Zero Resistance States from Surface Acoustic Waves

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Overview

- Surface Acoustic Waves
- Motivation(s)
- Classical Effects
  - Weiss Oscillations (spatial periodicity)
  - “Weiss-like” oscillations (spatial + temporal periodicity)
- Quantum Effects
  - Theories for ZRSs
  - Quantum Effects of SAWs
- Summary
Surface Acoustic Waves

\[ \omega = sq \quad s \simeq 3000 \text{ m/s} \]

\[ \phi(x, t) = \phi_0 \sin(qx - \omega t) \quad \Rightarrow \quad \vec{E} = E_0 \cos(qx - \omega t) \hat{x} \]

\[ \sigma_{xx}(\omega, q) \Rightarrow \text{acoustic attenuation & velocity shift.} \]
Acoustic attenuation in \textit{B}-field \ [Pippard, 1957]

\[ \omega \simeq 0 \Rightarrow \text{Geometrical commensurability} \]

\[ 2R_c \simeq n\lambda \quad \Rightarrow \quad 2\frac{mv_F}{eB} \simeq n\lambda \]

Measurement of cyclotron radius of composite fermions close to \( \nu = 1/2 \) \ [Willett \textit{et al}.\, 1993]. \( (\omega \sim 2\pi \times 6\text{GHz}, \, \lambda \sim 0.5\mu\text{m}) \)
Motivation(s)

- Zero resistance states in microwave-irradiated GaAs devices. [Mani et al., Zudov et al.]

- High-\(\omega\) SAW-generation in microwave-irradiated GaAs devices. [I.V. Kukushkin et al.]

- SAWs + MWs \(\Rightarrow\) probing collective excitations. [V.I. Talyanskii, I.V. Kukushkin]

Need to understand response of high quality 2DEGs on small lengthscales (\(\lambda_{\text{SAW}} \ll l_{\text{mfp}}\)), and the resulting non-equilibrium states.

We study the effects of SAWs on the d.c. resistivity.

[Typically \(\lambda_{\text{SAW}} \gg a_B, \lambda_F\) and \(s = \frac{\omega}{q} \ll v_F \Rightarrow\) some simplifications.]
Microwave-induced Resistance Oscillations

[Mani et al., Nature 420, 646 (2002)]

[Zudov et al., PRL 90, 046807 (2003)]

“Classical” magnetic fields (many Landau levels), $k_B T \gtrsim \hbar \omega_c$.
Multiple peaks $\Rightarrow$ translational invariance is broken (Kohn’s Theorem).
Classical Effects

**Weiss oscillations**: resistivity correction due to a *static* periodic potential. Due to geometric commensurability.

[Weiss *et al.*, Europhys. Lett. 8, 179 (1989)]

[Beenakker, PRL 62, 2020 (1989)]
Guiding centre drift due to SAWs

\[ m \ddot{\vec{r}} = -e \left( \vec{E} + \vec{v} \times \vec{B} \right) \quad \vec{B} = B \hat{z} \]

Guiding centre co-ordinates:

\[ X = x - \frac{v_y}{\omega_c}, \quad Y = y + \frac{v_x}{\omega_c}. \]

\( \omega_c = \frac{eB}{m} \)

\[ \dot{X} = \frac{E_y(\vec{r}, t)}{B} \quad \dot{Y} = -\frac{E_x(\vec{r}, t)}{B} \]

SAW potential:

\[ \vec{E}(\vec{r}, t) = E_{\omega q} \cos[qx - \omega t] \hat{x} \]
Perturbation theory:

\[ \dot{Y} \approx -\frac{E\omega_q}{B} \cos[qx_0(t) - \omega t], \quad \dot{X} \approx 0 \]

where \( x_0(t) \) is the trajectory in the absence of the SAW. e.g. ignoring disorder:

\[ x_0(t) = X_0 + R_c \cos (\omega_c t - \psi), \quad y_0(t) = Y_0 + R_c \sin (\omega_c t - \psi) \]

Resistivity change: SAW induced drift of electron cyclotron orbit – enhanced diffusion

\[ \delta D_{yy} = \int_0^\infty \langle \dot{Y}(t)\dot{Y}(0) \rangle dt \xrightarrow{\text{short-range scattering}} \int_0^\infty e^{-t/\tau} \langle \dot{Y}(t)\dot{Y}(0) \rangle x_0,\psi dt \]

