Correlated Phases of Atomic Bose Gases on a Rotating Lattice

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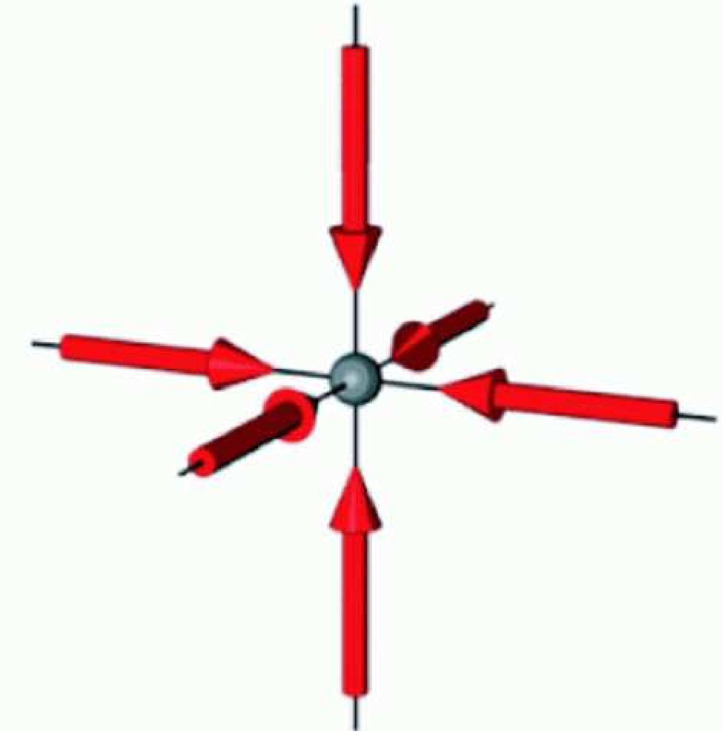
[Gunnar Möller & NRC, arXiv:0904.3097]

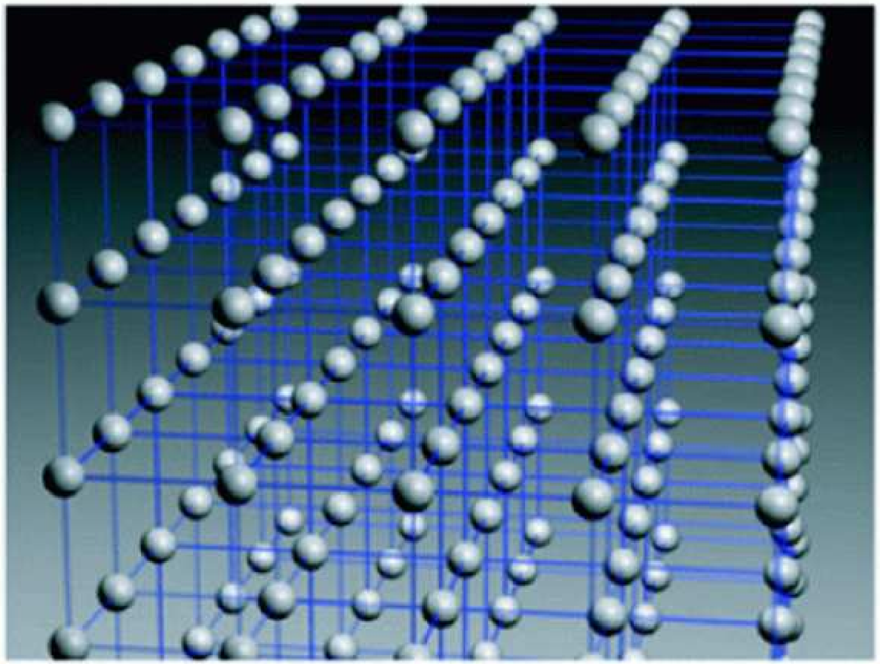
Overview

- Strongly Correlated Phases of Ultracold Atomic Bose Gases
- Atomic Bose Gases on a “Rotating Lattice”
- Strongly Correlated Phases: Numerical Evidence
- Summary

