

Topological Bandstructures for Ultracold Atoms

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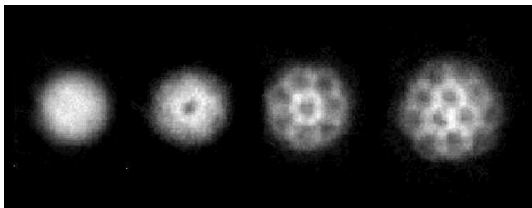
Topological Insulators and Superconductors
KITP Santa Barbara, 22 Sept 2011

NRC, PRL **106**, 175301 (2011)
Benjamin Béri & NRC, arXiv:1105.1252
NRC & Jean Dalibard, EPL **95**, 66004 (2011)

Motivation: fractional quantum Hall regime

Rotating BECs

$$n_\phi = \frac{2M\Omega}{h}$$



[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

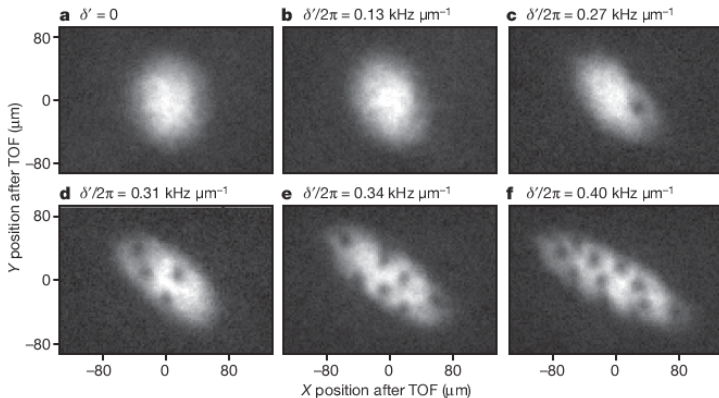
FQH states of bosons for $\nu \equiv \frac{n_{2D}}{n_\phi} \lesssim 6$ [NRC, Wilkin & Gunn, PRL (2001)]

[Laughlin, composite fermion, Moore-Read and Read-Rezayi]

$$\Omega \simeq 2\pi \times 100\text{Hz} \Rightarrow n_\phi \lesssim 2 \times 10^7 \text{cm}^{-2}$$

Optically Induced Gauge Fields

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto and I.B. Spielman, *Nature* **462**, 628 (2009)]



“Optical Flux Lattices”

[NRC, PRL **106**, 175301 (2011); NRC & Jean Dalibard, EPL **95**, 66004 (2011)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Narrow bands with non-zero Chern number
 $n_\phi \simeq 10^9 \text{cm}^{-2} \Rightarrow$ FQH states at high particle densities

- Distinct from previous tight-binding proposals

[Jaksch & Zoller (2003); Mueller (2004); Sørensen, Demler & Lukin (2005); Gerbier & Dalibard (2010)]

- Generalizes to \mathbb{Z}_2 topological invariant [Benjamin Béri & NRC, arXiv:1105.1252]

- “Nearly free electron” approach to topological bands

Outline

Optically Induced Gauge Fields

Optical Flux Lattices

\mathbb{Z}_2 Topological Insulators

Optically Induced Gauge Fields

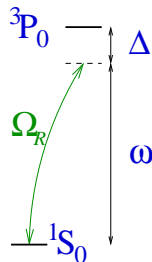
[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, arXiv:1008.5378]

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r})$: optical coupling of N internal states

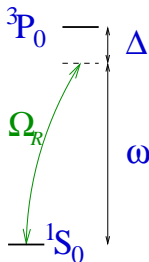
e.g. 1S_0 and 3P_0 for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP 12, 033007 (2010)]



e.g. 1S_0 and 3P_0 for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP 12, 033007 (2010)]



$$\hat{V} = \hbar \begin{pmatrix} 0 & \frac{1}{2} (\Omega_R e^{i\omega t} + \Omega_R^* e^{-i\omega t}) \\ \frac{1}{2} (\Omega_R^* e^{-i\omega t} + \Omega_R e^{i\omega t}) & \omega_0 \end{pmatrix}$$

$$\rightarrow \hbar \begin{pmatrix} -\frac{\Delta}{2} & \frac{1}{2} (\Omega_R + \Omega_R^* e^{-2i\omega t}) \\ \frac{1}{2} (\Omega_R^* + \Omega_R e^{2i\omega t}) & \frac{\Delta}{2} \end{pmatrix}$$

