Quantum Phases of Vortices in Atomic Bose Condensates

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"Flux, Charge, Topology and Statistics",
Amsterdam, 26 June, 2001

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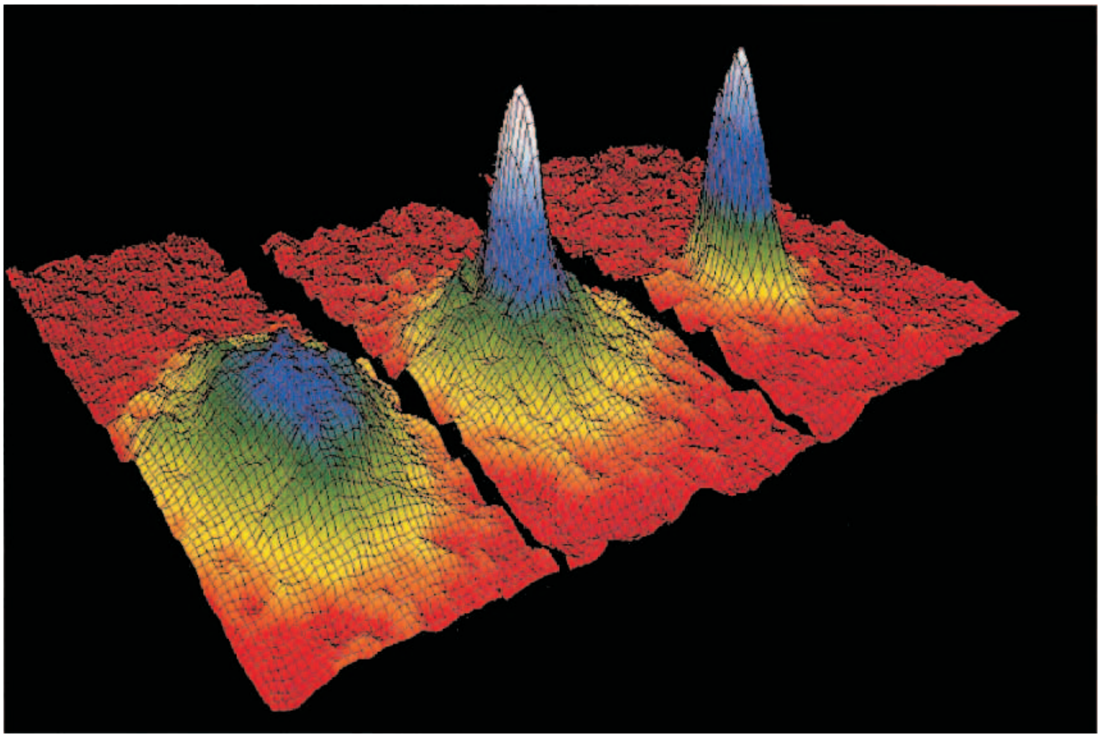
Overview

- Atomic Bose-Einstein Condensates
- Large Healing Length: the Lowest Landau Level
- Vortex Lattices or Vortex Liquids?
- Summary

Atomic Bose-Einstein Condensates

^{87}Rb (JILA, 1995)
^{7}Li (Rice, 1995)
^{23}Na (MIT, 1995)
^{1}H (MIT, 1998)
^{85}Rb (JILA, 2000)
^{4}He* (Orsay, 2001)

$\bar{n} \sim 10^{14}\text{cm}^{-3} \Rightarrow T_{\text{c}} \sim 100\text{nK}$
$a_{\text{ho}} \sim 1\mu\text{m}$
$N \sim 10^{3} - 10^{6}$
Lifetimes $\sim 100 - 1000\text{s}$