Strongly Correlated Phases of Atomic Bose Gases

(1) Optical Lattice [Bloch, Dalibard & Zwerger, RMP **80**, 885 (2008)]





Bose-Hubbard model [Jaksch *et al.*, PRL **81**, 3108 (1998)]

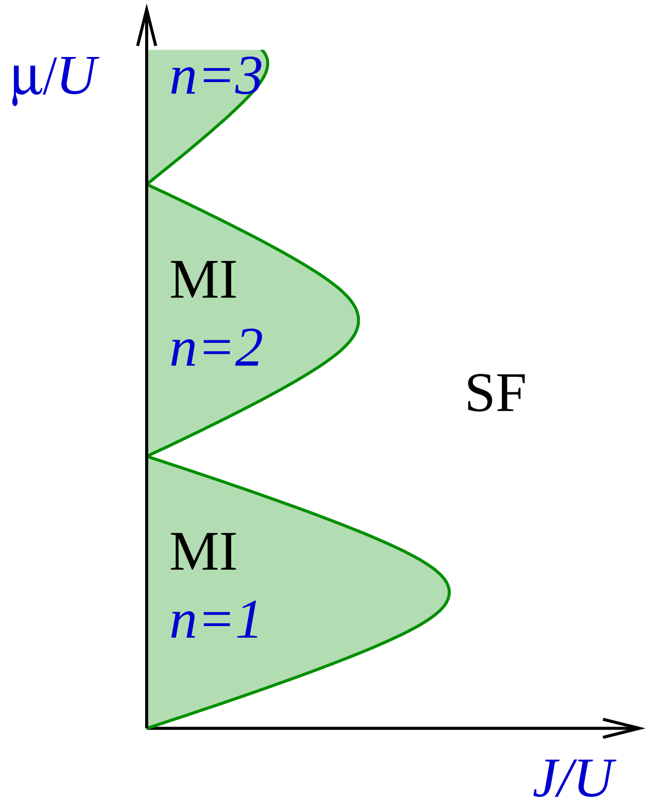
$$H = -J \sum_{\langle \alpha,\beta \rangle} \left[\hat{b}^{\dagger}_{\alpha} \hat{b}_{\beta} + h.c. \right] + \frac{1}{2} U \sum_{\alpha} \hat{n}_{\alpha} (\hat{n}_{\alpha} - 1) - \mu \sum_{\alpha} \hat{n}_{\alpha}$$

Strongly correlated regime for $U/J \gg 1$
at particle density $n \sim 1$.

$T = 0$: competition between

- superfluid (BEC)
- Mott insulators, at $n = 1, 2, \ldots$

[Fisher *et al.*, PRB **40**, 546 (1989)]



Transition to Mott insulator observed in experiment [Greiner *et al.*, Nature **415**, 39 (2002)]

