Theory of Nuclear Spin Relaxation in Semiconductor Quantum Wires

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Séminaire "Nanoélectronique Quantique"
Grenoble, 20 Jan 2009.

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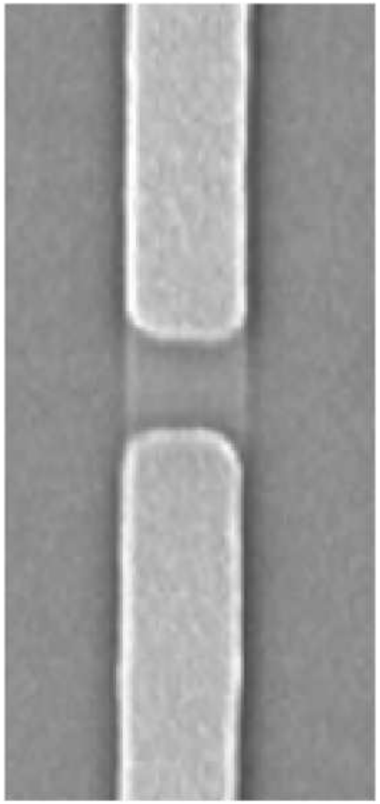
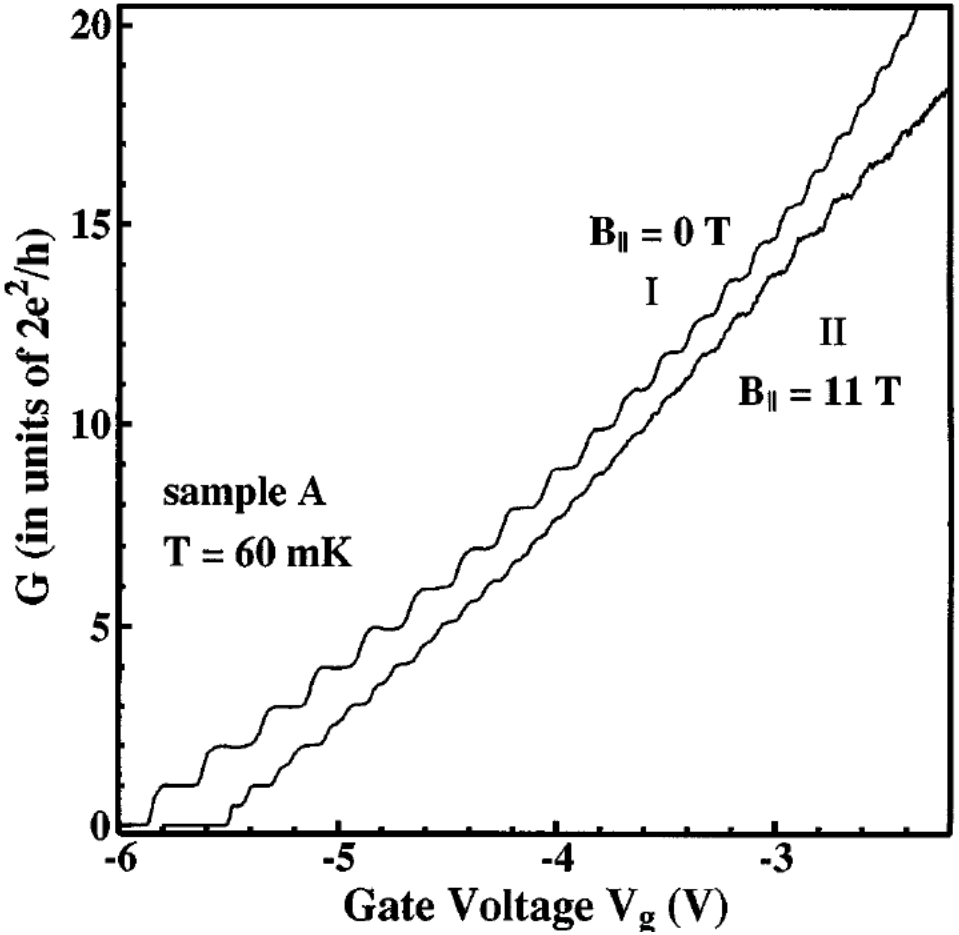
Semiconductor Quantum Point Contact Devices