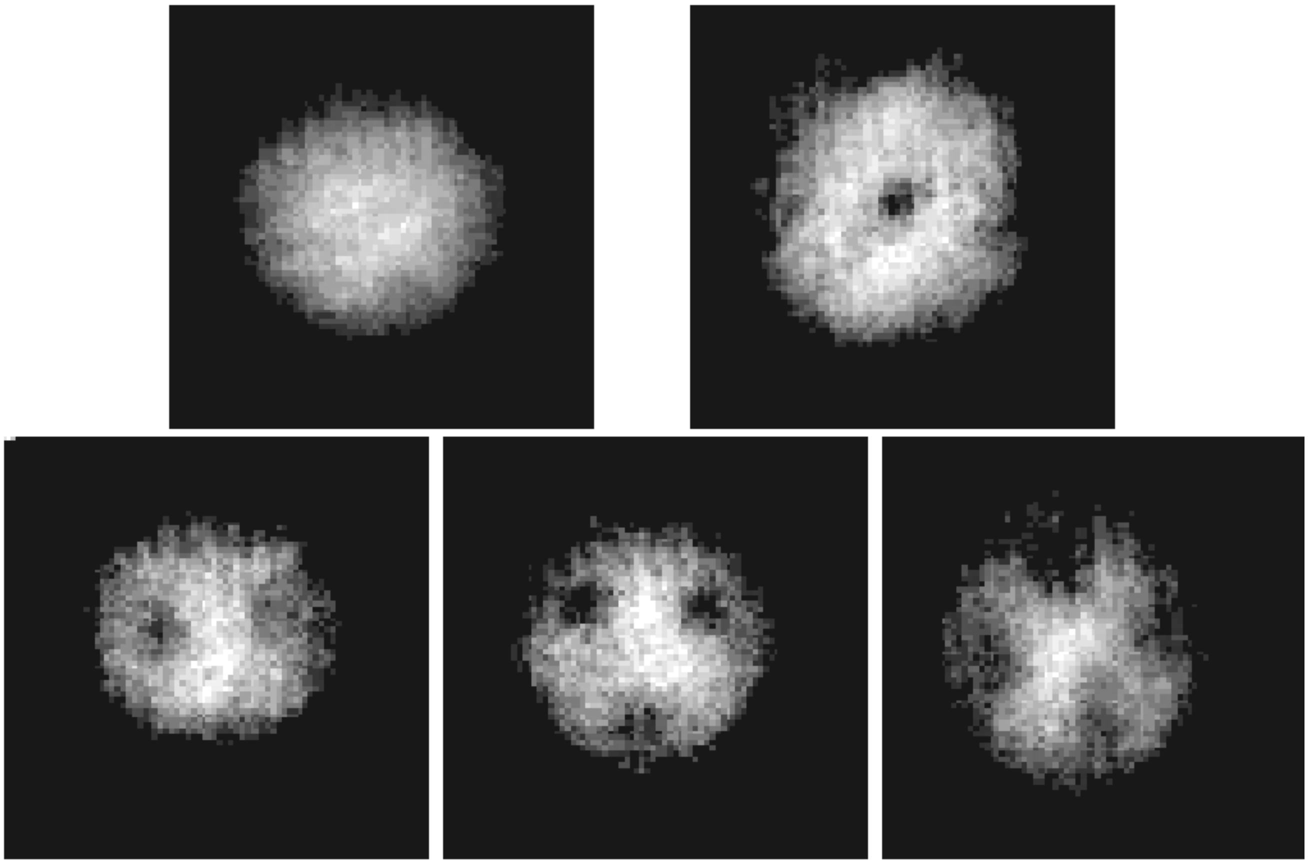


[Anderson *et. al.* [JILA], Science **269**, 198 (1995).]