RWA $\omega \gg \Delta, \Omega_R$ $\hat{V} \rightarrow \frac{\hbar}{2} \begin{pmatrix} -\Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & \Delta \end{pmatrix}$

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, arXiv:1008.5378]

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r}) \Rightarrow$ local spectrum $E_n(\mathbf{r})$ and dressed states $|n_{\mathbf{r}}\rangle$

$$|\psi(\mathbf{r})\rangle = \sum_n \psi_n(\mathbf{r}) |n_{\mathbf{r}}\rangle$$

Adiabatic motion $H_n \psi_n = \langle n_{\mathbf{r}} | \hat{H} \psi_n | n_{\mathbf{r}} \rangle$

$$H_n = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M} + V_n(\mathbf{r}) \quad q\mathbf{A} = i\hbar \langle n_{\mathbf{r}} | \nabla | n_{\mathbf{r}} \rangle$$

Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_r | \nabla 0_r \rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{\hbar}{\lambda}$

Cloud of radius $R \gg \lambda$

$$N_\phi \equiv \int n_\phi d^2\mathbf{r} = \frac{q}{h} \int \nabla \times \mathbf{A} \cdot d\mathbf{S} = \frac{q}{h} \oint \mathbf{A} \cdot d\mathbf{r} \lesssim \frac{1}{\lambda} (2\pi R)$$

$$\Rightarrow \bar{n}_\phi \equiv \frac{N_\phi}{\pi R^2} \lesssim \frac{1}{R\lambda} \simeq 2 \times 10^7 \text{cm}^{-2} \quad [R \simeq 10\mu\text{m} \quad \lambda \simeq 0.5\mu\text{m}]$$

Maximum flux density: Carefully this time!

Optical wavelength $\lambda \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

\mathbf{A} can have *singularities* – if the optical fields have vortices.

e.g. $\Omega_R(\mathbf{r}) \sim (x + iy)$

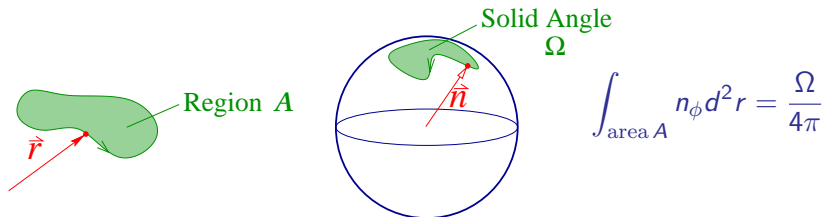
Vanishing net flux. Can be removed by a gauge transformation.

[cf. “Dirac strings”]

Gauge-independent approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_{\mathbf{r}} | \hat{\sigma} | 0_{\mathbf{r}} \rangle$

$$n_{\phi} = \frac{1}{8\pi} \epsilon_{ijk} \epsilon_{\mu\nu} n_i \partial_{\mu} n_j \partial_{\nu} n_k \quad |n_{\phi}| \lesssim \frac{1}{\lambda^2}$$



The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere

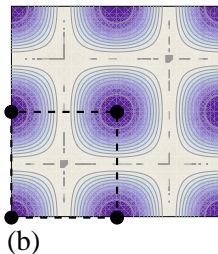
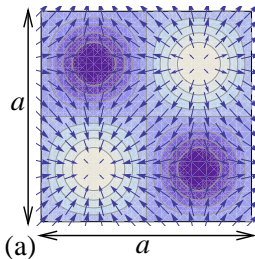
Optical flux lattices

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

Spatially periodic light fields which cause the Bloch vector to wrap the sphere a nonzero integer number, N_ϕ , times in each unit cell.

$$\bar{n}_\phi = \frac{N_\phi}{A_{\text{cell}}} \sim \frac{1}{\lambda^2} \simeq 10^9 \text{cm}^{-2}$$

vectors (n_x, n_y)
 contours n_z
 $N_\phi = 2$



contours n_ϕ

Optical Flux Lattice: One-Photon Implementation

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \mathcal{V} \hat{M}(\mathbf{r}) \quad \hat{M} = \vec{M}(\mathbf{r}) \cdot \hat{\sigma}$$

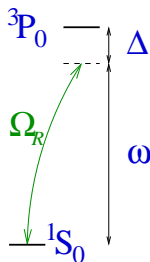
e.g. 1S_0 and 3P_0 for Yb or alkaline earth atom