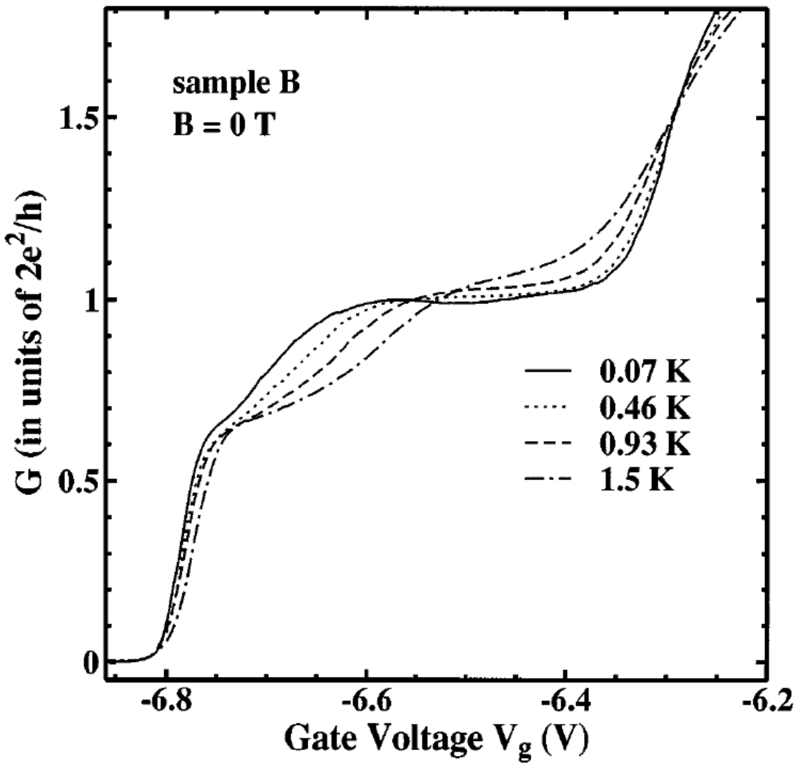
Conductance quantization $G = n \times \frac{e^2}{h}$

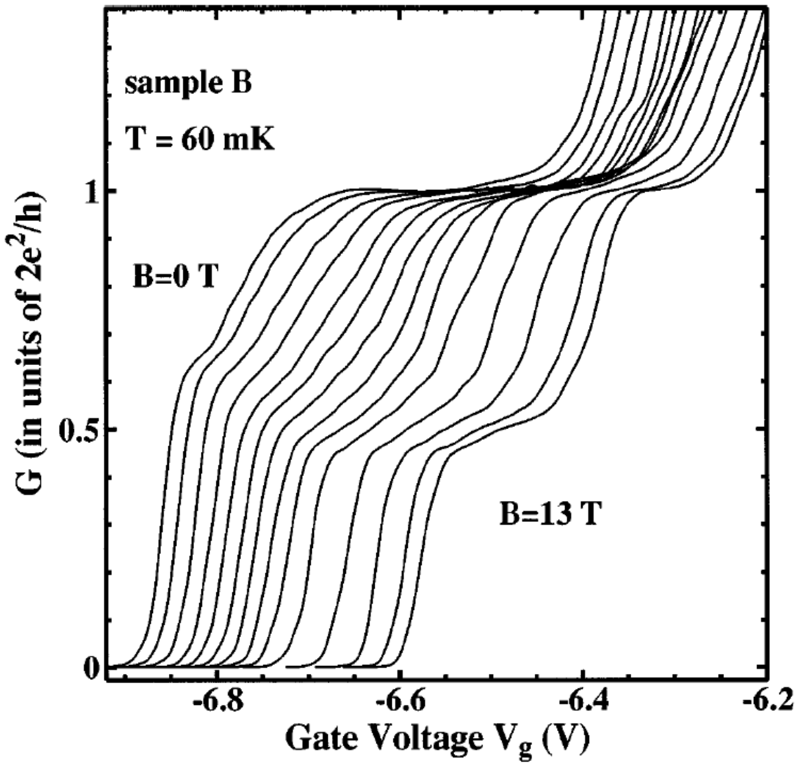
[van Wees *et al.*, PRL **60**, 848 (1988); Wharam *et al.*, J. Phys. C **21**, L209 (1988)]



The "0.7 Effect"

[K. J. Thomas *et al.*, PRL **77**, 135 (1996)]





An interaction effect – what is the mechanism?

Spontaneous spin-polarization; Kondo effect; spin-incoherent Luttinger liquid...

Overview

- "NMR" in a Quantum Point Contact
- Signatures of the 0.7 Effect
- Summary

“NMR” in a Quantum Point Contact

Hyperfine contact interaction

$$H_{\text{hyperfine}} = A_{\text{s}} \sum_i \vec{I}_i \cdot \vec{S}(\mathbf{R}_i)$$

Nuclear Spin relaxation rate

$$T_1^{-1}(\mathbf{R}) = \frac{A_{\text{s}}^2}{2\hbar^2} \int_{-\infty}^{\infty} dt \, \langle S^+(\mathbf{R}, t) S^-(\mathbf{R}, 0) \rangle$$

(i) Local non-equilibrium nuclear polarization.