(Averaging over location, \( X_0 \), and phase, \( \psi \), of the orbit.)
Final Result:

\[
\frac{\delta \rho_{xx}}{\rho_0} = \frac{\delta D_{yy}}{D_0} = 2 \left( \frac{v_F \tau e E \omega_q}{\epsilon_F} \right)^2 \sum_{p=-\infty}^{\infty} \frac{J_p(qR_c)^2}{1 + (\omega - p\omega_c)^2 \tau^2}
\]

\[\delta \rho_{xy} = \delta \rho_{yy} = 0\]

When \( qR_c \gg 1 \)

\[
\frac{\delta \rho_{xx}}{\rho_{xx}} \sim 4 \left( \frac{v_F \tau e E \omega_q}{\pi qR_c \epsilon_F^2} \right)^2 \sum_{p=-\infty}^{\infty} \frac{\cos^2(qR_c + \frac{p\pi}{2} - \frac{\pi}{4})}{1 + (\omega - p\omega_c)^2 \tau^2}
\]
**Kinetic equation**

- Distribution function

\[ f(t, x, \phi, \epsilon) = f_T(\epsilon) + \sum_{\omega q} e^{-i\omega t + iqx} \sum_m f_{\omega q}^m(\epsilon)e^{im\phi} \]

- Kinetic equation in relaxation-time approximation (short-range scattering)

\[ \hat{L} f(t, x, \phi, \epsilon) = -\frac{f - f^0}{\tau} - \frac{f^0 - f_T(\epsilon - \epsilon_F(t, x))}{\tau_{\text{in}}} \]

\[ \hat{L} = \frac{\partial}{\partial t} + v \cos \phi \frac{\partial}{\partial x} + \left[ \omega - \epsilon \frac{eE}{mv} \sin \phi \right] \frac{\partial}{\partial \phi} + evE \cos \phi \frac{\partial}{\partial \epsilon} \]

- Elastic scattering time \( \tau(\epsilon) \), inelastic scattering time \( \tau_{\text{in}} \)
Solution:

i) Assume $\tau(\epsilon) = \tau$ is energy-independent and $\tau_{in} \gg \tau$

ii) Solve for $f_{\omega q}(\phi)$

iii) Use solution for $f_{\omega q}(\phi)$ to find correction to $f_{00}(\phi)$

iv) $f_{00}$ gives dc current, and hence dc resistance

Resistivity correction:

$$\frac{\delta \rho_{xx}}{\rho_0} = 2 (v_F \tau q \mathcal{E})^2 \Re \left\{ \frac{K}{1 - K} \right\}$$

$$\mathcal{E} \equiv \frac{e a_{sc} E_{\omega q}}{\epsilon_F} \quad K \equiv \sum_{p=-\infty}^{\infty} \frac{J_p(q R_c)^2}{1 + i \tau (p \omega_c - \omega)}$$

[Thomas-Fermi screening.] [Simplifies to the result found previously if $q R_c \gg 1$.]
Classical Results

Comparison with Weiss oscillations

\[ q_l = 600, \, \varepsilon = 0.01, \, \omega \tau = 0, 20 \]
Summary of classical results

- Resonances at $\omega/\omega_c \simeq \text{integer}$ modulated by geometric commensurability.
- Even (odd) integer peaks are in (out of) phase with geometric resonances.

Comparison of static and dynamic potentials and their effects on $E \times B$ drift

Classical resistance is increased by SAWs
Quantum Effects: Theories for the microwave-induced resistance oscillations

Quantum Mechanics, $\omega_c T \gg 1 \Rightarrow$ oscillatory density of states

Durst, Sachdev, Read & Girvin: disorder-induced scattering

[PRL 91, 086803 (2003); see also Vavilov & Aleiner PRB 69, 035303 (2004)]
Dmitriev, Mirlin & Polyakov: oscillatory structure in distribution function

[PRL 91, 226802 (2003); Dmitriev, Vavilov, Aleiner, Mirlin & Polyakov, cond-mat/0310668]

