

Theory of Nuclear Spin Relaxation in Semiconductor Quantum Wires

Nigel Cooper
T.C.M. Group, Cavendish Laboratory,
University of Cambridge

Séminaire “Nanoélectronique Quantique”
Grenoble, 20 Jan 2009.

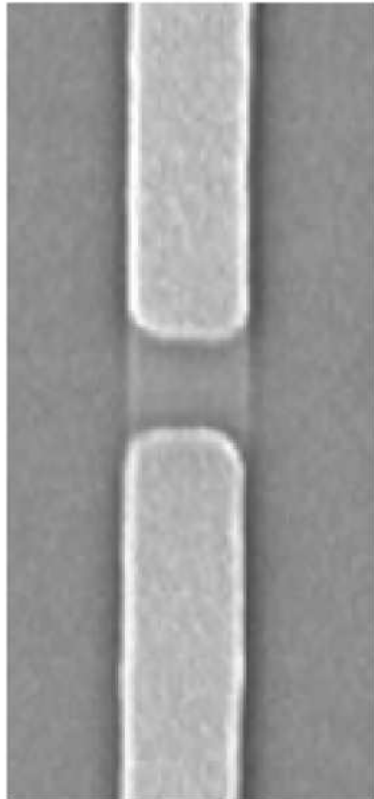
Vikram Tripathi (Tata Institute)

K. Thomas & Antonio Corcoles (Cambridge)

EPSRC

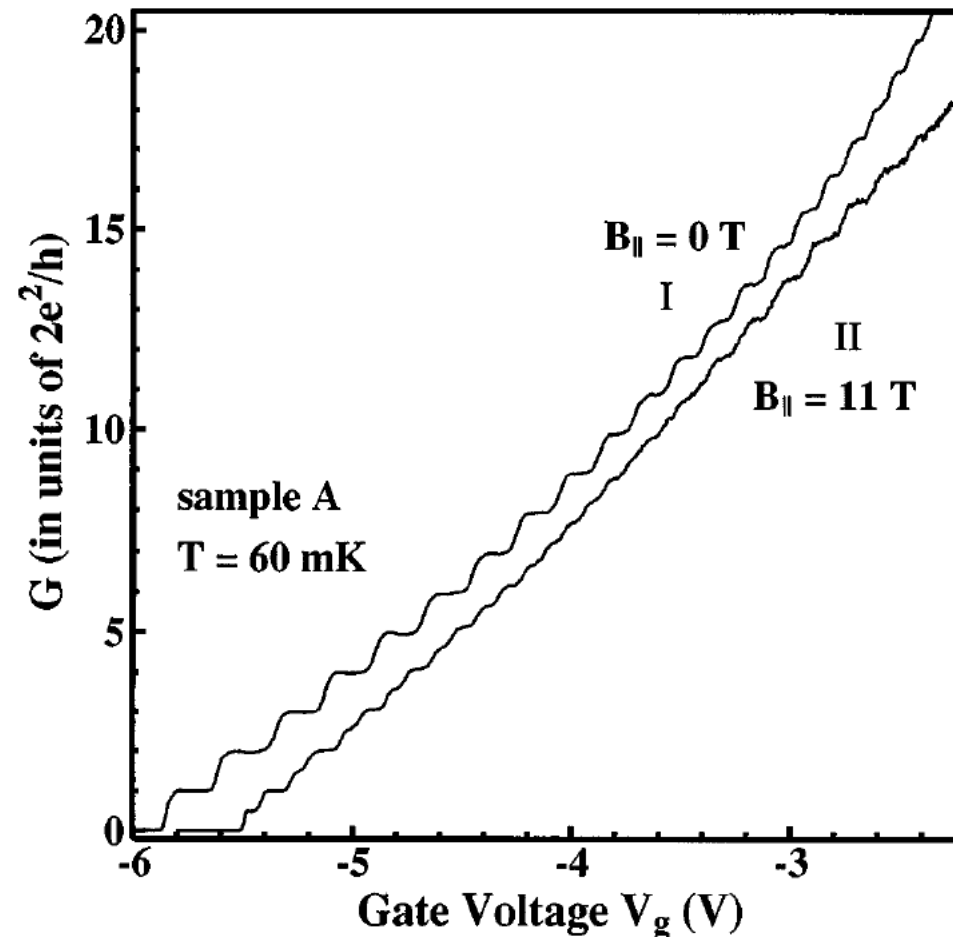
Engineering and Physical Sciences
Research Council

