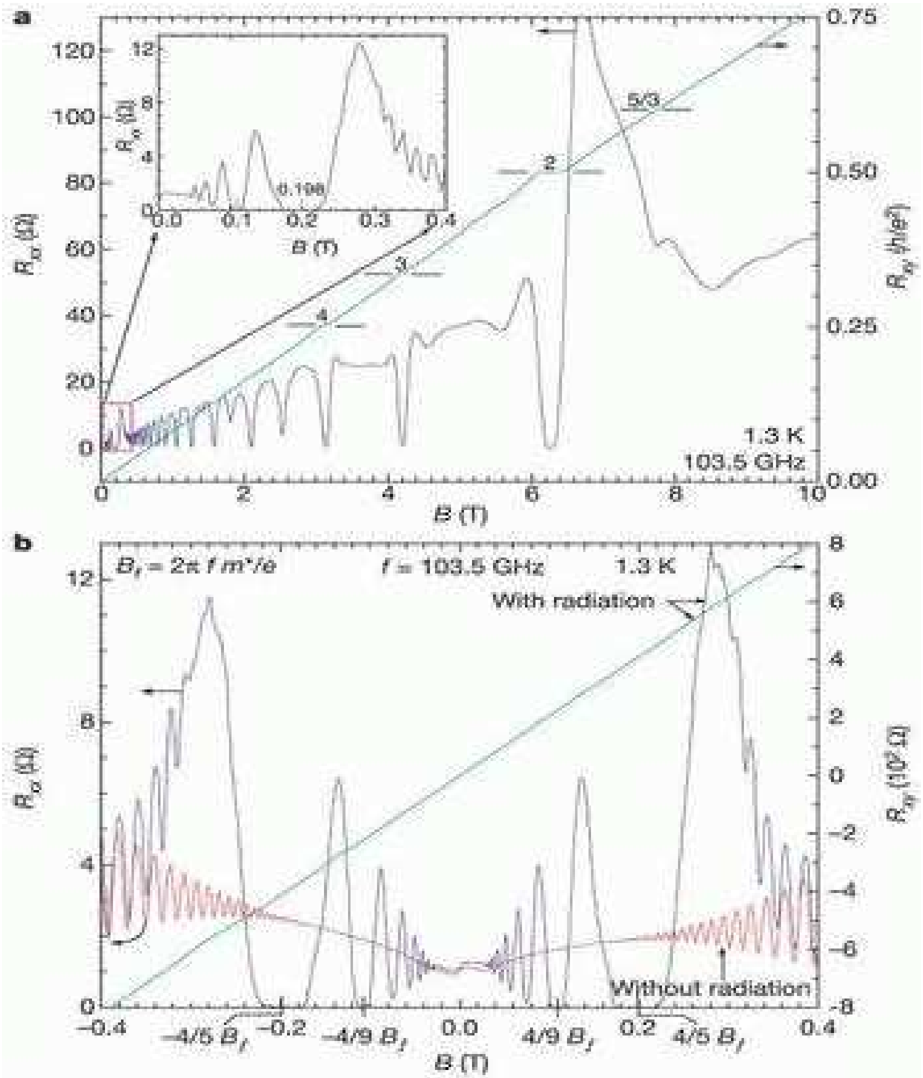
Need to understand response of high quality 2DEGs on small lengthscales ($\lambda_{\mathrm{SAW}} \ll l_{\mathrm{mfp}}$), and the resulting non-equilibrium states.

We study the effects of SAWs on the d.c. resistivity.