Vortices in Atomic Bose Condensates

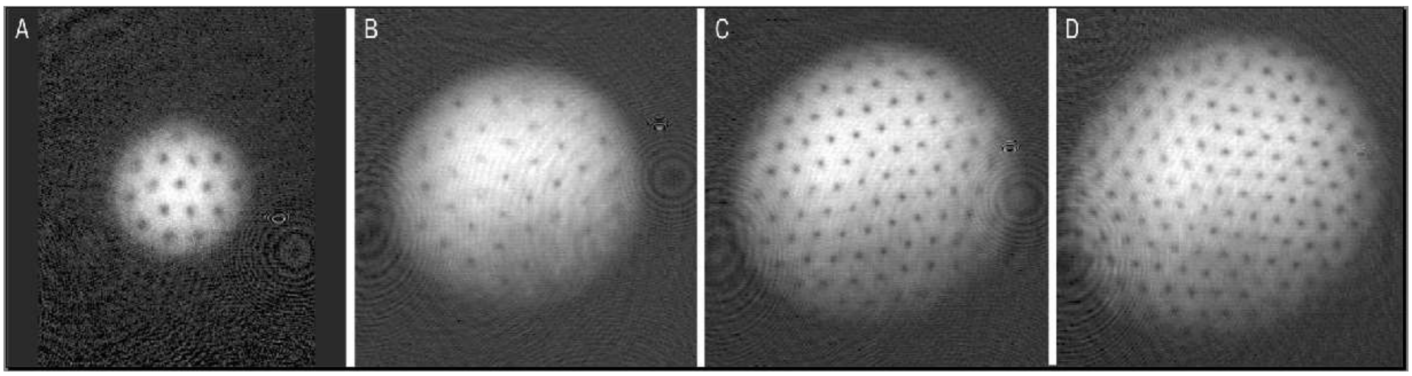
^{87}Rb

[Madison *et al.*[ENS], PRL **84**, 806 (2000)]



^{23}Na

[Abo-Shaeer *et al.*[MIT], Science **476**, 476 (2001)]



$\xi \gg a_{\mathrm{ho}}$ = The Lowest Landau Level

$$H = \sum_{i=1}^{N}\left[\frac{p_i^2}{2m} + \frac{1}{2}m\omega_0^2|\vec{r}_i|^2\right] + \eta\sum_{i<j}\delta(\vec{r}_i - \vec{r}_j)$$

Rotating Frame: $H_\Omega = H - \vec{\Omega}\cdot\vec{L}$

One-Body Terms

$$H_\Omega^{(1)} = \frac{|\vec{p}|^2}{2m} + \frac{1}{2}m\omega_0^2|\vec{r}|^2 - \vec{\Omega}\cdot\vec{r}\times\vec{p}$$

$$= \frac{|\vec{p} - m\vec{\Omega}\times\vec{r}|^2}{2m} + \frac{1}{2}m(\omega_0^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2}m\omega_0^2 z^2$$

$$\boxed{q^*\vec{B}^* = 2m\vec{\Omega}}$$

$+$ Interactions

[Wilkin, Gunn & Smith (1998)]