Semiconductor Quantum Point Contact Devices



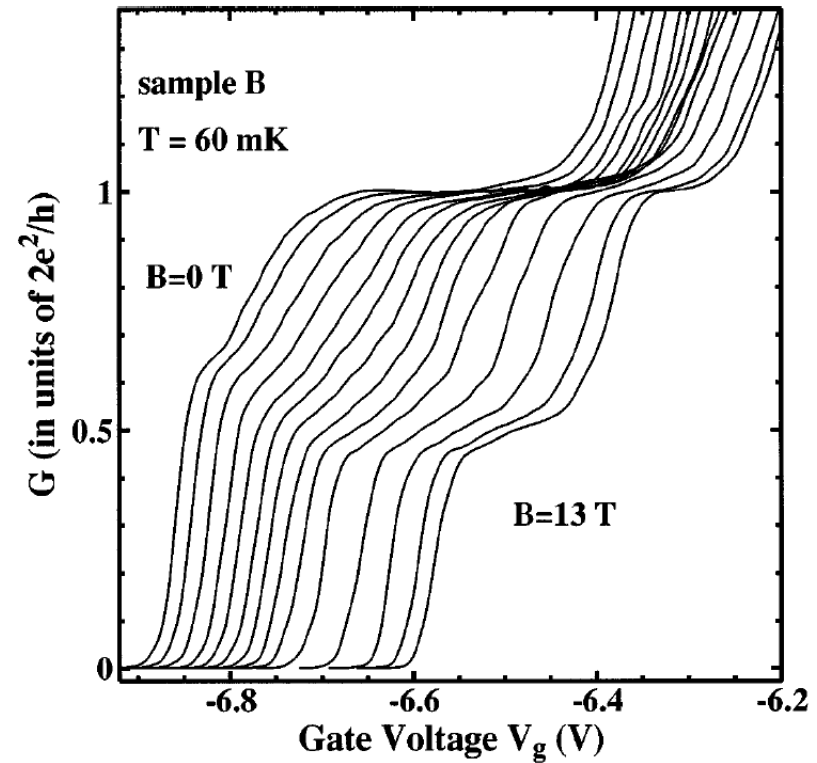
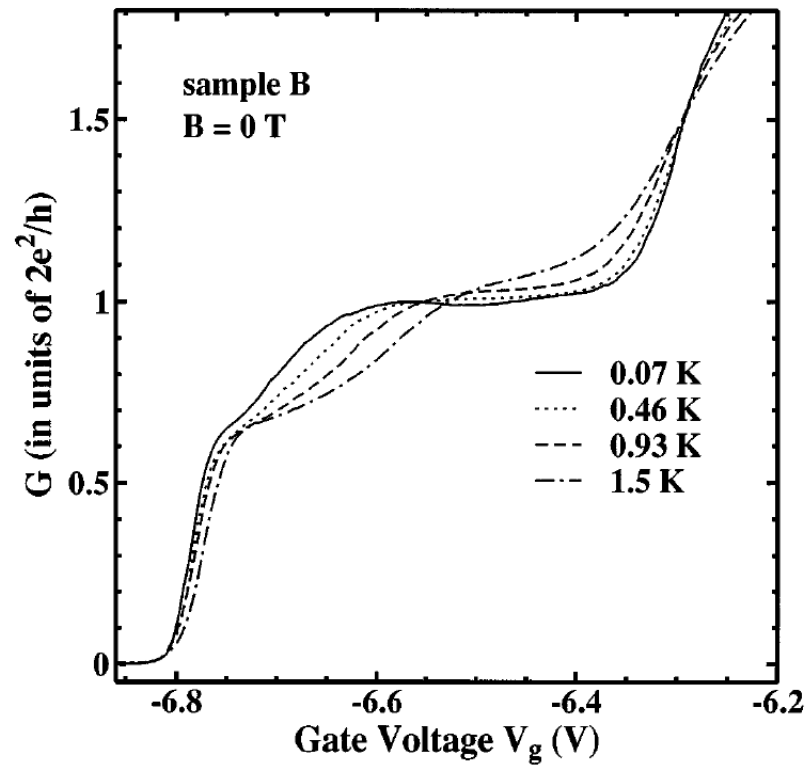
Conductance quantization $G = n \times \frac{e^2}{h}$

[van Wees *et al.*, PRL **60**, 848 (1988); Wharam *et al.*, J. Phys. C **21**, L209 (1988)]



The “0.7 Effect”

[K. J. Thomas *et al.*, PRL **77**, 135 (1996)]



An interaction effect – what is the mechanism?

Spontaneous spin-polarization; Kondo effect; spin-incoherent Luttinger liquid...

Overview

- “NMR” in a Quantum Point Contact
- Signatures of the 0.7 Effect
- Summary

“NMR” in a Quantum Point Contact

Hyperfine contact interaction

$$H_{\text{hyperfine}} = A_s \sum_i \vec{I}_i \cdot \vec{S}(\mathbf{R}_i)$$

Nuclear Spin relaxation rate

$$T_1^{-1}(\mathbf{R}) = \frac{A_s^2}{2\hbar^2} \int_{-\infty}^{\infty} dt \langle S^+(\mathbf{R}, t) S^-(\mathbf{R}, 0) \rangle$$

- (i) Local non-equilibrium nuclear polarization.
- (ii) Resistive detection.