Strongly Correlated Phases of Atomic Bose Gases

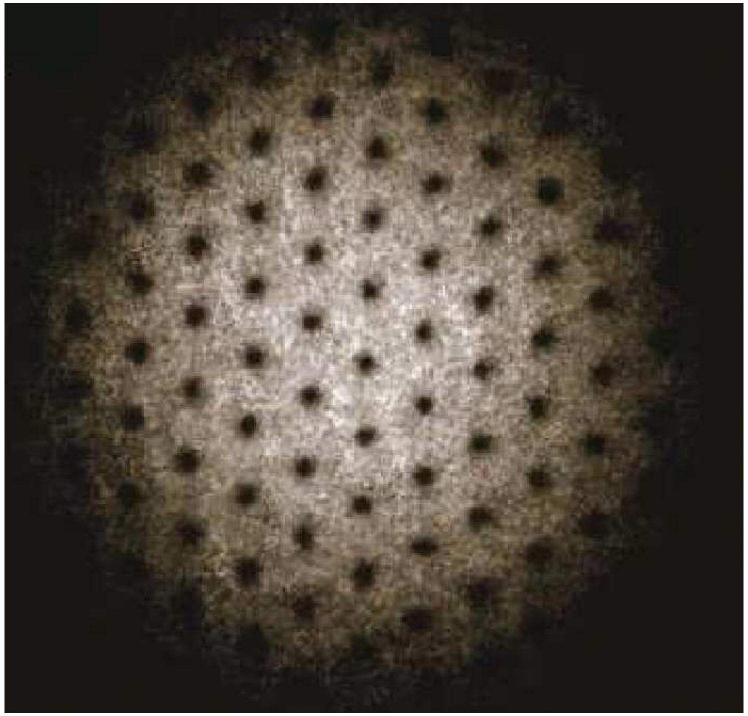
(2) Rapid Rotation

[Bloch, Dalibard & Zwerger, RMP **80**, 885 (2008)]

Rotation frequency, Ω

Quantized vortices

Vortex density $n_\mathrm{v} = \frac{2M\Omega}{h}$



[Coddington *et al.* [JILA], PRA **70**, 063607 (2004)]

Strong correlation regime for $\Omega \rightarrow \omega_\perp$

Filling Factor $\nu \equiv \frac{n_{2\mathrm{d}}}{n_{\mathrm{v}}}$ [NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

Critical filling factor $\nu_{\mathrm{c}} \simeq 6$

- $\nu > \nu_{\mathrm{c}}$: Vortex Lattice (BEC)
- $\nu < \nu_{\mathrm{c}}$: *Bosonic* versions of fractional quantum Hall states:

Laughlin, hierarchy/CF, Moore-Read & Read-Rezayi phases, smectic +...?

[For a review, see: NRC, Adv. Phys. **57**, 539 (2008)]

Experimental challenges:

- the interaction scale at $\nu \sim 1$ is small
- rotating gas susceptible to “heating” by static perturbations.

Atomic Bose Gases on a "Rotating Lattice"

- Rotating lattice [Tung, Schweikhard, Cornell (2006); Williams *et al.* (2008)]
- Tunneling phases [Jaksch & Zoller (2003); Mueller (2004); Sørensen, Demler & Lukin (2005)]

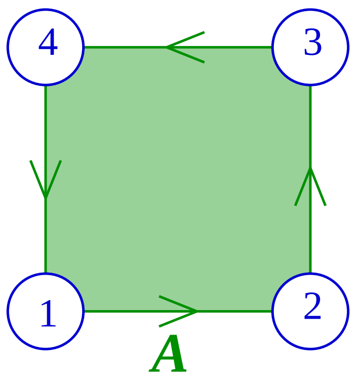
Bose-Hubbard model with "magnetic field" (2D square lattice)

$$H = -J \sum_{\langle \alpha,\beta \rangle} \left[\hat{b}^\dagger_\alpha \hat{b}_\beta e^{iA_{\alpha\beta}} + h.c. \right] + \frac{1}{2} U \sum_\alpha \hat{n}_\alpha (\hat{n}_\alpha - 1) - \mu \sum_\alpha \hat{n}_\alpha$$