$\eta\bar{n} \ll \hbar\omega_0 \Rightarrow$the Lowest Landau Level

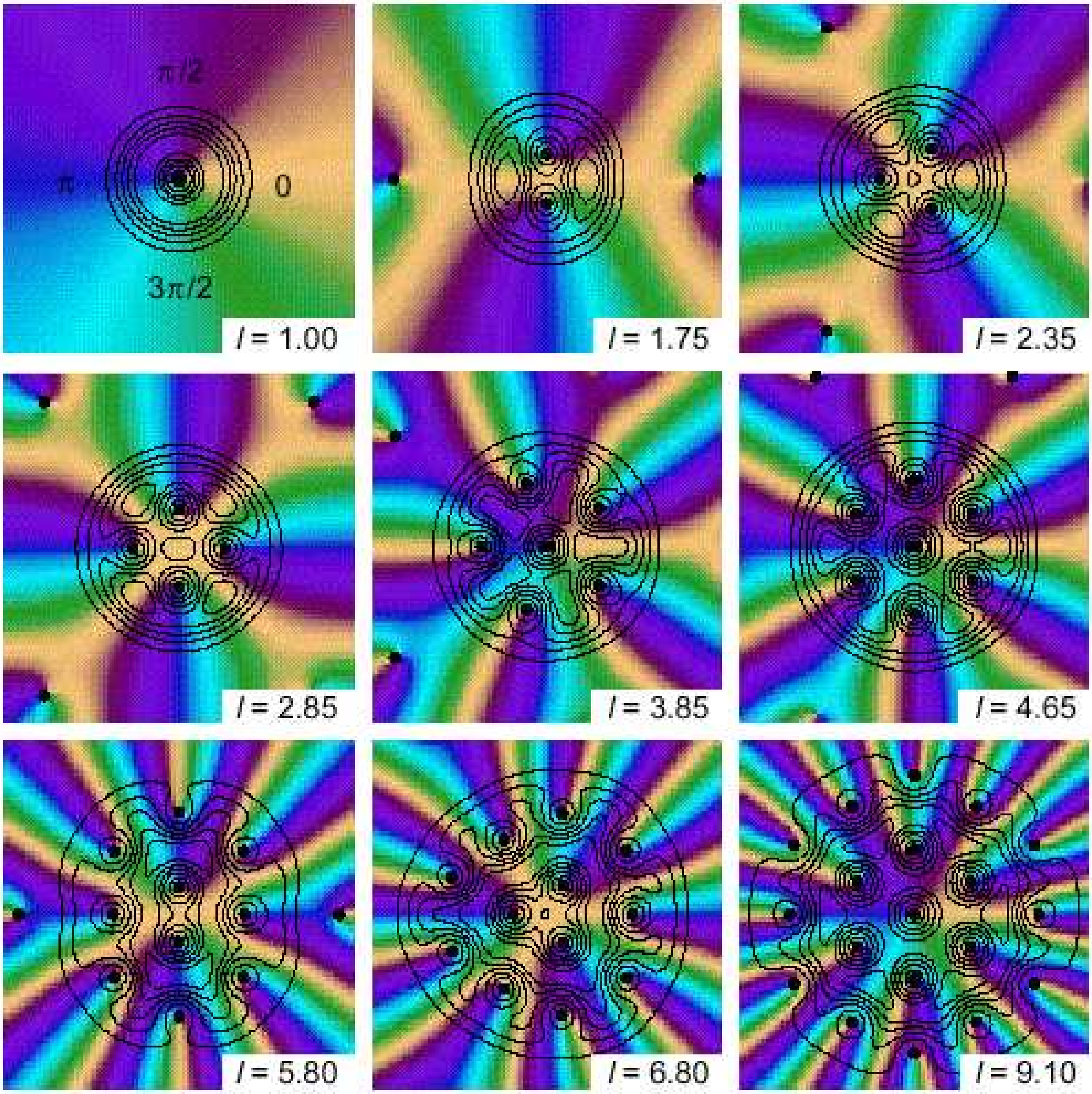
$\langle x, y|m\rangle \propto z^m e^{-|z|^2/4}$ $\left[z \equiv (x+iy)/\ell; \ell \equiv \sqrt{\frac{\hbar}{2m\omega_0}}\right]$

Bosonic version of the FQH problem (quantum dot).

Gross-Pitaevskii Mean-Field Theory

$$\Psi(\{r_i\}) = \prod_i \phi(\vec{r_i})$$

Numerical results: [Butts & Rohksar, Nature **397**, 327 (1999)]



Exact Results

Conserved angular momentum: $L \equiv \sum_i m_i$

$\underline{L \leq N}$ Analytic results [Smith & Wilkin, PRA **62**, 061602 (2000)]

$$\Psi[L = N] \;\propto \prod_i (z_i - z_c)$$

$\underline{L \geq N(N-1)}$ Laughlin states (+ quasiholes)

$$\Psi[L = N(N-1)] \propto \prod_{i<j} (z_i - z_j)^2$$

$\underline{N < L < N(N-1)}$ Numerical exact diagonalisations

$\Rightarrow$ "Vortex Liquids" [Wilkin & Gunn, PRL **84**, 6 (2000)]