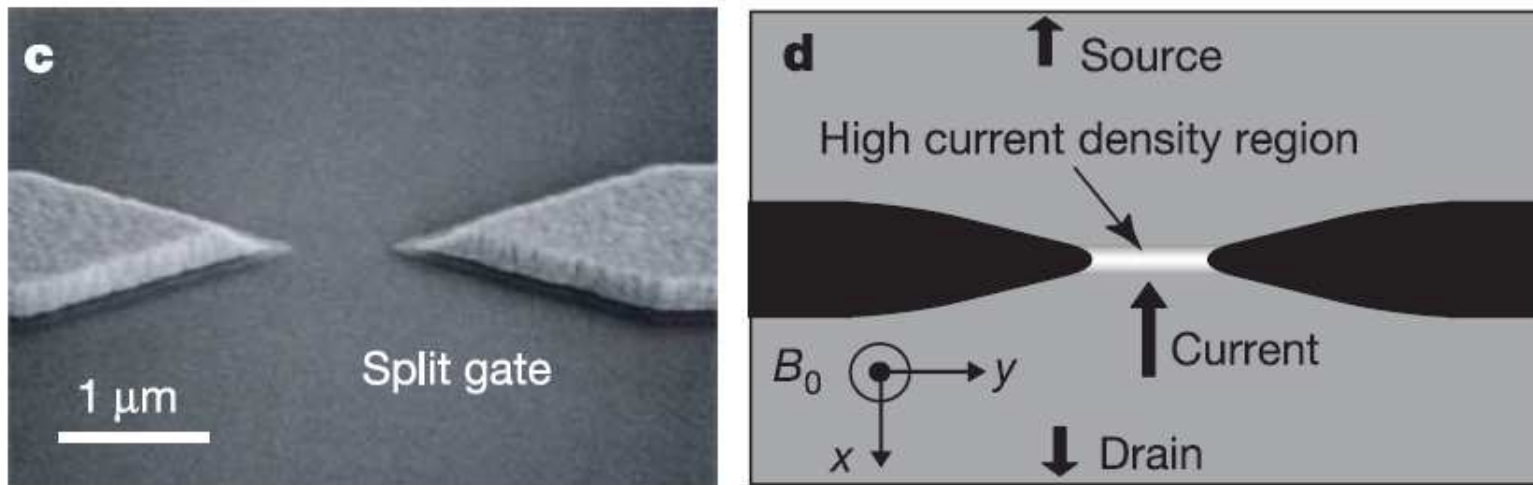
Electrical Pumping of Nuclear Polarization in QPCs

Breakdown of the $\nu = 2/3$ fractional quantum Hall state

[S. Kronmüller *et al.*, PRL **81**, 2526 (1998)]

QPC device at $\nu = 2/3$

[G. Yusa *et al.*, PRB **69**, 161302 (2004); Nature **434**, 1001 (2005).]



Current density is maximum close to the point contact \Rightarrow local breakdown.

Resistive Detection of Nuclear Polarization

[NRC & Tripathi, PRB **77**, 245324 (2008)]

Non-interacting electron gas

$$H = \sum_{s,k,\sigma} \left[\epsilon_s + \frac{\hbar^2 k^2}{2m} + \frac{\sigma}{2} g \mu_B B \right] \hat{c}_{nk\sigma}^\dagger \hat{c}_{nk\sigma} + A_s \sum_i \vec{I}_i \cdot \vec{S}(\mathbf{R}_i)$$