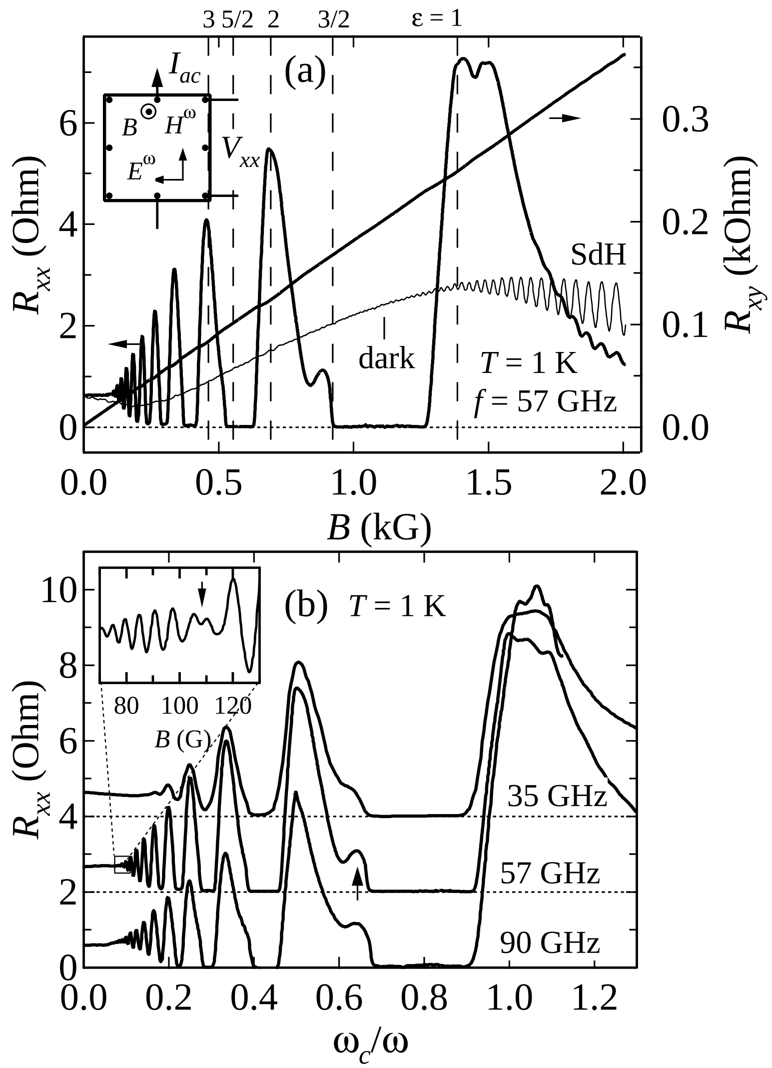
[Typically $\lambda_{\mathrm{SAW}} \gg a_B, \lambda_F$ and $s = \frac{\omega}{q} \ll v_F$ $\Rightarrow$some simplifications.]

Microwave-induced Resistance Oscillations

[Mani *et al.*, Nature **420**, 646 (2002)]



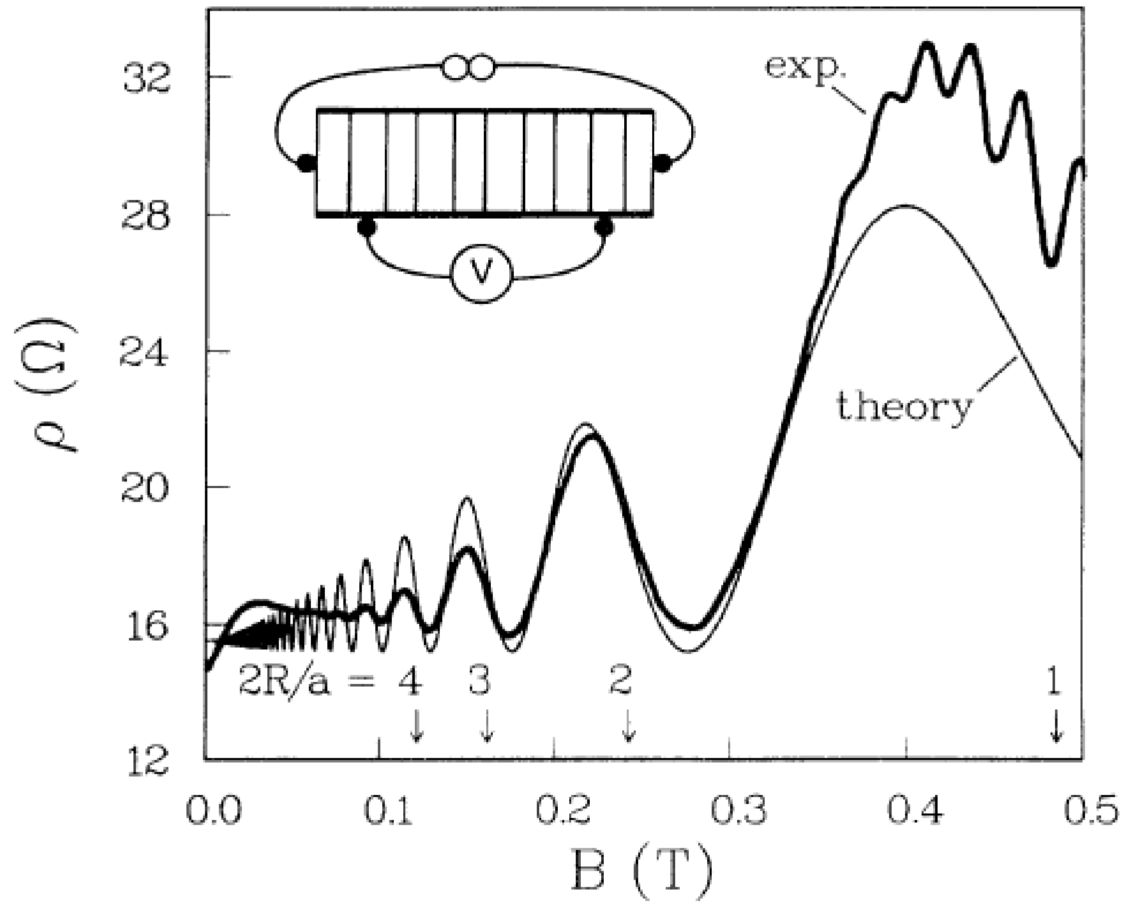
[Zudov *et al.*, PRL **90**, 046807 (2003)]