[Gerbier & Dalibard, *New Journal of Physics* **12**, 033007 (2010)]

M_x, M_y : Rabi coupling, $\omega \simeq \omega_0$

M_z : standing waves at “anti-magic” frequency, ω_{am}

$$\mathcal{V}M = \begin{pmatrix} -\frac{\hbar\Delta}{2} - V_{\text{am}}(\mathbf{r}) & \frac{\hbar\Omega(\mathbf{r})}{2} \\ \frac{\hbar\Omega^*(\mathbf{r})}{2} & \frac{\hbar\Delta}{2} + V_{\text{am}}(\mathbf{r}) \end{pmatrix}$$

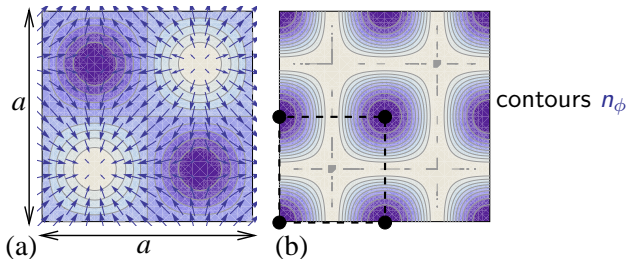


Square Lattice

$$\hat{M}_{\text{sq}} = \begin{pmatrix} \sin(\kappa x) \sin(\kappa y) & \cos(\kappa x) - i \cos(\kappa y) \\ \cos(\kappa x) + i \cos(\kappa y) & -\sin(\kappa x) \sin(\kappa y) \end{pmatrix}$$

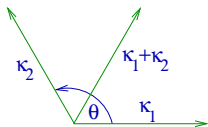
where $\kappa \equiv 2\pi/a$.

vectors (n_x, n_y)
 contours n_z
 $N_\phi = 2$

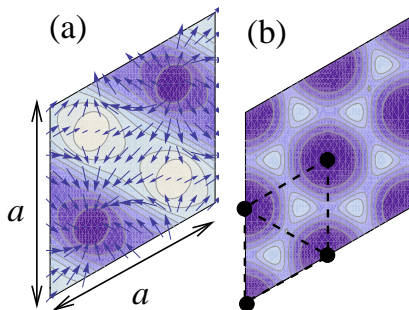


Triangular lattice

$$\hat{M}_{\text{tri}} = \begin{pmatrix} \cos[\mathbf{r} \cdot (\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2)] & \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_1) - i \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_2) \\ \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_1) + i \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_2) & -\cos[\mathbf{r} \cdot (\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2)] \end{pmatrix}$$



$\theta \simeq 2\pi/3$
 vectors: (n_x, n_y)
 contours: n_z
 $N_\phi = 2$

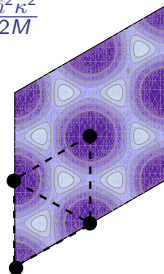
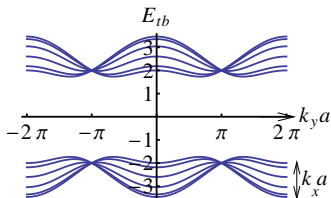


Bandstructure (Triangular Lattice)

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}} + \mathcal{V} [c_1 \hat{\sigma}_x + c_2 \hat{\sigma}_y + c_{12} \hat{\sigma}_z]$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}), \quad c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2) \cdot \mathbf{r}]$$

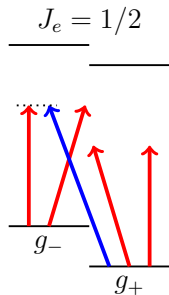
Tight-binding limit $\mathcal{V} \gtrsim E_R \equiv \frac{\hbar^2 \kappa^2}{2M}$



Lowest energy band has narrow width and Chern number of 1.