$$\epsilon_s \equiv \hbar \omega_y (s + 1/2)$$

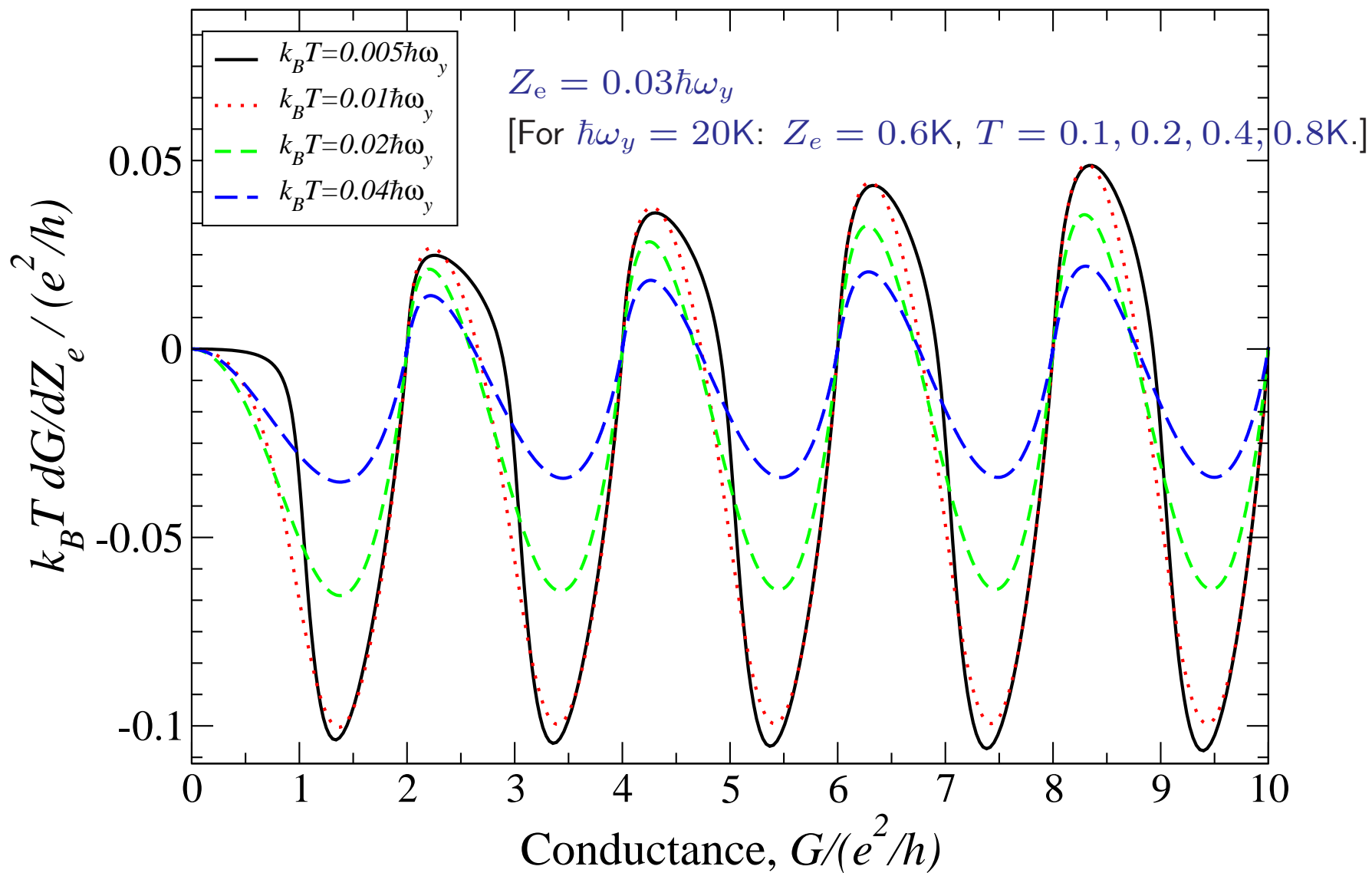
hyperfine coupling

Overhauser shift: $Z_e \equiv g \mu_B B + A_s n_{\text{nuc}} \langle I^z \rangle$ nuclear density, n_{nuc}

Conductance:

$$G(Z_e) = \frac{e^2}{h} \sum_{s,\sigma} f(\epsilon_s + \sigma Z_e/2) \quad \left[f(z) \equiv (e^{(z-\mu)/k_B T} + 1)^{-1} \right]$$

Gate voltage, $V \Rightarrow$ 1D electron density $n = c(V - V_0)/e \Rightarrow \left(\frac{\partial G}{\partial Z_e} \right)_n$



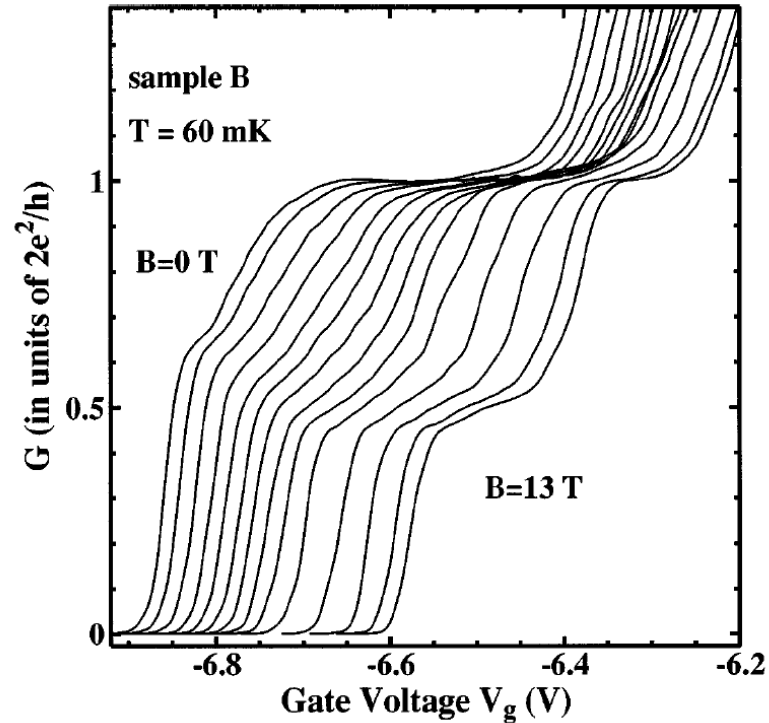
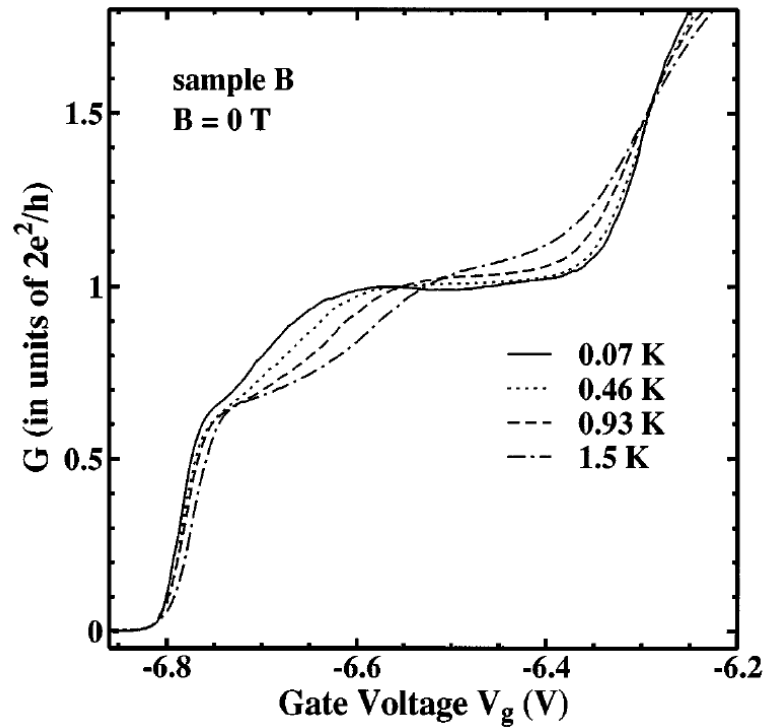
For GaAs at 50mK, $\Delta G \sim 0.01 e^2/h \Rightarrow \Delta I^z / (2I) \simeq 0.3\%$