Particle density, n

Interaction strength, U/J

Vortex density, $n_{\rm v}$



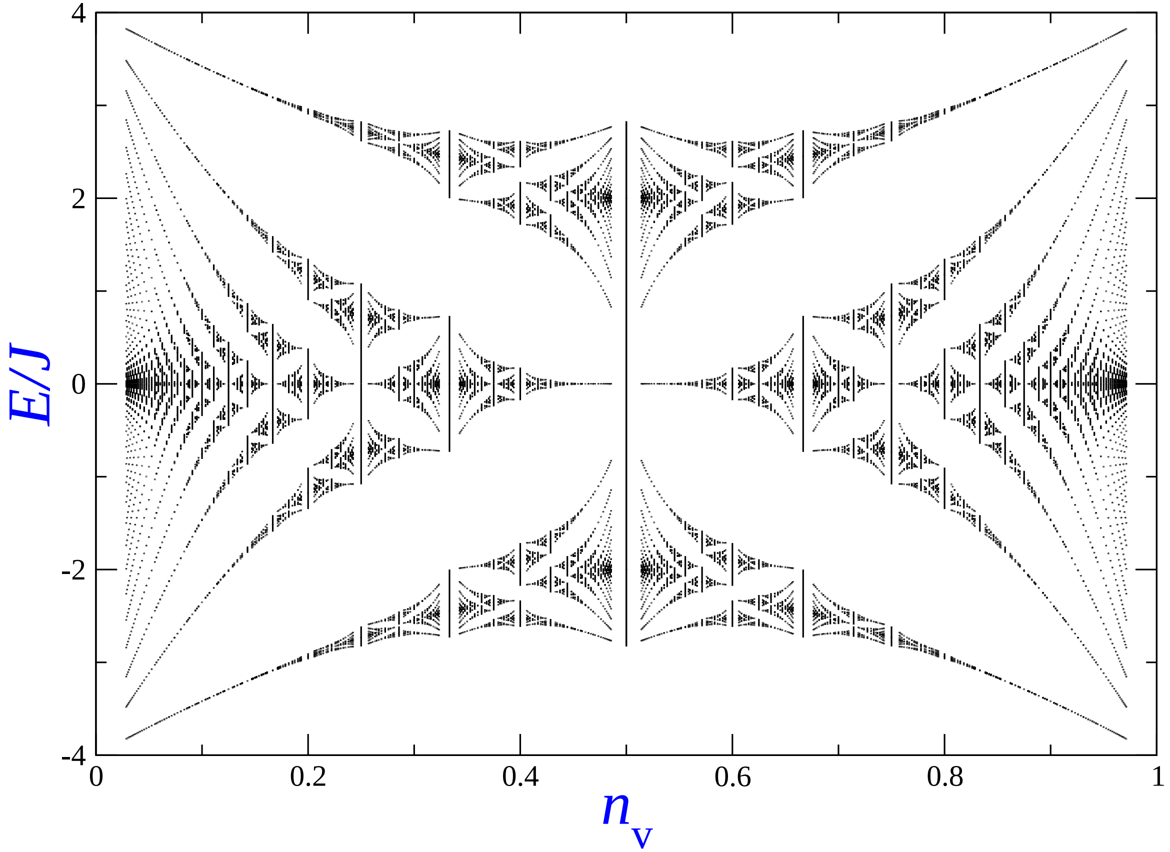
$\sum_{\rm plaquette} A_{\alpha\beta} = 2\pi n_{\rm v}$

$(0 \leq n_{\rm v} < 1)$

What are the groundstates of bosons on a "rotating lattice"?

Single particle spectrum is the "Hofstadter butterfly"

[Harper, Proc. Phys. Soc. Lond. A **68**, 874 (1955); Hofstadter, PRB **14**, 2239 (1976)]



$n, n_\mathrm{v} \ll 1 \Rightarrow$continuum limit [Sørensen, Demler & Lukin, PRL (2005); Hafezi *et al.*, PRA (2007)]

Are there new strongly correlated phases on the lattice for $n \sim n_\mathrm{v} \sim 1$?

Hard-core limit $U \gg J \Rightarrow 0 \leq n_\alpha \leq 1$ [frustrated spin-1/2 quantum magnet]

Composite Fermions

Rapidly rotating bosons in the continuum

Composite fermion = a bound state of a boson with *one vortex*.