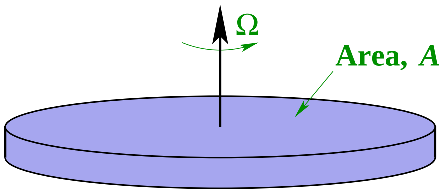
Closely related to FQH states

$$\psi_{\mathrm{B}}(\{\vec{r}_i\}) = \mathcal{P} \prod_{i<j} (z_i - z_j) \psi_{\mathrm{CF}}(\{\vec{r}_i\})$$

[NRC & Wilkin, PRB **60**, R16279 (1999)]

The Filling Fraction

FQHE: $\nu = n_e \frac{h}{eB}$



Here: $\nu = \frac{N}{A}\,\frac{h}{q^* B^*} = \frac{N}{A}\frac{h}{2m\Omega}$

Vortex density: $\frac{N_V}{A} = \frac{\vec{\nabla}\times\vec{v}}{h/m} = \frac{2m\Omega}{h}$

$$\boxed{\nu = \frac{N}{N_V}}$$

What is the nature of the groundstates at large ν?

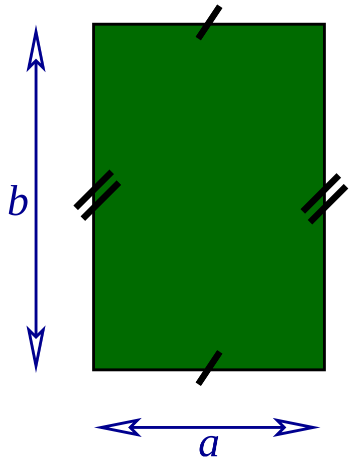
Vortex Lattice vs. Vortex Liquid

[NRC, Wilkin & Gunn, preprint]

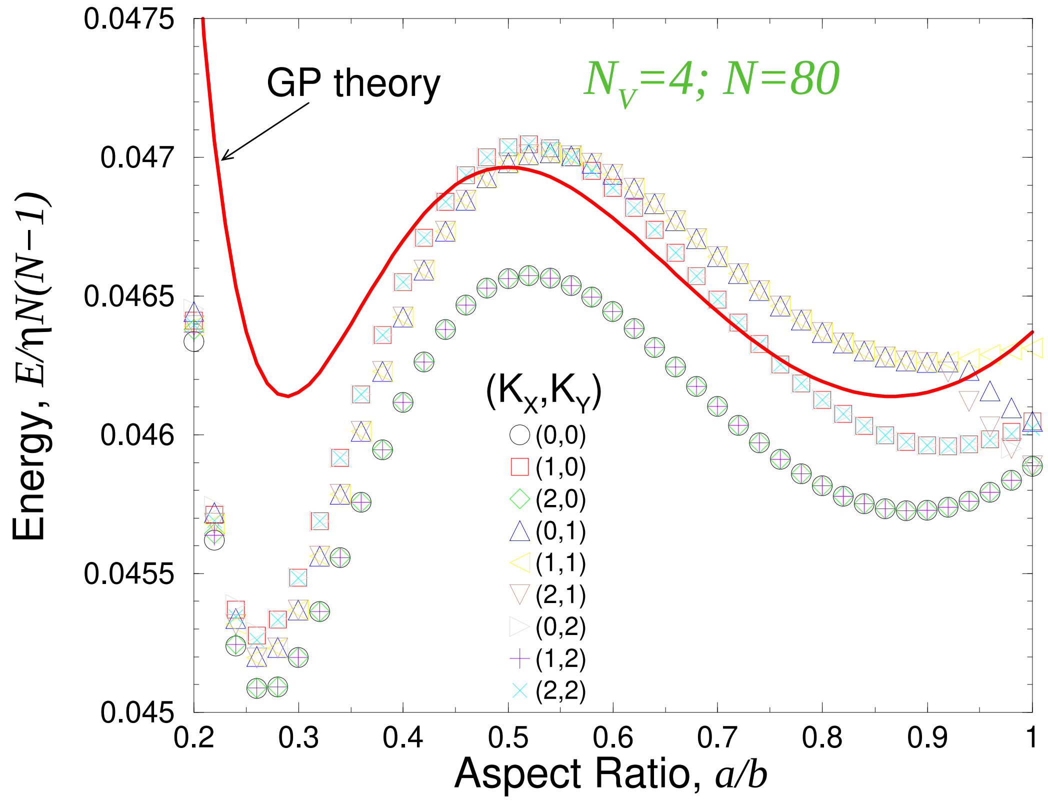
Exact diagonalisations on a torus

- Uniform system with well-defined ν;
- Consistent with a vortex lattice;
- Conserved 2D momentum