Overview

- “NMR” in a Quantum Point Contact
- Signatures of the 0.7 Effect
- Summary

Nuclear Spin Relaxation Rate of the 0.7 Effect

[NRC & Tripathi, PRB **77**, 245324 (2008)]



[K. J. Thomas *et al.*, PRL **77**, 135 (1996)]

Nuclear spin relaxation rate

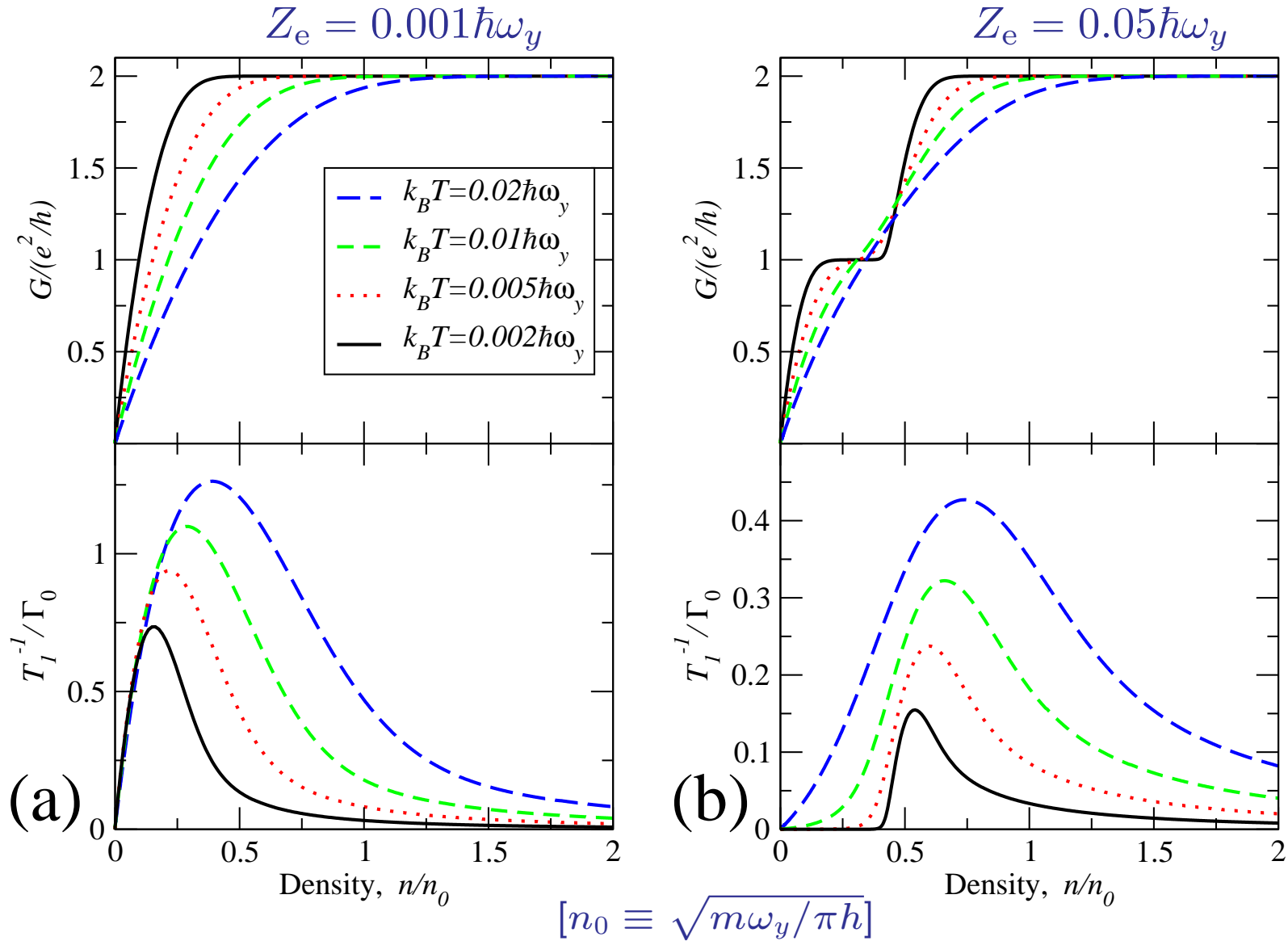
$$T_1^{-1}(\mathbf{R}) = \frac{A_s^2}{2\hbar^2} \int_{-\infty}^{\infty} dt \langle S^+(\mathbf{R}, t) S^-(\mathbf{R}, 0) \rangle$$

Fastest decay rate \Rightarrow nuclei close to the centre of the QPC.