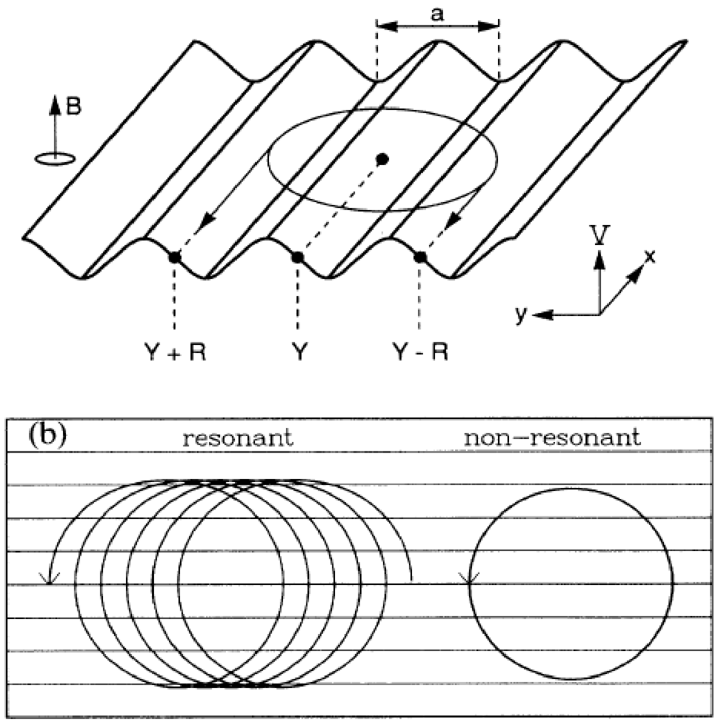
"Classical" magnetic fields (many Landau levels), $k_B T \gtrsim \hbar\omega_c$.
Multiple peaks $\Rightarrow$translational invariance is broken (Kohn's Theorem).

Classical Effects

Weiss oscillations: resistivity correction due to a *static* periodic potential.
Due to geometric commensurability. [Weiss *et al.*, Europhys. Lett. **8**, 179 (1989)]



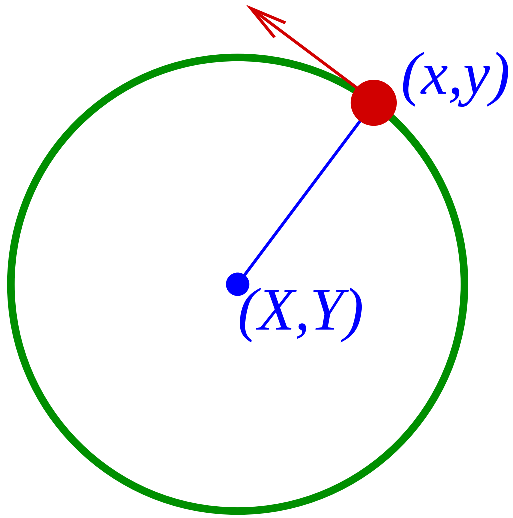
[Beenakker, PRL **62**, 2020 (1989)]