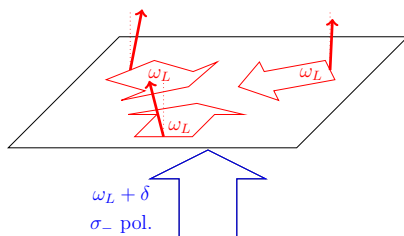
Two-Photon Dressed States

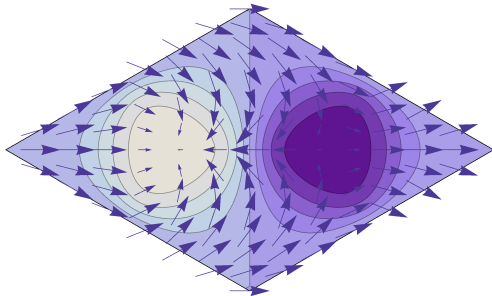
[NRC & Jean Dalibard, EPL 95, 66004 (2011)]



Light at two frequencies:

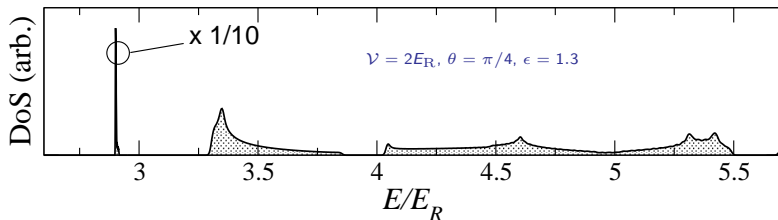
- ω_L with Rabi freqs. κ_m ($m = 0, \pm 1$)
- $\omega_L + \delta$ with Rabi freq. E in σ_-





Triangular lattice with $N_\phi = 1$ per unit cell.

Bandstructure, $J_g = 1/2$



- Narrow lowest energy band, with Chern number of 1
- Can also be applied to bosons $J_g = 1$ (e.g. ^{87}Rb)

Experimental Consequences

Non-interacting fermions (IQHE)

- Filled band has chiral edge state:



Precession of collective modes.

Interacting fermions/bosons

Strongly correlated phases if interactions large compared to bandwidth: likely candidates for FQHE states.

- Incompressible states (density plateaus).
- Chiral edge modes.

[M. Cazalilla, PRA 2003]

Outline

Optically Induced Gauge Fields

Optical Flux Lattices

\mathbb{Z}_2 Topological Insulators

\mathbb{Z}_2 Topological Insulators

[Hasan & Kane, RMP **82**, 3045 (2010); Qi & Zhang, arXiv:1008.2026]

$$\hat{H} = \frac{\mathbf{p}^2}{2M} \hat{\mathbb{I}}_N + \mathcal{V} \hat{M}(\mathbf{r})$$

[Benjamin Béri & NRC, arXiv:1105.1252]

Time-reversal $\hat{\theta} = i\hat{\sigma}_y \hat{K}$ TRS: $\hat{M} = \hat{\theta}^{-1} \hat{M} \hat{\theta} \Rightarrow N = 4$

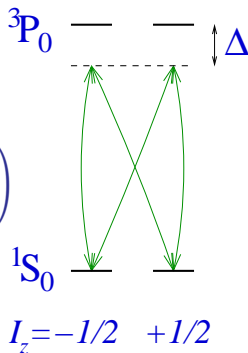
$$\begin{aligned} \hat{M} &= \begin{pmatrix} (A+B)\hat{\mathbb{I}}_2 & C\hat{\mathbb{I}}_2 - i\hat{\sigma} \cdot \vec{D} \\ C\hat{\mathbb{I}}_2 + i\hat{\sigma} \cdot \vec{D} & (A-B)\hat{\mathbb{I}}_2 \end{pmatrix} \\ &= A\hat{\mathbb{I}}_4 + B\hat{\Sigma}_3 + C\hat{\Sigma}_1 + \vec{D}\hat{\Sigma}_2\hat{\sigma} \end{aligned}$$

[$A, B, C, \vec{D} = (D_x, D_y, D_z)$ real]

Dressed states are Kramers doublets \Rightarrow non-Abelian gauge field.

^{171}Yb has nuclear spin $I = 1/2$

$$\nu \hat{M} = \begin{pmatrix} -(\frac{\hbar}{2}\Delta + V_{\text{am}}) \hat{\mathbb{I}}_2 & -i\hat{\sigma} \cdot \vec{\mathcal{E}} d_r \\ i\hat{\sigma} \cdot \vec{\mathcal{E}}^* d_r & (\frac{\hbar}{2}\Delta + V_{\text{am}}) \hat{\mathbb{I}}_2 \end{pmatrix}$$



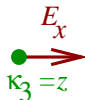
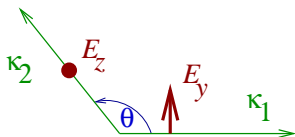
TRS preserved if all components of $\vec{\mathcal{E}}$ have a common phase.