Non-interacting electron gas

$$\begin{aligned}
 T_1^{-1} &= \frac{2\pi A_s^2}{\hbar} \sum_{i,f} p_i |\langle f|S^-|i\rangle|^2 \delta(E_i - E_f) \\
 &= \frac{2\pi A_s^2}{\hbar} \sum_{k,k'} f(\epsilon_k + Z_e/2) [1 - f(\epsilon_{k'} - Z_e/2)] \frac{1}{L_x^2 w_y^2 w_z^2} \delta(\epsilon_k - \epsilon_{k'} + Z_e) \\
 &= \Gamma_0 \int_{\hbar\omega_y/2 + |Z_e|/2}^{\infty} \frac{f(\epsilon) [1 - f(\epsilon)]}{\sqrt{(\epsilon - \hbar\omega_y/2)^2 - (Z_e/2)^2}} d\epsilon \quad w_y w_z \text{ rms area of s}
 \end{aligned}$$

$$\Gamma_0 \equiv \frac{2\pi A_s^2 m}{\hbar^3 w_y^2 w_z^2} \quad \text{typically } \Gamma_0 \simeq 0.5\text{Hz}$$



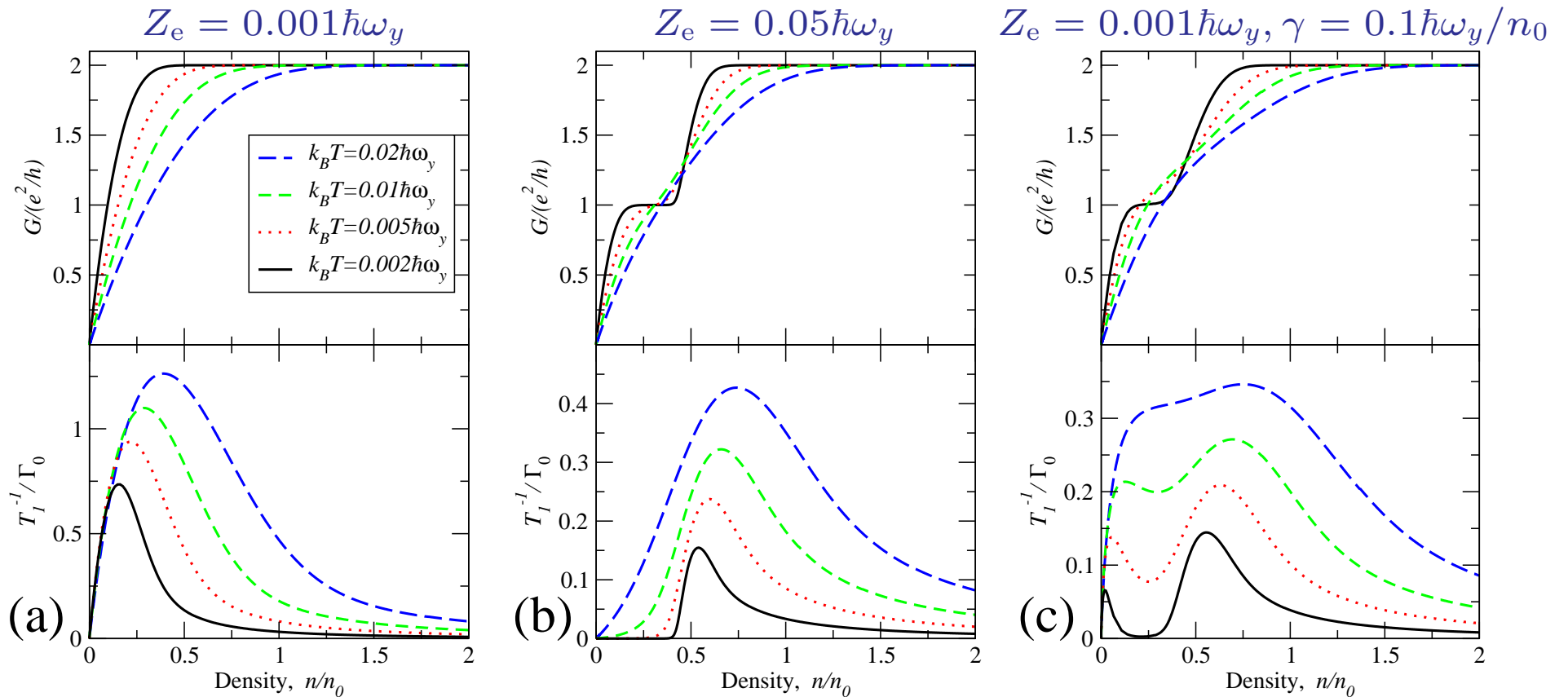
Korringa expression, $T_1^{-1} \propto \rho_{\uparrow}\rho_{\downarrow}T$, fails to account for main feature.