Guiding centre drift due to SAWs

$$m\ddot{\vec{r}} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{B} = B\hat{\vec{z}}$$

Guiding centre co-ordinates:



$$X = x - \frac{v_y}{\omega_c}, \qquad Y = y + \frac{v_x}{\omega_c}.$$

$$\left(\omega_c = \frac{eB}{m}\right)$$

$$\dot{X} = \frac{E_y(\vec{r},t)}{B} \quad \dot{Y} = -\frac{E_x(\vec{r},t)}{B}$$

SAW potential: $\vec{E}(\vec{r},t) = E_{\omega q}\cos[qx - \omega t]\hat{\vec{x}}$

Perturbation theory:

$$\dot{Y} \simeq -\frac{E_{\omega q}}{B} \cos[q x_0(t) - \omega t] \quad , \quad \dot{X} \simeq 0$$

where $x_0(t)$ is the trajectory in the $absence$ of the SAW.
$e.g.$ ignoring disorder:

$$x_0(t) = X_0 + R_c \cos\left(\omega_c t - \psi\right), \qquad y_0(t) = Y_0 + R_c \sin\left(\omega_c t - \psi\right)$$

Resistivity change: SAW induced drift of electron cyclotron orbit – enhanced diffusion

$$\delta D_{yy} = \int_0^\infty \langle \dot{Y}(t) \dot{Y}(0) \rangle dt \xrightarrow{\text{short-range scattering}} \int_0^\infty e^{-t/\tau} \langle \dot{Y}(t) \dot{Y}(0) \rangle_{X_0, \psi} dt$$

(Averaging over location, X_0, and phase, ψ, of the orbit.)

Final Result:

$$\frac{\delta\rho_{xx}}{\rho_0} = \frac{\delta D_{yy}}{D_0} = 2\left(\frac{v_F \tau e E_{\omega q}}{\epsilon_F}\right)^2 \sum_{p=-\infty}^{\infty} \frac{J_p(qR_c)^2}{1+(\omega - p\omega_c)^2\tau^2}$$

$$\delta\rho_{xy} = \delta\rho_{yy} = 0$$

When $qR_c \gg 1$

$$\frac{\delta\rho_{xx}}{\rho_{xx}} \simeq 4\frac{\left(v_F \tau e E_{\omega q}\right)^2}{\pi q R_c \epsilon_F^2} \sum_{p=-\infty}^{\infty} \frac{\cos^2(qR_c + \frac{p\pi}{2} - \frac{\pi}{4})}{1+(\omega - p\omega_c)^2\tau^2}$$

Kinetic equation

- Distribution function

$$f(t,x,\phi,\epsilon) = f_T(\epsilon) + \sum_{\omega q} e^{-i\omega t + iqx} \sum_m f^m_{\omega q}(\epsilon) e^{im\phi}$$

- Kinetic equation in relaxation-time approximation (short-range scattering)

$$\begin{aligned} \hat{\mathcal{L}} f(t,x,\phi,\epsilon) &= -\frac{f - f^0}{\tau} - \frac{f^0 - f_T(\epsilon - \epsilon_F(t,x))}{\tau_{\mathrm{in}}}, \\ \hat{\mathcal{L}} &= \frac{\partial}{\partial t} + v\cos\phi \frac{\partial}{\partial x} + \left[\omega_c - \frac{eE}{mv}\sin\phi\right]\frac{\partial}{\partial \phi} + evE\cos\phi\frac{\partial}{\partial \epsilon} \end{aligned}$$

- Elastic scattering time $\tau(\epsilon)$, inelastic scattering time τ_{in}

Solution:

i) Assume $\tau(\epsilon) = \tau$ is energy-independent and $\tau_{\rm in} \gg \tau$
ii) Solve for $f_{\omega q}(\phi)$
iii) Use solution for $f_{\omega q}(\phi)$ to find correction to $f_{00}(\phi)$
iv) f_{00} gives dc current, and hence dc resistance

Resistivity correction:

$$\frac{\delta\rho_{xx}}{\rho_0} = 2\left(v_F \tau q \mathcal{E}\right)^2 \,\Re\left\{\frac{K}{1-K}\right\}$$