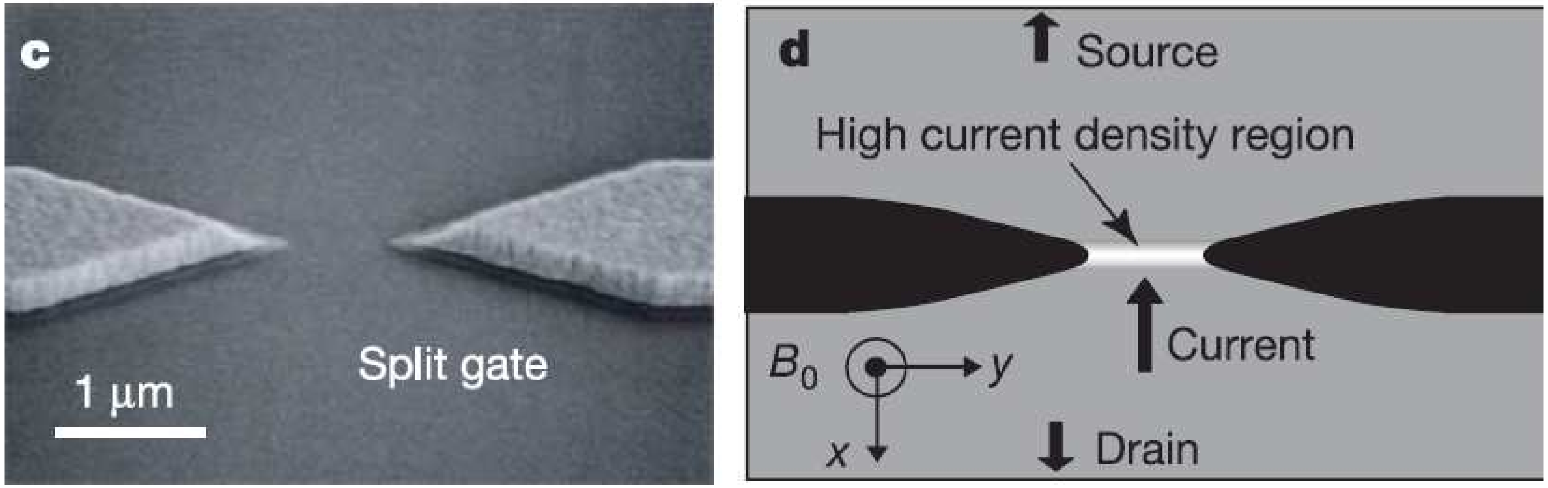
(ii) Resistive detection.

Electrical Pumping of Nuclear Polarization in QPCs

Breakdown of the $\nu = 2/3$ fractional quantum Hall state

[S. Kronmüller *et al.*, PRL **81**, 2526 (1998)]

QPC device at $\nu = 2/3$ [G. Yusa *et al.*, PRB **69**, 161302 (2004); Nature **434**, 1001 (2005).]



Current density is maximum close to the point contact $\Rightarrow$local breakdown.

Resistive Detection of Nuclear Polarization

[NRC & Tripathi, PRB **77**, 245324 (2008)]

Non-interacting electron gas

$$H = \sum_{s,k,\sigma} \left[\epsilon_s + \frac{\hbar^2 k^2}{2m} + \frac{\sigma}{2} g\mu_B B \right] \hat{c}^\dagger_{nk\sigma} \hat{c}_{nk\sigma} + A_\mathrm{s} \sum_i \vec{I}_i \cdot \vec{S}(\mathbf{R}_i)$$

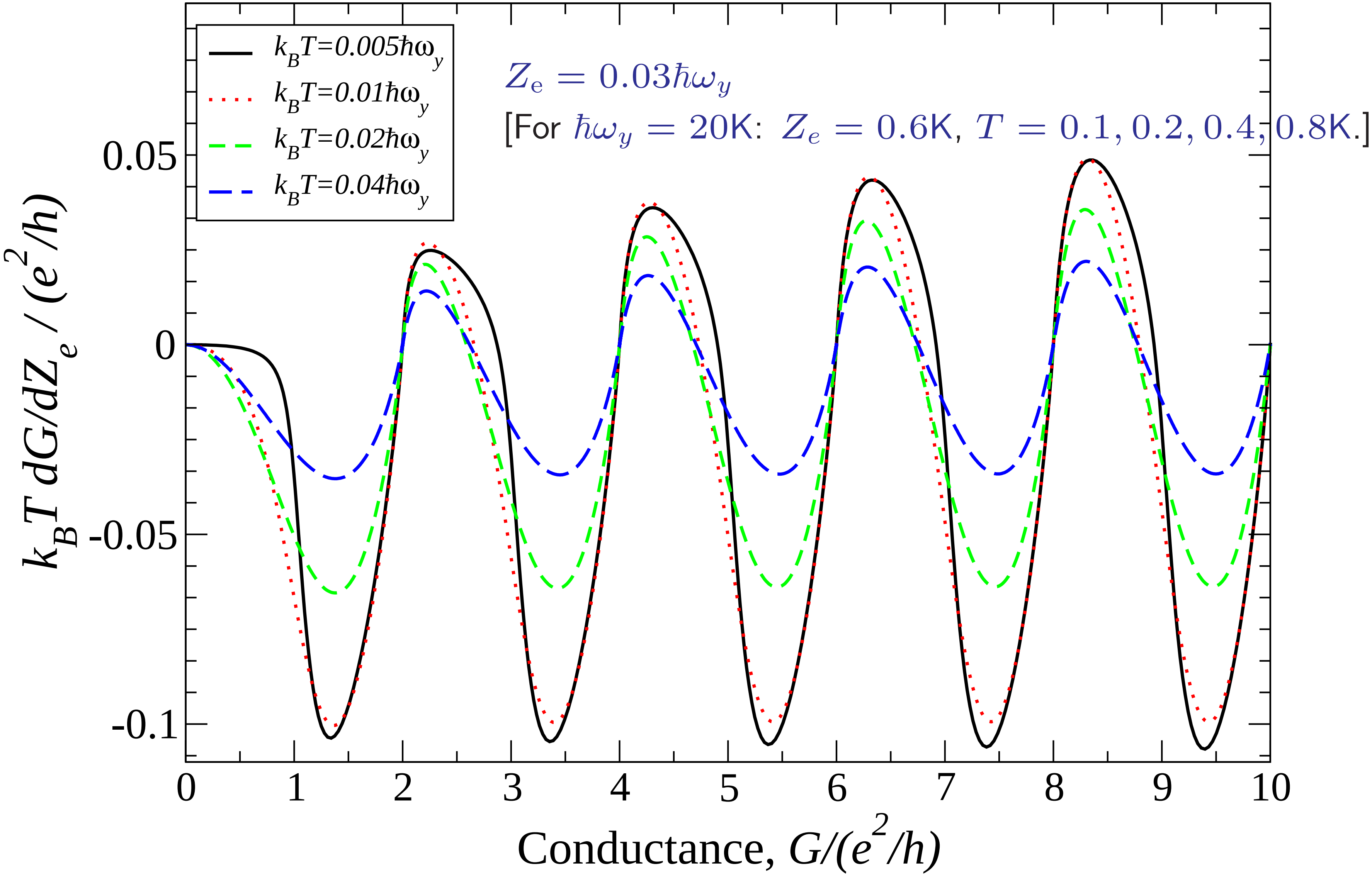
$\epsilon_s \equiv \hbar\omega_y(s + 1/2)$ hyperfine coupling