Exchange-enhanced spin-splitting

[Wang & Berggren (1996); Bruus, Cheianov & Flensberg (2001); Spivak & Zhou (2000); Reilly *et al.* (2002).]

Phenomenological model: $Z_{\text{eff}} = Z_e + \gamma n$

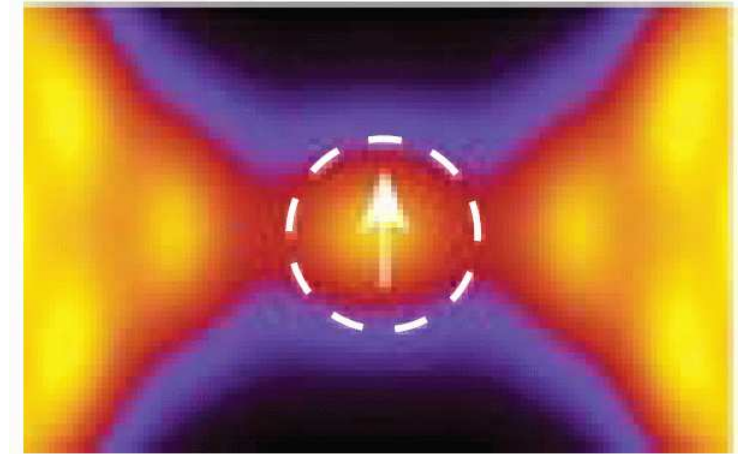
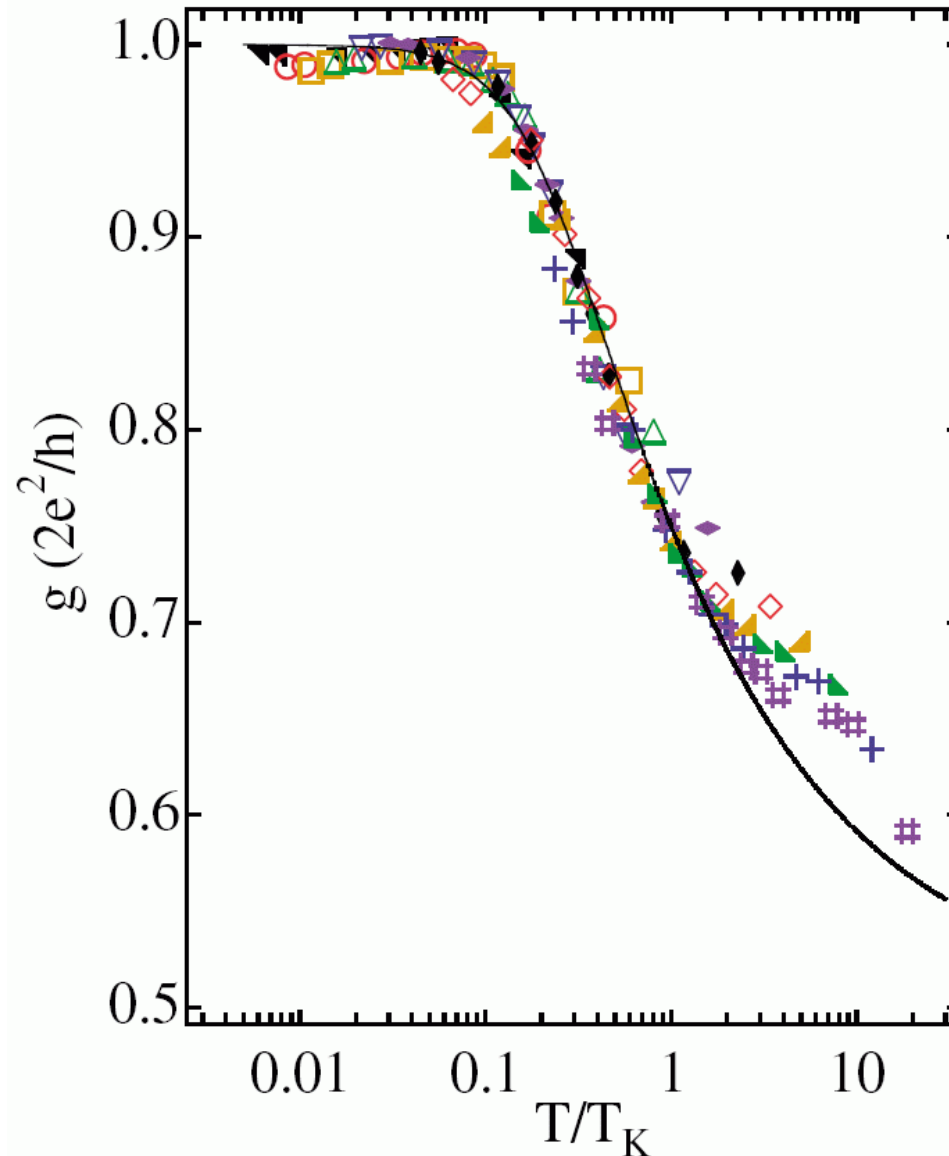
[D. J. Reilly, PRB **72**, 033309 (2005)]



The double peak is a signature of a density dependent exchange-enhanced spin-splitting.