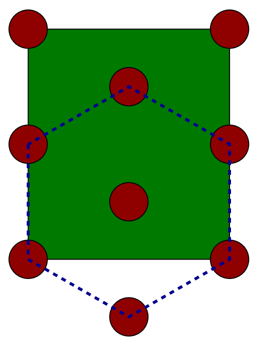
  [Haldane, PRL **55**, 2095 (1985)].



$\nu \to \infty$

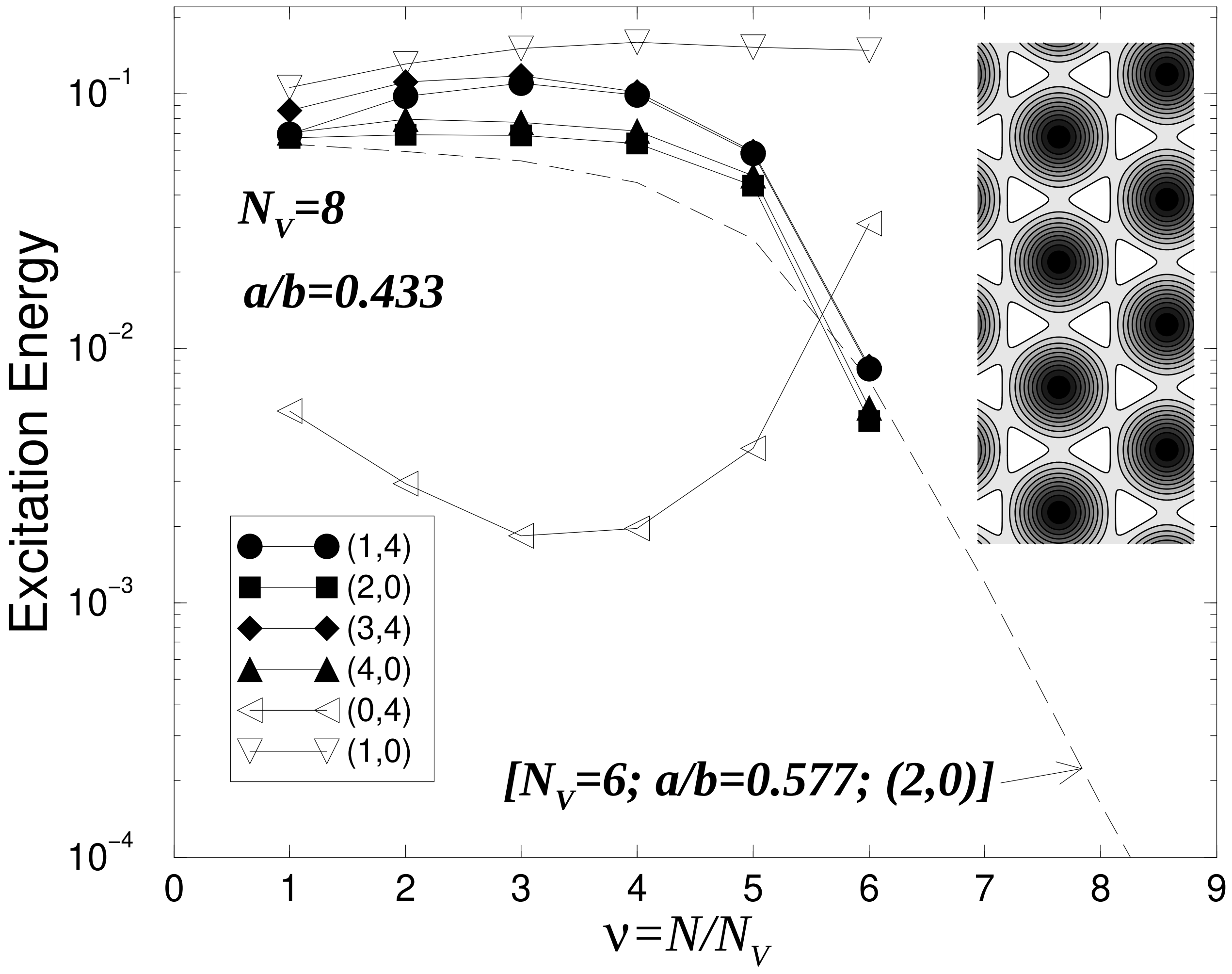


Broken Translational Symmetry



Phase Transition?