Overhauser shift: $Z_\mathrm{e} \equiv g\mu_B B + A_\mathrm{s} n_\mathrm{nuc} \langle I^z \rangle$ nuclear density, n_nuc

Conductance:

$$G(Z_\mathrm{e}) = \frac{e^2}{h} \sum_{s,\sigma} f\left(\epsilon_s + \sigma Z_\mathrm{e}/2\right) \qquad \left[f(z) \equiv (e^{(z-\mu)/k_B T} + 1)^{-1} \right]$$

Gate voltage, V $\Rightarrow$ 1D electron density $n = c(V - V_0)/e$ $\Rightarrow \left(\frac{\partial G}{\partial Z_\mathrm{e}} \right)_n$



For GaAs at 50mK, $\Delta G \sim 0.01e^2/h \Rightarrow \Delta I^z/(2I) \simeq 0.3\%$

Overview

- “NMR” in a Quantum Point Contact
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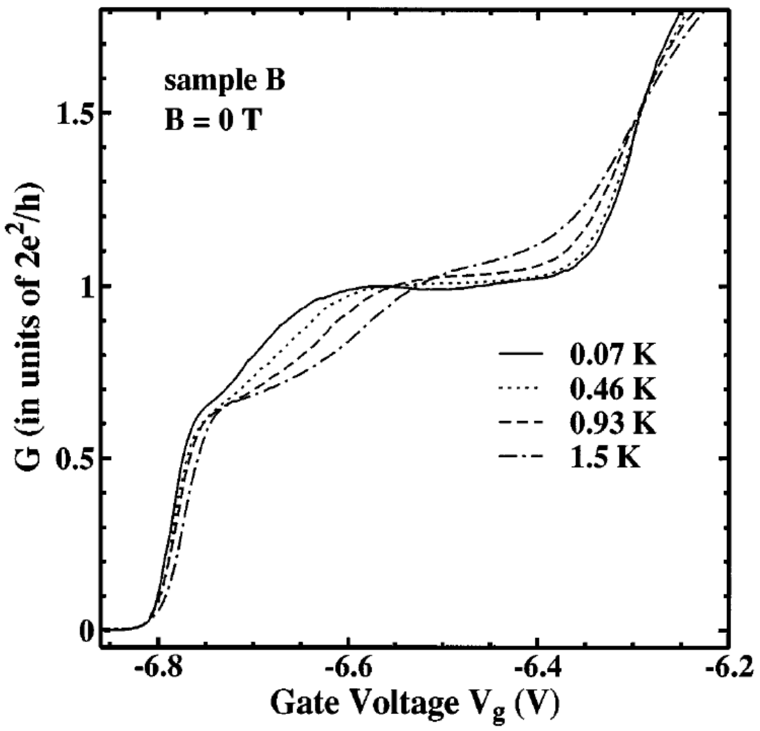
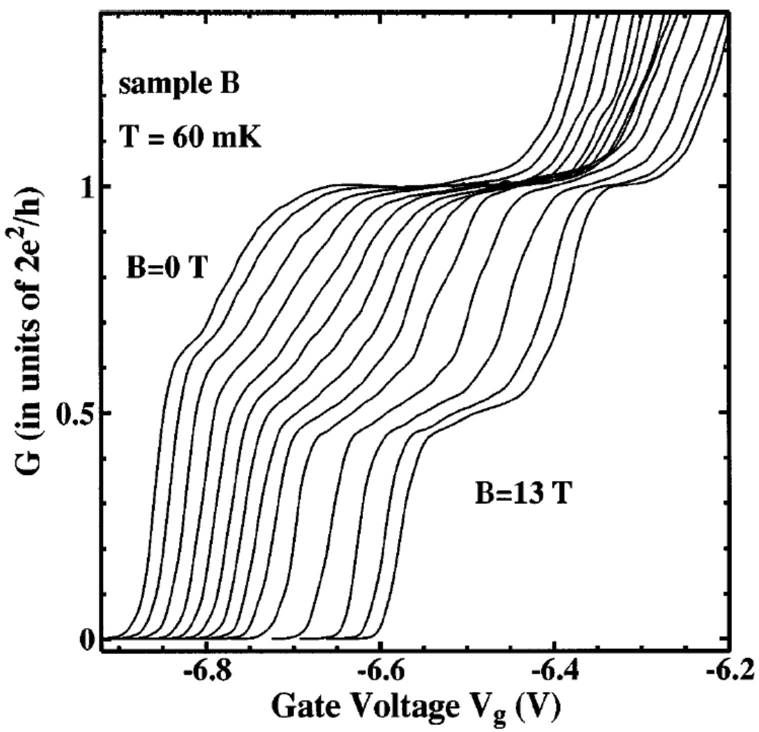
Nuclear Spin Relaxation Rate of the 0.7 Effect