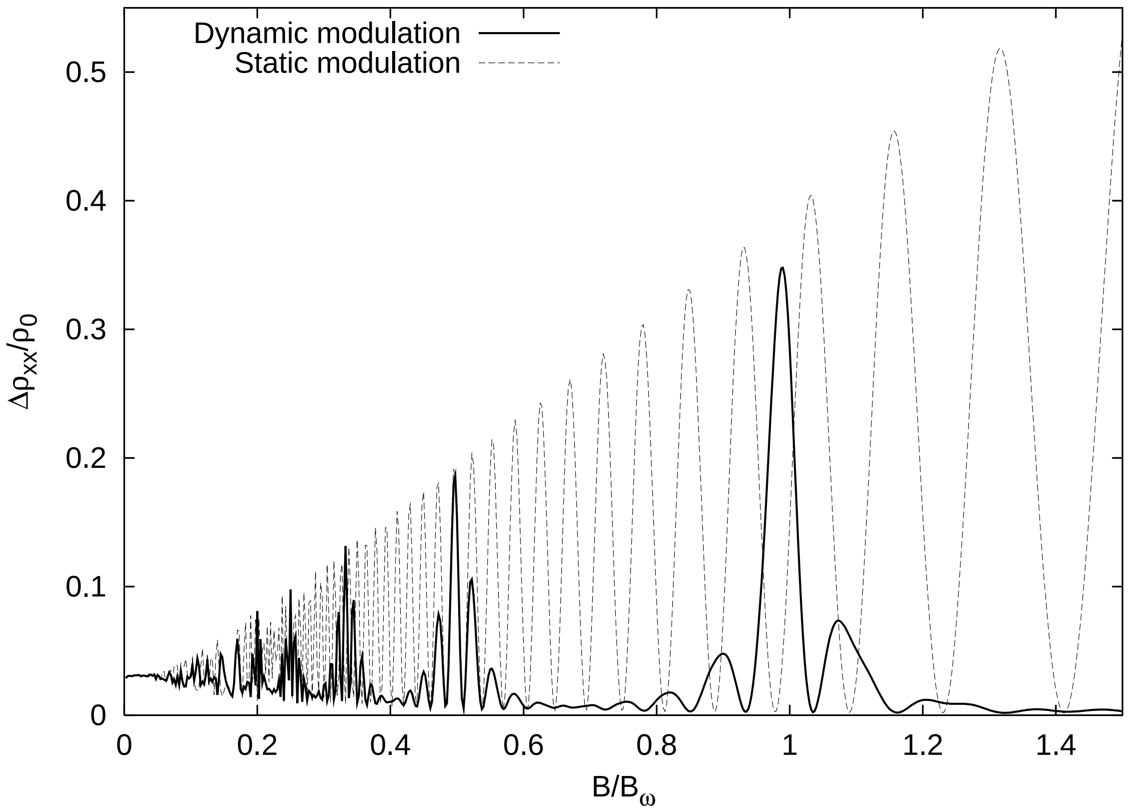
$$\mathcal{E} \equiv \frac{e a_{\rm sc} E_{\omega q}}{\epsilon_F} \qquad\qquad K \equiv \sum_{p=-\infty}^{\infty} \frac{J_p(qR_c)^2}{1 + i\tau(p\omega_c - \omega)}$$

[Thomas-Fermi screening.]

[Simplifies to the result found previously if $qR_c \gg 1$.]