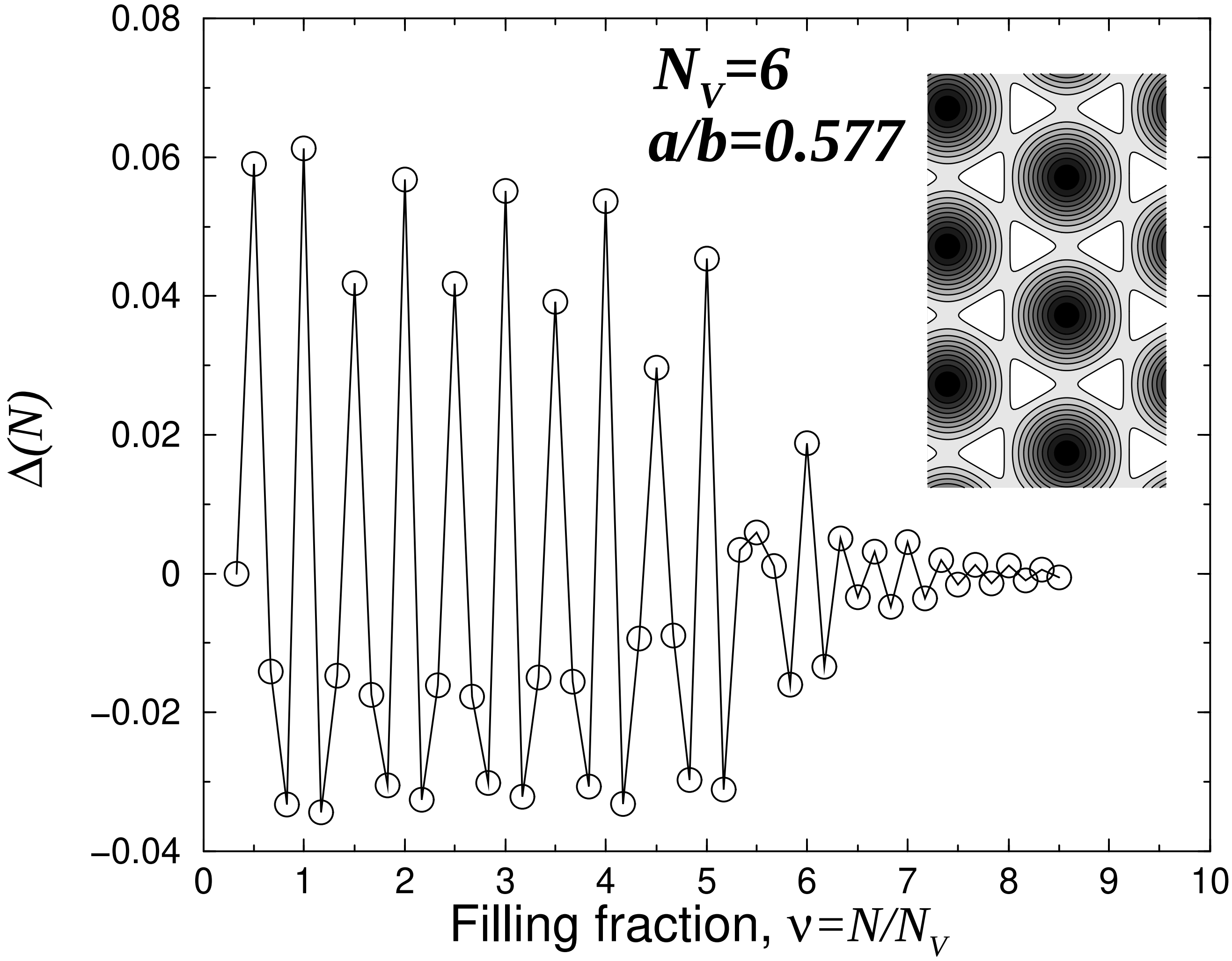
Translational Symmetry Breaking



Transition to a triangular vortex lattice for $\nu \gtrsim 6$.

Charged Excitation Gaps

$$\Delta(N) \equiv N\left[\frac{E(N+1)}{N+1} + \frac{E(N-1)}{N-1} - 2\frac{E(N)}{N}\right]$$



Incompressible liquids at $\nu = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 5$

Read-Rezayi "Parafermion" states

[Read & Rezayi, PRB **59**, 8084 (1999)]

$(k+1)$-body repulsion

$$V^{(k)} = \sum_{i_1 < i_2 \ldots < i_{k+1}} \delta(\vec{r}_{i_1} - \vec{r}_{i_2})\delta(\vec{r}_{i_2} - \vec{r}_{i_3}) \ldots \delta(\vec{r}_{i_k} - \vec{r}_{i_{k+1}})$$

zero-energy eigenstates

$$\Psi^{(k)}(\{z_i\}) = \mathcal{S}\left[\prod_{i<j\in A}^{N/k}(z_i - z_j)^2 \prod_{l<m\in B}^{N/k}(z_l - z_m)^2 \ldots\right]$$

$$\nu^{(k)} = \frac{k}{2}$$

$k=1$: Laughlin state ($\nu = 1/2$)
$k=2$: Moore-Read ("Pfaffian") state ($\nu = 1$).

- The dominant sequence of incompressible states
- Large overlaps with the two-body groundstates (up to $\nu = 3$).

$N_V = 6; a/b = 1/\sqrt{3}$

k	ν	$(K_x, K_y)\times$degeneracy	$\lvert\langle\Psi^{(k)}\lvert\Psi\rangle\rvert$	$\lvert\langle\Psi^{GP}\lvert\Psi\rangle\rvert$