[NRC & Tripathi, PRB **77**, 245324 (2008)]





[K. J. Thomas *et al.*, PRL **77**, 135 (1996)]

Nuclear spin relaxation rate

$$T_1^{-1}(\mathbf{R}) = \frac{A_{\rm s}^2}{2\hbar^2}\int_{-\infty}^{\infty} dt\, \langle S^+(\mathbf{R},t)S^-(\mathbf{R},0)\rangle$$

Fastest decay rate $\Rightarrow$nuclei close to the centre of the QPC.

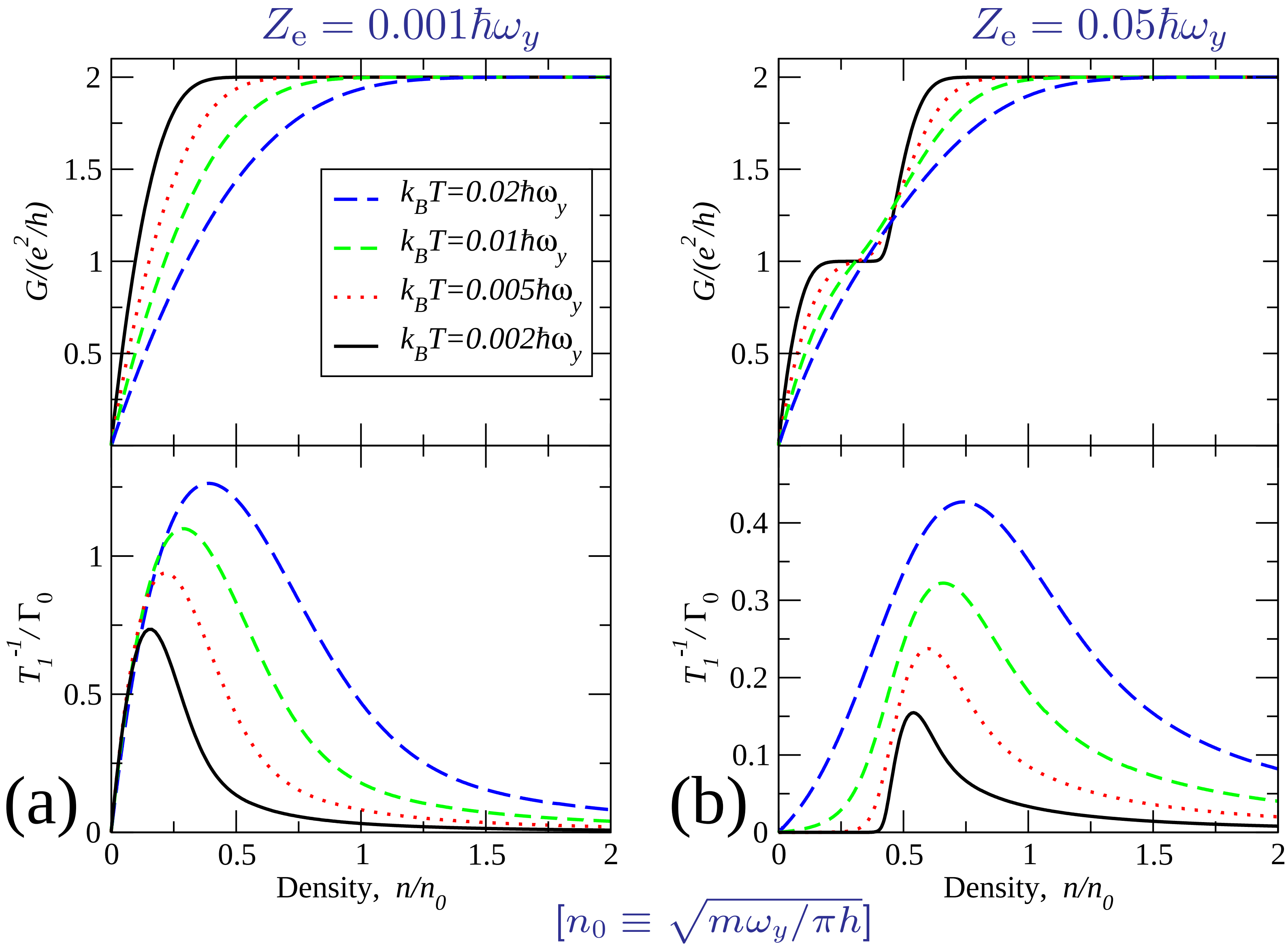
Non-interacting electron gas

$$
\begin{aligned}
{T_1}^{-1} &= \frac{2\pi A_s^2}{\hbar} \sum_{i,f} p_i \left| \langle f | S^- | i \rangle \right|^2 \delta(E_i - E_f) \\
&= \frac{2\pi A_s^2}{\hbar} \sum_{k,k'} f(\epsilon_k + Z_\mathrm{e}/2)[1 - f(\epsilon_{k'} - Z_\mathrm{e}/2)] \frac{1}{L_x^2 w_y^2 w_z^2} \delta(\epsilon_k - \epsilon_{k'} + Z_\mathrm{e}) \\