[NRC & Wilkin, PRB **80**, 16279 (1999)]

$$\Psi_{\rm B}(\{\boldsymbol{r}_i\}) \propto \mathcal{P}_{LLL} \prod_{i<j} (z_i - z_j)\, \psi_{\rm CF}(\{\boldsymbol{r}_i\})$$

$$n_{\rm v}^{\rm CF} = n_{\rm v} - n$$

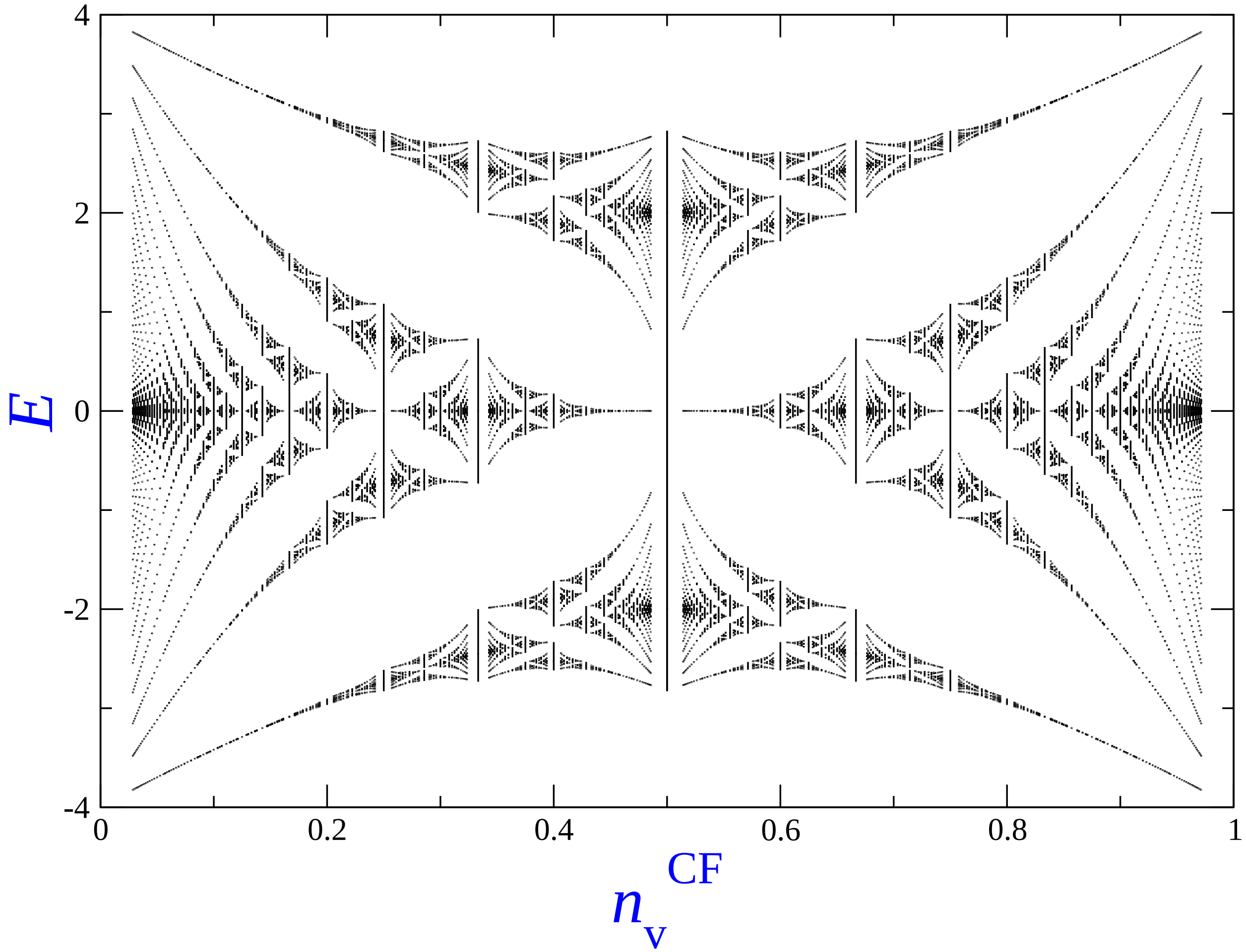
CFs fill p Landau levels for

$$\frac{n}{n_{\rm v}^{\rm CF}} = \pm p \qquad \Rightarrow \qquad \nu = \frac{n}{n_{\rm v}} = \frac{p}{p \pm 1}$$

$\Rightarrow$(trial) incompressible states of interacting bosons, describe exact groundstates well for $\nu = 1/2, 2/3, (3/4)$

[NRC & Wilkin, PRB (1999); Regnault & Jolicoeur, PRL (2003); . . .]