Stationary kinetic equation

\[ E^2 \sigma^D_\omega \frac{\sum \tilde{\gamma}(\epsilon \pm \omega)[f(\epsilon \pm \omega) - f(\epsilon)]}{2\omega^2\gamma^2} = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{in}} \]

Assume weak DOS modulations:

\[ \tilde{\gamma}(\epsilon) = \left[ 1 - \Gamma \cos \left( \frac{2\pi \epsilon}{\omega_c} \right) \right] \gamma \quad \Gamma = 2e^{-\frac{\pi}{\omega_c \tau_q}} \ll 1 \]

This implies oscillations in the elastic scattering rate \( \tau^{-1}(\epsilon) = \tau^{-1}\tilde{\gamma}(\epsilon) / \gamma \).

Consider \( k_B T \gg \omega_c \Rightarrow \) No SdH oscillations.

Nevertheless, there are microwave-induced corrections to the d.c. conductivity. Larger than those from the disorder-induced scattering mechanism by \( \tau_{in}/\tau_q \).
Analysis for SAWs

• For $\omega \ll \omega_c$ the balance equation is

$$\sum_{\pm \omega q} |E_{\omega q}|^2 \frac{\sigma_{\omega q}}{\tilde{\gamma}} \partial_\epsilon \left[ \frac{\tilde{\gamma}^2}{\gamma^2} \partial_\epsilon (f_{00} + f_T) \right] = \frac{f_{00}}{\tau_{in}}$$

• Distribution acquires an oscillatory part $f_{00} = \delta f^0(\epsilon)$. To lowest order in $|E_{\omega q}|^2$

$$\delta f^0(\epsilon) = 4\tau_{in} \frac{|E_{\omega q}|^2 \sigma_{\omega q}}{\gamma^2} \partial_\epsilon \tilde{\gamma} \partial_\epsilon f_T$$

• Average over energy $\Rightarrow isotropic$ magneto-oscillations ($1/\tau \lesssim \omega \ll \omega_c$)

$$\frac{\delta^q \rho_{\alpha\alpha}}{\rho_0} = \frac{\delta^q \sigma_{\alpha\alpha}}{\sigma_0} = -\frac{2\tau_{in}}{\tau} \left| \frac{4\pi \Gamma \epsilon_F}{\hbar \omega_c} \right|^2 \mathcal{E}^2 J_0^2(qR_c)$$
Comparison of classical and quantum contributions

• Quantum correction to resistance is isotropic, classical is anisotropic.

• Quantum correction has opposite sign to classical contribution.

Final results for $\tau^{-1} \lesssim \omega \ll \omega_c$ are

$$\frac{\delta \rho_{xx}}{\rho_0} \approx \frac{\delta \sigma_{yy}}{\sigma_0} = 2J_0^2 (qR_c) E^2 \left[ \frac{v_F^2}{S^2} - \frac{\tau_{in}}{\tau} (2\pi \Gamma \nu)^2 \right],$$

$$\frac{\delta \rho_{yy}}{\rho_0} \approx \frac{\delta \sigma_{xx}}{\sigma_0} = -2J_0^2 (qR_c) E^2 \times \frac{\tau_{in}}{\tau} (2\pi \Gamma \nu)^2,$$

[filling fraction, $\nu = 2\epsilon_F/(\hbar \omega_c)$]

The parameter controlling the resistance anisotropy is $\eta = \frac{2\pi \Gamma \nu s}{v_F} \sqrt{\frac{\tau_{in}}{\tau}}$.

$\Rightarrow$ possibility of zero resistance states at $\omega \ll \omega_c$. 
Conclusions

• New experimental regimes of SAW frequency and 2DEG quality require new theoretical investigations.

• We have studied the effects of SAWs on the d.c. magnetoresistance of a 2DEG in the regime $v_F \gg \omega_c \tau \gg 1$, and $\lambda_{SAW} \gg a_B, \lambda_F$.

• The classical results can be understood in terms of the guiding centre drift. We find combined geometrical and temporal resonances ($qR_c$ and $\omega/\omega_c$).

• We have computed the quantum corrections arising from the oscillatory non-equilibrium distribution function (the leading effect for $\tau_{in} \gg \tau$).

• We find the possibility of SAW-induced zero resistance states when $\omega \lesssim \omega_c$, which will show geometric oscillations as a function of $qR_c$. 