&= \Gamma_0 \int_{\hbar\omega_y/2 + |Z_\mathrm{e}|/2}^{\infty} \frac{f(\epsilon)\,[1 - f(\epsilon)]}{\sqrt{(\epsilon - \hbar\omega_y/2)^2 - (Z_\mathrm{e}/2)^2}} d\epsilon
\end{aligned}
$$

$w_y w_z$ rms area of s

$$\Gamma_0 \equiv \frac{2\pi A_\mathrm{s}^2 m}{\hbar^3 w_y^2 w_z^2}$$

typically $\Gamma_0 \simeq 0.5\mathrm{Hz}$

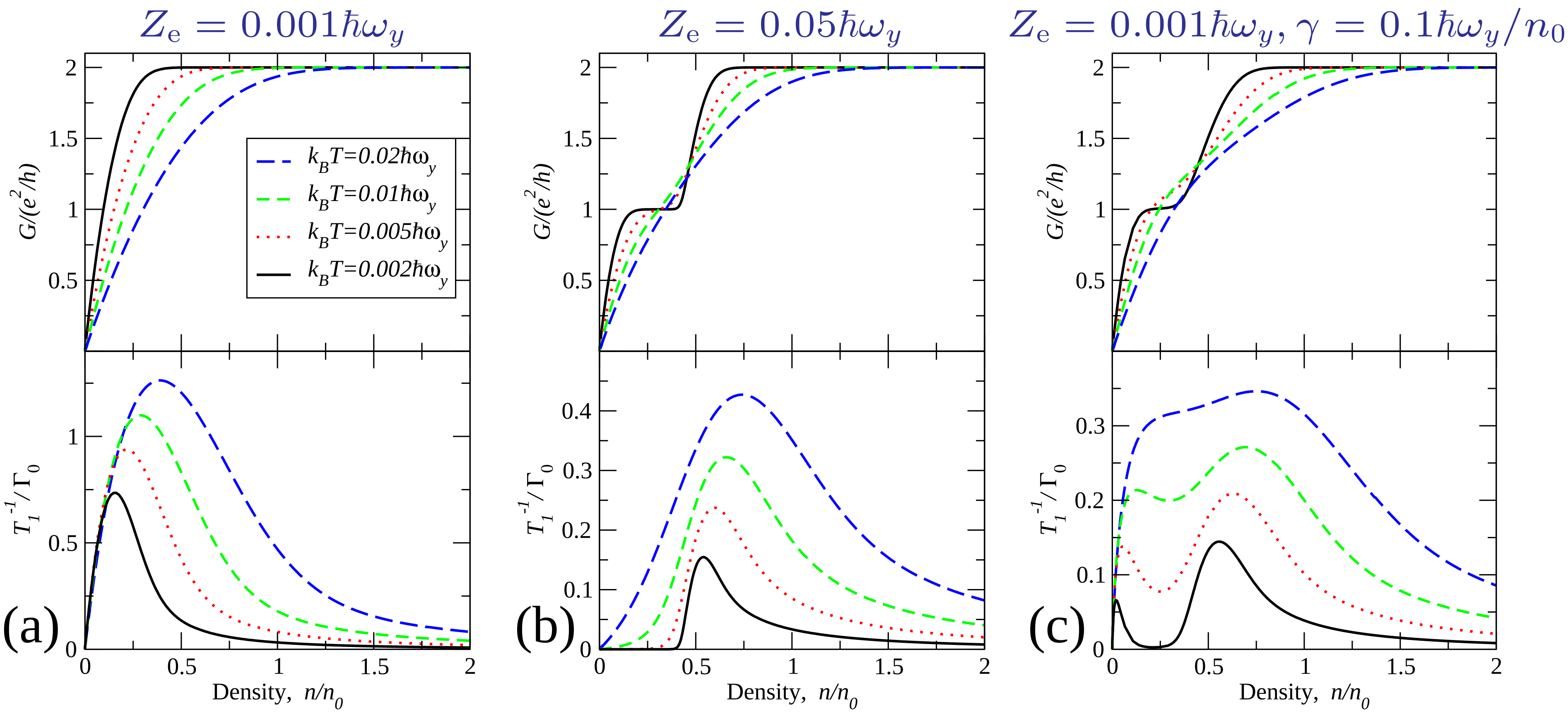


Korringa expression, $T_1^{-1} \propto \rho_\uparrow \rho_\downarrow T$, fails to account for main feature.

Exchange-enhanced spin-splitting

[Wang & Berggren (1996); Bruus, Cheianov & Flensberg (2001); Spivak & Zhou (2000); Reilly *et al.* (2002).]