Classical Results

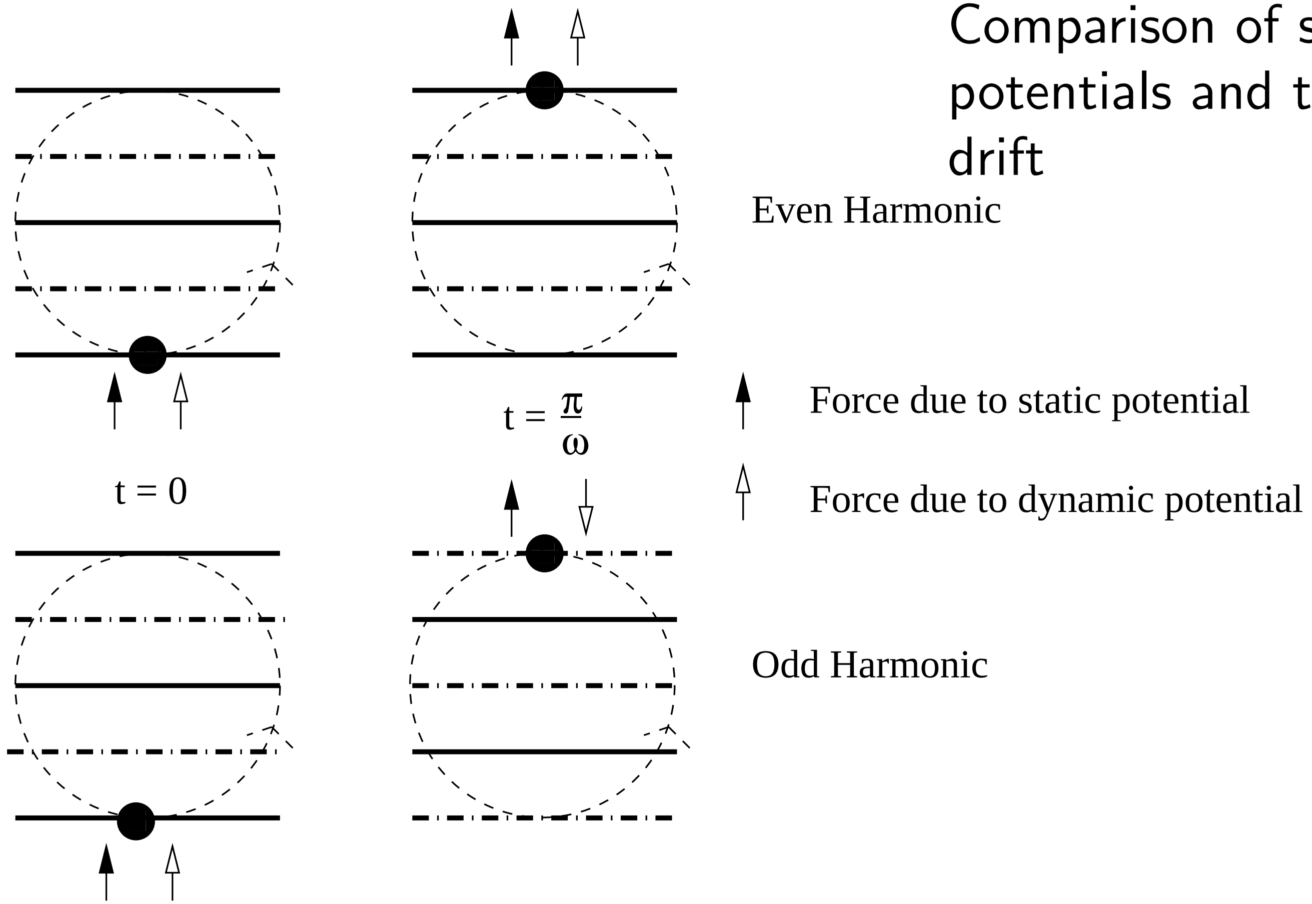
Comparison with Weiss oscillations



$ql = 600,\ \mathcal{E} = 0.01,\ \omega\tau = 0, 20$

Summary of classical results

- Resonances at $\omega/\omega_c \simeq$ integer modulated by geometric commensurability.
- Even (odd) integer peaks are in (out of) phase with geometric resonances.



Comparison of static and dynamic potentials and their effects on $E \times B$ drift