Kondo model

[S. M. Cronenwett *et al.*, PRL **88**, 226805 (2002)]



[Meir, Hirose & Wingreen, PRL **89**, 196802 (2002);
Rejec & Meir, Nature **442**, 900 (2006)]

Exchange, $J_K \Rightarrow k_B T_K \sim \epsilon_F e^{-1/J_K \rho(\epsilon_F)}$

The nuclear spin relaxation in the QPC is dominated by the fluctuations of the impurity spin, $A_d \sim A_s/(w_x w_y w_z)$.

$$T_1^{-1} = k_B T \left(\frac{A_d}{\hbar g_s \mu_B} \right)^2 \operatorname{Im} \frac{\chi_{\text{imp}}^{+-}(\omega)}{\omega} \Big|_{\omega \rightarrow 0}.$$

$T \gg T_K$ Weak coupling limit

[W. Götze and P. Wölfle, J. Low Temp. Phys. **5**, 575 (1971)]

$$T_1^{-1} = 2 \frac{A_d^2 S(S+1)}{3\pi \hbar (k_B T) [J_K \rho(\epsilon_F)]^2}, \quad [S = 1/2]$$

$T \ll T_K$ Local Fermi liquid

[H. Shiba, Prog. Theor. Phys. **54**, 967 (1975)]

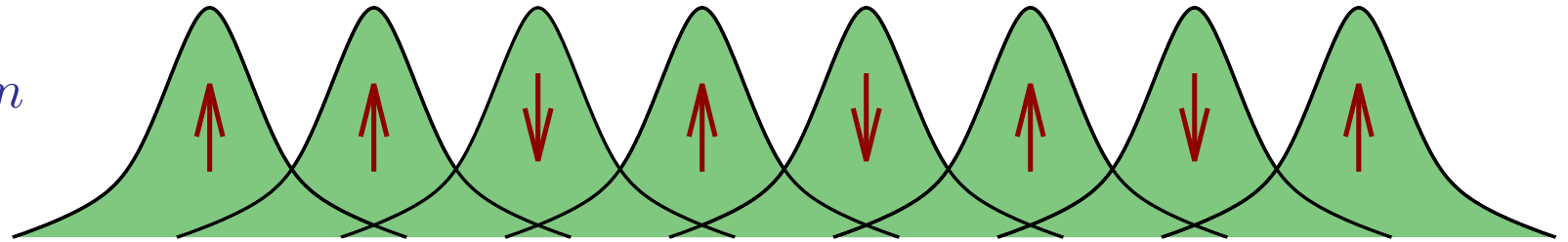
$$T_1^{-1} = \frac{2\pi (k_B T) A_d^2}{\hbar (g_s \mu_B)^4} \chi_{\text{imp}}^2, \quad \chi_{\text{imp}} = \text{static Kondo impurity susceptibility.}$$

The non-monotonic temperature dependence of T_1^{-1} is characteristic of Kondo physics.

Luttinger Liquid

Strongly interacting 1D electron gas

$$\frac{\hbar^2 n^2}{m} \ll \frac{e^2}{\epsilon} n$$



Exchange interaction $J_{LL} \ll \epsilon_F$

[K. A. Matveev, PRL **92**, 106801 (2004)]

(1) $k_B T \ll J_{LL} \ll \epsilon_F$

Luttinger Liquid (coupled to FL leads)

$$G = 2e^2/h \quad [\text{Maslov \& Stone, PRB } \mathbf{52}, \text{ R5539 (1995)}]$$

Relaxation rate: [Chitra & Giamarchi, PRB **55**, 5816 (1997); Zavidonov & Brinkmann, PRB **61**, 3282 (2000)]

$$T_1^{-1} \sim T^{K_\rho} \quad \text{with} \quad K_\rho < 1$$

The nuclear spin relaxation rate is sensitive to the formation of a Luttinger liquid.

(2) $J_{LL} \ll k_B T \ll \epsilon_F$

“spin-incoherent” Luttinger liquid

$$G \simeq e^2/h$$

[K. A. Matveev, PRL **92**, 106801 (2004)]

Low energy spin-flip excitations \Rightarrow

Spin-chain, lattice constant $1/n$, bandwidth J_{LL} , $k_B T \gg J_{LL}$

$$T_1^{-1} \sim \Gamma_{\text{SILL}} \equiv \frac{A_s^2 n^2}{\hbar \omega_y^2 \omega_z^2 J_{LL}}$$

cf. non-interacting electron gas

$$\frac{\Gamma_{\text{SILL}}}{\Gamma_0} \sim \frac{\epsilon_F}{J_{LL}} \gg 1$$

In the spin-incoherent LL regime T_1^{-1} is expected to be large and weakly temperature-dependent.

Summary

- The possibility of generating local non-equilibrium nuclear polarization in semiconductor QPCs opens the way to NMR of nanoscale semiconductor devices.
- The two-terminal conductance of the QPC provides a sensitive way to detect the nuclear polarization.
- The nuclear spin relaxation rate, T_1^{-1} , will show clear and distinctive signatures of electron-electron interaction effects that may be at play in the 0.7 effect.
[Exchange-enhancement to the Zeeman energy; Kondo Effect; Luttinger liquid; Spin-incoherent Luttinger Liquid.]