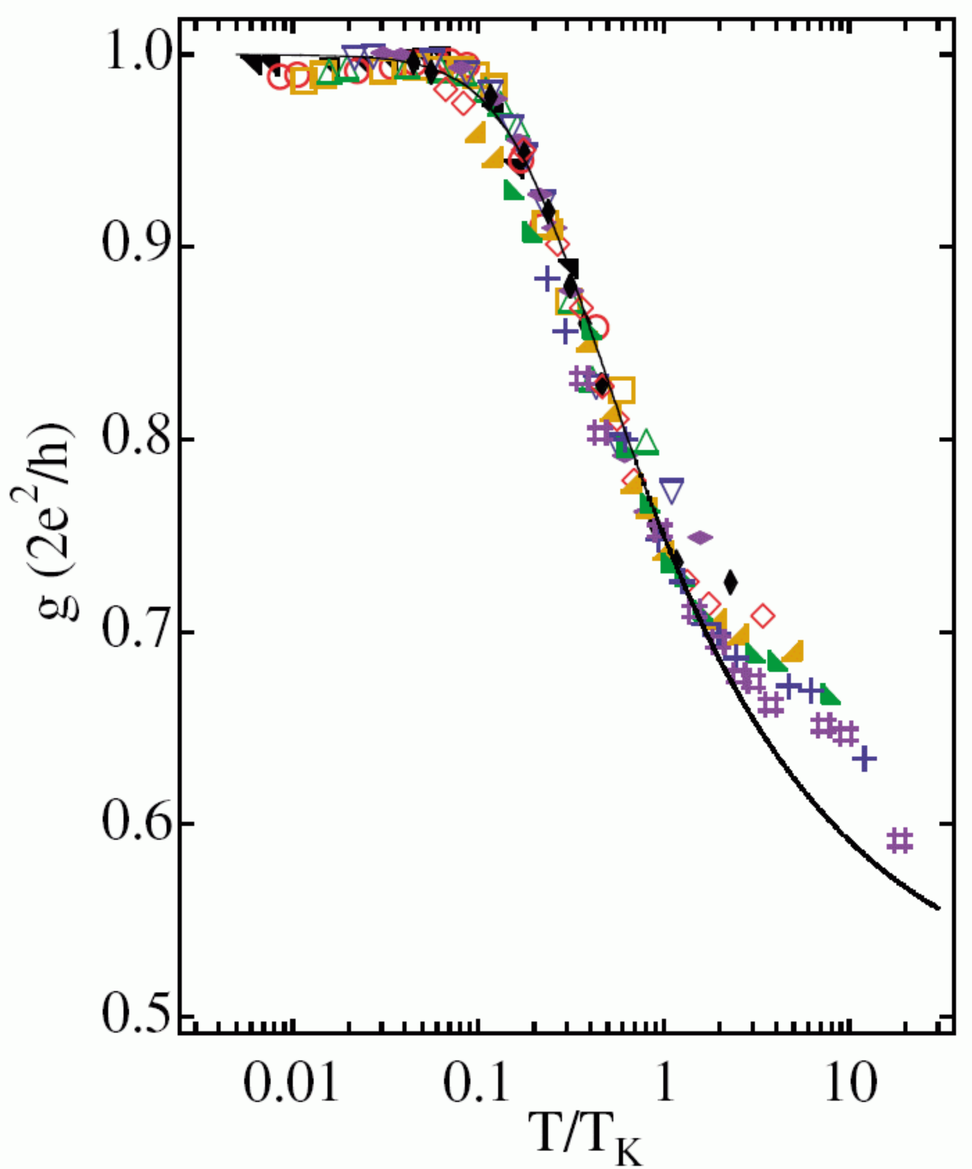
Phenomenological model: $Z_{\rm eff} = Z_{\rm e} + \gamma n$ [D. J. Reilly, PRB **72**, 033309 (2005)]

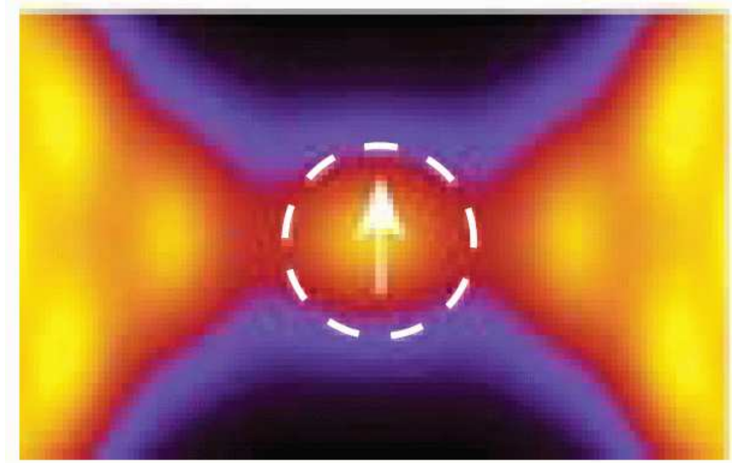


The double peak is a signature of a density dependent exchange-enhanced spin-splitting.