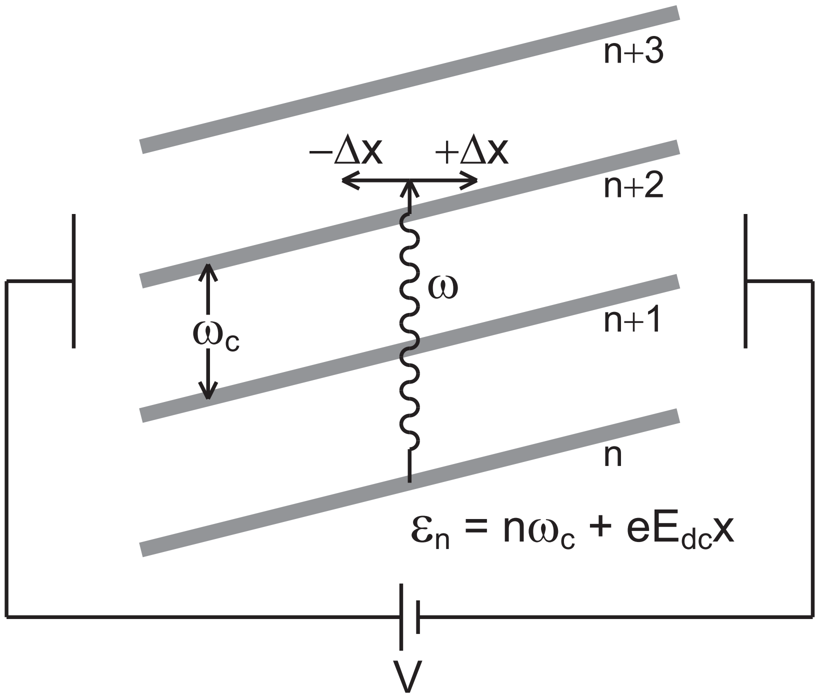
Classical resistance is *increased* by SAWs

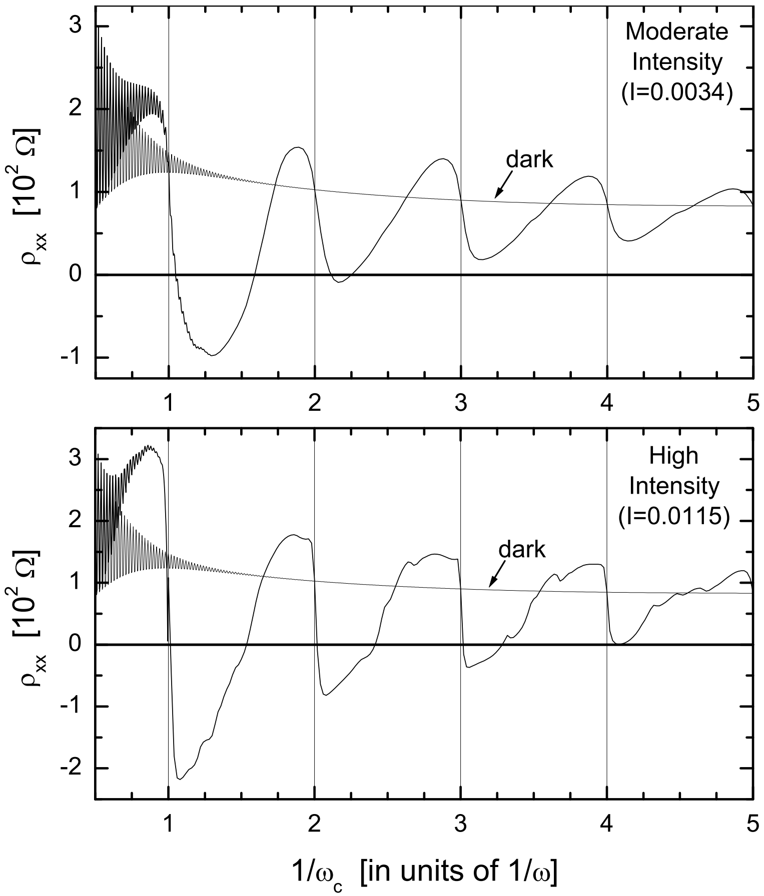
Quantum Effects: Theories for the microwave-induced resistance oscillations

Quantum Mechanics, $\omega_c\tau \gg 1$ $\Rightarrow$oscillatory density of states

Durst, Sachdev, Read & Girvin: disorder-induced scattering

[PRL **91**, 086803 (2003); see also Vavilov & Aleiner PRB **69**, 035303 (2004)]





Dmitriev, Mirlin & Polyakov: oscillatory structure in distribution function

[PRL **91**, 226802 (2003); Dmitriev, Vavilov, Aleiner, Mirlin & Polyakov, cond-mat/0310668]

Stationary kinetic equation

$$E^2 \frac{\sigma^D(\omega)}{2\omega^2\gamma^2} \sum_{\pm} \tilde{\gamma}(\epsilon \pm \omega)[f(\epsilon \pm \omega) - f(\epsilon)] = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{\rm in}}$$

Assume *weak* DOS modulations:

$$\tilde{\gamma}(\epsilon) = \left[1 - \Gamma \cos\left(\frac{2\pi\epsilon}{\omega_c}\right)\right]\gamma \qquad \Gamma = 2e^{-\frac{\pi}{\omega_c \tau_q}} \ll 1$$

This implies oscillations in the elastic scattering rate $\tau^{-1}(\epsilon) = \tau^{-1}\frac{\tilde{\gamma}(\epsilon)}{\gamma}$.