Two Dimensions

$$d_r \vec{\mathcal{E}} = \mathcal{V} (\delta, \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_1), \cos(\mathbf{r} \cdot \boldsymbol{\kappa}_2))$$

$$\boldsymbol{\kappa}_1 = (1, 0, 0)\kappa$$

$$\boldsymbol{\kappa}_2 = (\cos \theta, \sin \theta, 0)\kappa$$



$$\frac{\hbar}{2} \Delta + V_{\text{am}}(\mathbf{r}) = -\mathcal{V} \cos[\mathbf{r} \cdot (\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2)]$$

For Yb, $\theta \simeq 2\pi/3$

$$\hat{U} = 2^{-1/2}(\hat{\mathbb{I}}_4 - i\hat{\Sigma}_3\hat{\sigma}_2)$$

$$\hat{M}' = \hat{U}^\dagger \hat{M} \hat{U} = c_1 \hat{\Sigma}_1 + c_2 \hat{\Sigma}_2 \hat{\sigma}_3 + c_{12} \hat{\Sigma}_3 + \delta \hat{\Sigma}_2 \hat{\sigma}_1.$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}), \quad c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2) \cdot \mathbf{r}]$$

(i) Decoupled spins, $\delta = 0$

$$\hat{M}' = c_1 \hat{\Sigma}_1 \pm c_2 \hat{\Sigma}_2 + c_{12} \hat{\Sigma}_3$$

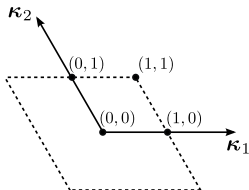
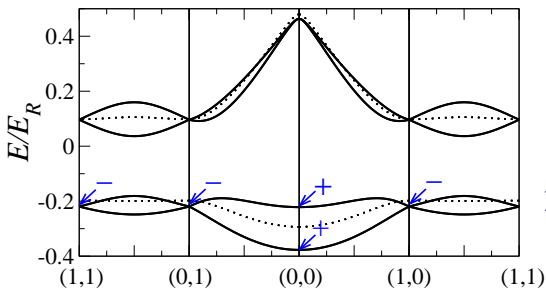
OFLs of opposite flux for spin $\sigma_3 = \pm 1$.

$\sigma_3 = \pm 1$ bands are degenerate, but with opposite Chern numbers.

\Rightarrow “quantum spin Hall” system

[e.g. Levin & Stern, PRL (2009)]

(ii) "Spin-orbit coupling", $\delta \neq 0$



$$\Gamma_{nm} = \frac{1}{2}(n\kappa_1 + m\kappa_2)$$

Inversion symmetry

[Fu & Kane, PRB (2007)]

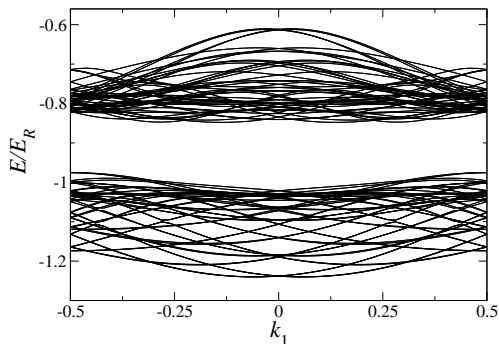
$$\prod_{n,m=0,1} \prod_{\alpha \in \text{filled}} \xi_{nm}^{(\alpha)} = -1$$

Three Dimensions

This nearly-free electron viewpoint leads to a general method to construct \mathbb{Z}_2 non-trivial bands in 3D.

[Benjamin Béri & NRC, arXiv:1105.1252]

e.g. $\delta \rightarrow \delta_0 \cos(\kappa_3 \cdot \mathbf{r}) \quad c_{12} \rightarrow c_{12} + \delta_0(\mu + c_{13} + c_{23})$



$$\mathcal{V} = 0.9E_R$$

$$\delta_0 = 1, \mu = -0.4$$

Summary

- ▶ Simple forms of optical dressing lead to “optical flux lattices”: periodic magnetic flux with high mean density, $n_\phi \sim 1/\lambda^2$.
- ▶ The low energy bands are analogous to the lowest Landau level of a charged particle in a uniform magnetic field.
- ▶ The approach can be generalized to generate \mathbb{Z}_2 nontrivial bandstructures in 2D and 3D.
- ▶ Ultracold atomic gases can readily be used to explore strong correlation phenomena in topological bands.