Lattice: CF spectrum is the "Hofstadter butterfly" [Kol & Read, PRB **48**, 8890 (1993)]

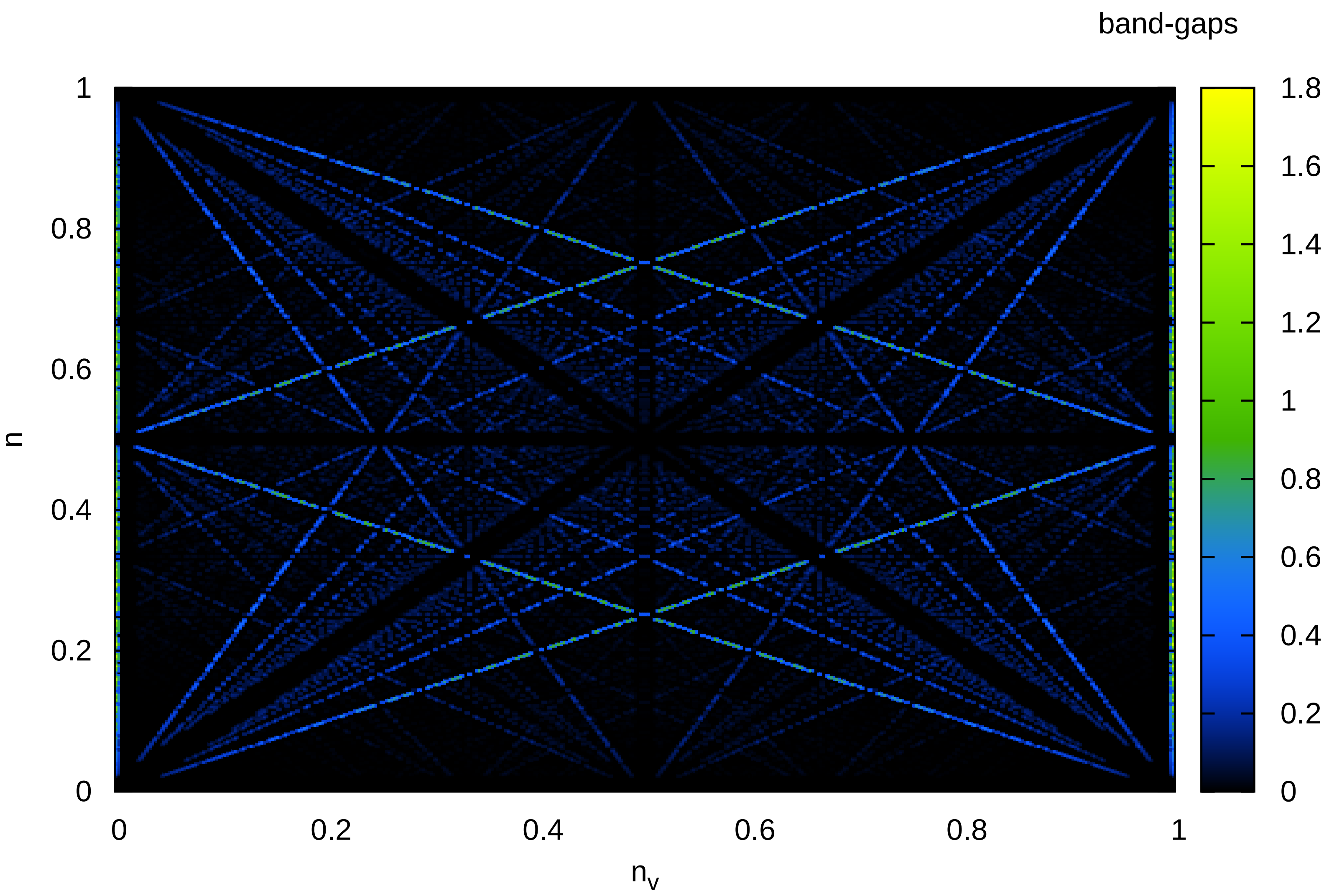


Filled band of CFs at $(n, n_{\rm v}^{\rm CF})$ $\Rightarrow$trial incompressible state of bosons at $(n, n_{\rm v})$

There can exist incompressible states with no counterpart in the continuum

Gaps for non-interacting CFs

[G. Möller & NRC, arXiv:0904.3097]



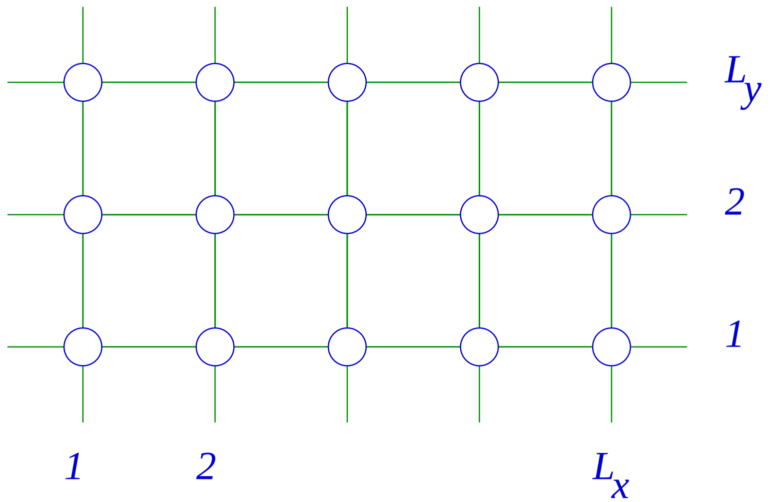
Do these new phases describe the exact groundstates?

Numerical Methods

- Exact Diagonalization

$L_x \times L_y$ square lattice, with periodic boundary conditions (torus).

$$N = nL_xL_y$$
$$N_{\rm v} = n_{\rm v}L_xL_y$$



- Low-energy spectrum (Lanczos) for hard-core interactions $U \gg J$.

- Limited by finite size effects, $N \leq 6$.

Composite Fermion Wavefunction

Continuum

$$\Psi_{\rm B}(\{\boldsymbol{r}_i\}) \propto \mathcal{P}_{LLL} \underbrace{\prod_{i<j}(z_i - z_j)\ \psi_{\rm CF}(\{\boldsymbol{r}_i\})}$$

Slater det. of lowest Landau level wavefunctions:
$\nu = 1$ state of fermions.

Lattice

[G. Möller & NRC, arXiv:0904.3097]

$$\Psi_{\rm B}(\{\boldsymbol{r}_i\}) \propto \underbrace{\psi_{\rm J}^{(\phi_x,\phi_y)}(\{\boldsymbol{r}_i\})}\ \psi_{\rm CF}^{(-\phi_x,-\phi_y)}(\{\boldsymbol{r}_i\})$$

$\nu = 1$ state of fermions.