Consider $k_B T \gg \omega_c$ $\Rightarrow$No SdH oscillations.

Nevertheless, there *are* microwave-induced corrections to the d.c. conductivity. Larger than those from the disorder-induced scattering mechanism by $\tau_{\rm in}/\tau_{\rm q}$.

Analysis for SAWs

- For $\omega \ll \omega_c$ the balance equation is

$$\sum_{\pm\omega q} |E_{\omega q}|^2 \frac{\sigma_{\omega q}}{\tilde{\gamma}}\, \partial_\epsilon \left[\frac{\tilde{\gamma}^2}{\gamma^2} \partial_\epsilon (f_{00}^0 + f_T) \right] = \frac{f_{00}^0}{\tau_{\text{in}}}$$

- Distribution acquires an oscillatory part $f_{00}^0 = \delta f^0(\epsilon)$.

To lowest order in $|E_{\omega q}|^2$

$$\delta f^0(\epsilon) = 4\tau_{\text{in}} \frac{|E_{\omega q}|^2 \sigma_{\omega q}}{\gamma^2}\, \partial_\epsilon \tilde{\gamma}\, \partial_\epsilon f_T$$

- Average over energy $\Rightarrow$ *isotropic* magneto-oscillations ($1/\tau \lesssim \omega \ll \omega_c$)

$$\frac{\delta^q \rho_{\alpha\alpha}}{\rho_0} = \frac{\delta^q \sigma_{\alpha\alpha}}{\sigma_0} = -\frac{2\tau_{\text{in}}}{\tau} \left| \frac{4\pi\Gamma\epsilon_F}{\hbar\omega_c} \right|^2 \mathcal{E}^2 J_0^2(qR_c)$$

Comparison of classical and quantum contributions

- Quantum correction to resistance is isotropic, classical is anisotropic.
- Quantum correction has opposite sign to classical contribution.

Final results for $\tau^{-1} \lesssim \omega \ll \omega_c$ are

$$
\begin{aligned}
\frac{\delta\rho_{xx}}{\rho_0} &\approx \frac{\delta\sigma_{yy}}{\sigma_0} = 2J_0^2\left(qR_c\right)\mathcal{E}^2\left[\frac{v_F^2}{s^2} - \frac{\tau_{\mathrm{in}}}{\tau}\left(2\pi\Gamma\nu\right)^2\right], \\
\frac{\delta\rho_{yy}}{\rho_0} &\approx \frac{\delta\sigma_{xx}}{\sigma_0} = -2J_0^2\left(qR_c\right)\mathcal{E}^2 \times \frac{\tau_{\mathrm{in}}}{\tau}\left(2\pi\Gamma\nu\right)^2,
\end{aligned}
$$

[filling fraction, $\nu = 2\epsilon_F/(\hbar\omega_c)$]

The parameter controlling the resistance anisotropy is $\eta = \frac{2\pi\Gamma\nu s}{v_F}\sqrt{\frac{\tau_{\mathrm{in}}}{\tau}}$.

$\Rightarrow$possibility of zero resistance states at $\omega \ll \omega_c$.

Conclusions

• New experimental regimes of SAW frequency and 2DEG quality require new theoretical investigations.

• We have studied the effects of SAWs on the d.c. magnetoresistance of a 2DEG in the regime $v_F \gg s$, $\omega_c \tau \gg 1$, and $\lambda_{\mathrm{SAW}} \gg a_B, \lambda_F$.

• The *classical* results can be understood in terms of the guiding centre drift. We find combined geometrical and temporal resonances (qR_c and ω/ω_c).

• We have computed the *quantum* corrections arising from the oscillatory non-equilibrium distribution function (the leading effect for $\tau_{\mathrm{in}} \gg \tau$).

• We find the possibility of SAW-induced zero resistance states when $\omega \lesssim \omega_c$, which will show geometric oscillations as a function of qR_c.