Kondo model

[S. M. Cronenwett *et al.*, PRL **88**, 226805 (2002)]





[Meir, Hirose & Wingreen, PRL **89**, 196802 (2002); Rejec & Meir, Nature **442**, 900 (2006)]

Exchange, $J_{\rm K} \Rightarrow k_B T_{\rm K} \sim \epsilon_F e^{-1/J_{\rm K}\rho(\epsilon_F)}$

The nuclear spin relaxation in the QPC is dominated by the fluctuations of the impurity spin, $A_\mathrm{d} \sim A_\mathrm{s}/(w_x w_y w_z)$.

$$T_1^{-1} = k_B T \left(\frac{A_\mathrm{d}}{\hbar g_s \mu_B}\right)^2 \mathrm{Im} \frac{\chi_\mathrm{imp}^{+-}(\omega)}{\omega}\bigg|_{\omega\to 0}.$$

$T \gg T_\mathrm{K}$ Weak coupling limit [W. Götze and P. Wölfle, J. Low Temp. Phys. **5**, 575 (1971)]

$$T_1^{-1} = 2\frac{A_\mathrm{d}^2 S(S+1)}{3\pi\hbar(k_B T)[J_\mathrm{K}\rho(\epsilon_F)]^2}, \qquad [S = 1/2]$$

$T \ll T_\mathrm{K}$ Local Fermi liquid [H. Shiba, Prog. Theor. Phys. **54**, 967 (1975)]

$$T_1^{-1} = \frac{2\pi(k_B T)A_\mathrm{d}^2}{\hbar(g_s\mu_B)^4}\chi_\mathrm{imp}^2,$$

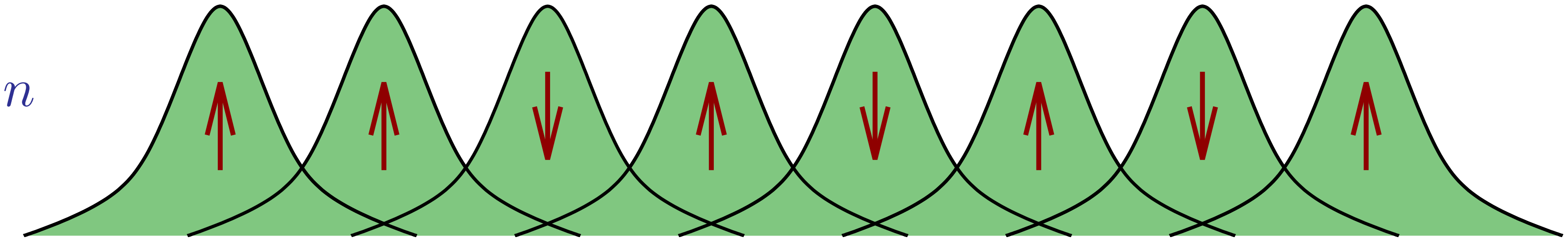
$\chi_\mathrm{imp} =$ static Kondo impurity susceptibility.

The non-monotonic temperature dependence of T_1^{-1} is characteristic of Kondo physics.

Luttinger Liquid

Strongly interacting 1D electron gas

$$\frac{\hbar^2 n^2}{m} \ll \frac{e^2}{\epsilon} n$$



Exchange interaction $J_{\mathrm{LL}} \ll \epsilon_F$

[K. A. Matveev, PRL **92**, 106801 (2004)]

(1) $k_B T \ll J_{\rm LL} \ll \epsilon_F$

Luttinger Liquid (coupled to FL leads)

$$G = 2e^2/h$$

[Maslov & Stone, PRB **52**, R5539 (1995)]

Relaxation rate: [Chitra & Giamarchi, PRB **55**, 5816 (1997); Zavidonov & Brinkmann, PRB **61**, 3282 (2000)]

$$T_1^{-1} \sim T^{K_\rho} \qquad \text{with} \quad K_\rho < 1$$

The nuclear spin relaxation rate is sensitive to the formation of a Luttinger liquid.

(2) $J_{\rm LL} \ll k_B T \ll \epsilon_F$

"spin-incoherent" Luttinger liquid

$$G \simeq e^2/h$$

[K. A. Matveev, PRL **92**, 106801 (2004)]

Low energy spin-flip excitations $\Rightarrow$

Spin-chain, lattice constant $1/n$, bandwidth $J_{\rm LL}$, $k_B T \gg J_{\rm LL}$

$$T_1{}^{-1} \sim \Gamma_{\rm SILL} \equiv \frac{A_{\rm s}^2 n^2}{\hbar w_y^2 w_z^2 J_{\rm LL}}$$

cf. non-interacting electron gas

$$\frac{\Gamma_{\rm SILL}}{\Gamma_0} \sim \frac{\epsilon_F}{J_{\rm LL}} \gg 1$$

In the spin-incoherent LL regime T_1^{-1} is expected to be large and weakly temperature-dependent.

Summary

- The possibility of generating local non-equilibrium nuclear polarization in semiconductor QPCs opens the way to NMR of nanoscale semiconductor devices.

- The two-terminal conductance of the QPC provides a sensitive way to detect the nuclear polarization.

- The nuclear spin relaxation rate, T_1^{-1}, will show clear and distinctive signatures of electron-electron interaction effects that may be at play in the 0.7 effect. [Exchange-enhancement to the Zeeman energy; Kondo Effect; Luttinger liquid; Spin-incoherent Luttinger Liquid.]