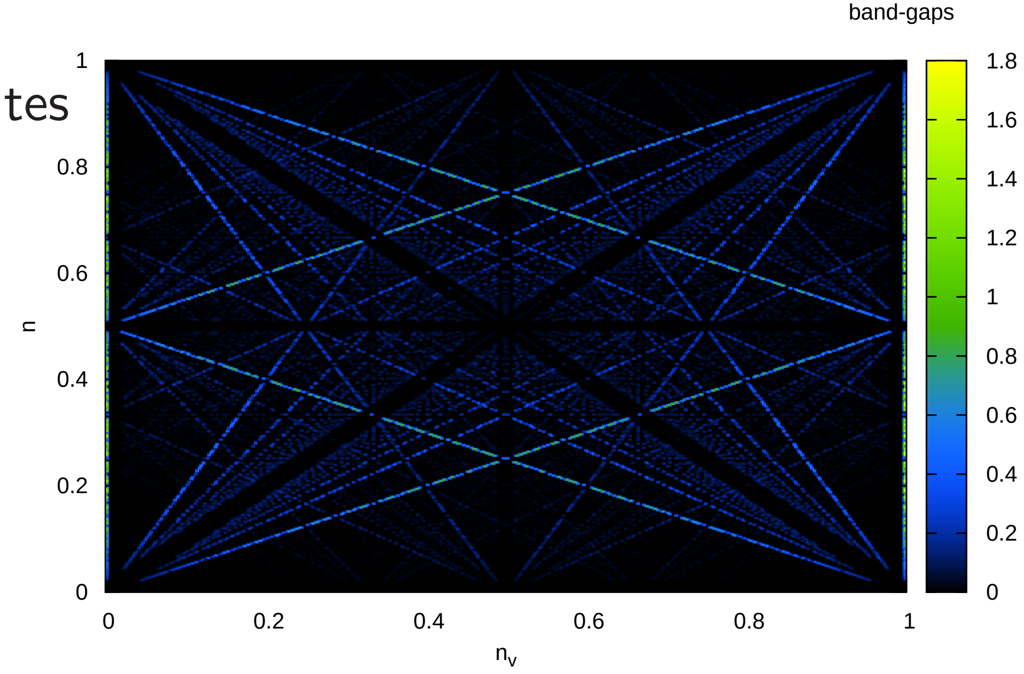
- Hard-core bosons.
- Generalized periodic boundary conditions: phases (ϕ_x, ϕ_y).

  - Recovers the two $\nu = 1/2$ Laughlin wavefunctions in continuum limit.

[Haldane & Rezayi, PRB **31**, 2529 (1985)]

CF states stabilized by the lattice

Evidence for strongly correlated states at a series of these new cases.



On $n_\mathrm{v} = \frac{1}{2}(1-n)$:

Groundstate is consistent with the CF state for $n \lesssim 1/5$

Overlap with trial CF state

n	n_ϕ	N	L_x	L_y	$\lvert\langle\Psi\vert\Psi_{\text{trialCF}}\rangle\rvert^2$	$\dim(\mathcal{H})$
1/7	3/7	2	2	7	0.437	91
1/7	3/7	3	3	7	0.745	1330
1/7	3/7	4	4	7	0 [0.2753]	20.5k
1/7	3/7	5	5	7	0.5631	324k
1/7	3/7	6	6	7	0.3284	5.2M
1/9	4/9	2	2	9	0.3603	153
1/9	4/9	3	3	9	0.8407	2925
1/9	4/9	4	4	9	0 [0.1515]	58.9k
1/9	4/9	4	6	6	0.3061	58.9k
1/9	4/9	5	5	9	0.4585	1.2M
1/9	4/9	6	6	9	0 [0.1957]	25.8M

- Sizeable overlap with CF state (no free parameters!)
- Correct groundstate degeneracy on the torus (1).
- Correct Chern number (2), tested for $N \leq 5$.

Evidence for wider applicability of CF ansatz.

Summary

- Ultracold atomic Bose gases on a rotating lattice offer the possibility to explore novel aspects of the FQHE: the FQHE of bosons; the interplay of the FQHE and lattice periodicity.

- A generalized composite fermion construction leads to the prediction of strongly correlated incompressible phases of bosons at certain $(n, n_{\rm v})$, including states which are stabilized by the lattice.

- We find numerical evidence for uncondensed incompressible fluids for several of the predicted cases. This shows a wider applicability of the CF construction than its continuum formulation.

- There are many other cases $(n, n_{\rm v})$ to understand: points of failure of the CF construction can help to identify other competing phases.