1	1/2 (Laughlin)	(0,0)$\times 2$	1.000	0.555
2	1 (Moore-Read)	(3,3)$\times 1$	0.982	N/W
2	1 (Moore-Read)	(3,0)$\times 1$	0.982	0.408
2	1 (Moore-Read)	(0,3)$\times 1$	0.981	0.493
3	3/2	(0,0)$\times 4$	0.967	0.234
4	2	(0,0)$\times 2$	0.956	0.242
4	2	(3,0)$\times 1$	0.966	N/W
4	2	(0,3)$\times 1$	0.935	N/W
4	2	(3,3)$\times 1$	0.844	0.547
5	5/2	(0,0)$\times 6$	0.955	0.163
6	3	(3,3)$\times 2$	0.960	N/W
6	3	(3,0)$\times 2$	0.944	0.198
6	3	(0,3)$\times 2$	0.744	0.534
6	3	(0,0)$\times 1$	0.852	N/W

Summary

- Owing to the weak interparticle repulsion, the vortex cores in atomic Bose condensates are large.

- We have studied the groundstates of a rotating boson system in the limit of weak interactions. The problem maps to interacting bosons in the lowest Landau level.

- Using exact diagonalisation studies, we find that for large filling fractions $\nu \equiv N/N_V \gtrsim 6$ the groundstate appears to be a triangular $vortex\ lattice$.

- For $\nu \lesssim 6$ there appear $vortex\ liquid$ states. The groundstates at $\nu = k/2$ have large overlaps with the Read-Rezayi parafermion states.