Signatures of the FFLO State in the Collective Modes of a Trapped Fermi gas

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[Jonathan Edge & NRC, PRL **103** 065301 (2009); arXiv:0912.5275]

EPSRC Engineering and Physical Sciences Research Council

Outline

- The FFLO State
- Collective modes of a Homogeneous System
- Collective modes of a Trapped System
  Signatures of the FFLO State
- Modulation of Coupling Constant
- Summary and Outlook

Two-component Atomic Fermi Gas in 3D

$$H = \sum_{\sigma=\uparrow,\downarrow} \sum_{i=1}^{N_\sigma} \left[\frac{\mathbf{p}_{i\sigma}^2}{2m} + V_T(\mathbf{r}_{i\sigma}) \right] + \sum_{i=1}^{N_\uparrow} \sum_{j=1}^{N_\downarrow} V(\mathbf{r}_{i\uparrow} - \mathbf{r}_{j\downarrow})$$

s-wave scattering length close to a Feshbach resonance

balanced densities $n_\uparrow = n_\downarrow \Leftrightarrow k_{F\uparrow} = k_{F\downarrow}$

$\Rightarrow$ superfluid of s-wave fermionic pairs

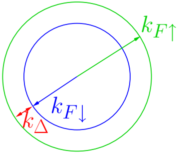
BCS order parameter: $\Delta \sim \langle \hat{c}_{\mathbf{k}\uparrow} \hat{c}_{-\mathbf{k}\downarrow} \rangle$

Pair wavefunction: $\langle \hat{c}_{\mathbf{r}_\uparrow\uparrow} \hat{c}_{\mathbf{r}_\downarrow\downarrow} \rangle \sim g(\mathbf{r}_\uparrow - \mathbf{r}_\downarrow)$

The FFLO State

Unconventional superfluid state when $k_{F\uparrow} \neq k_{F\downarrow}$.

[Fulde & Ferrell (1964); Larkin & Ovchinnikov (1964)]



$$\langle \hat{c}_{\mathbf{k}_{F\uparrow}\uparrow} \hat{c}_{-\mathbf{k}_{F\downarrow}\downarrow} \rangle \neq 0$$

$$\langle \hat{c}_{\mathbf{r}_\uparrow\uparrow} \hat{c}_{\mathbf{r}_\downarrow\downarrow} \rangle \sim g(\mathbf{r}_\uparrow - \mathbf{r}_\downarrow) \exp\left[i\,(\mathbf{k}_{F\uparrow} - \mathbf{k}_{F\downarrow}) \cdot \left(\frac{\mathbf{r}_\uparrow + \mathbf{r}_\downarrow}{2} \right) \right]$$

Inhomogeneous gap parameter (FF)

$$\Delta(\mathbf{r}) = \Delta \exp(i\mathbf{Q}\cdot\mathbf{r}) \qquad |\mathbf{Q}| = k_\Delta \equiv k_{F\uparrow} - k_{F\downarrow}$$

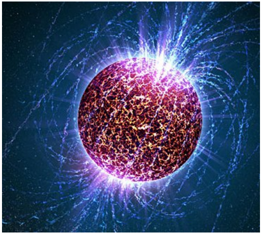
Standing waves (LO)

$$\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} \exp(i\mathbf{Q}\cdot\mathbf{r})$$

fixed by anharmonic terms in the Ginzburg-Landau expansion
$(F \sim a|\Delta|^2 + b|\Delta|^4 \dots)$

Why study the FFLO State?

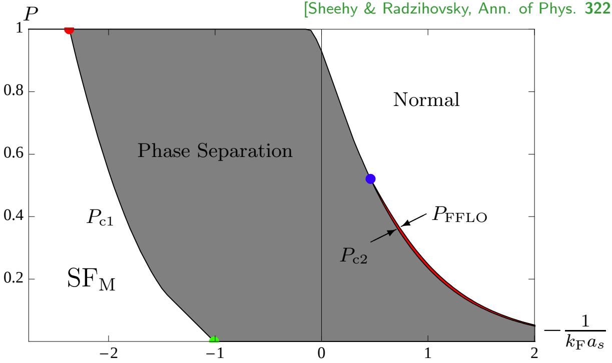
- A central concept in the theory of superconductivity:
  - solid-state superconductors
  - particle physics: colour QCD
  - astrophysics: neutron star glitches
  - nuclear physics: neutron-proton superfluids



- So far no unambiguous evidence in experiment.

Mean-Field Phase Diagram (3D)

[Sheehy & Radzihovsky, Ann. of Phys. **322**, 1790 (2007)]



FFLO studied within a Ginzburg-Landau expansion.

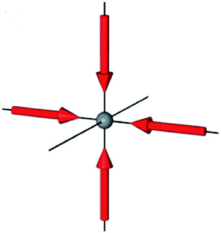
Broader range of FFLO? [A. Bulgac & M.M. Forbes, PRL **101**, 215302 (2008)]

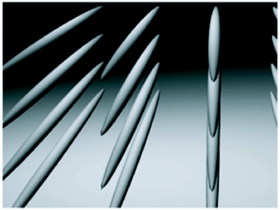
No experimental evidence of FFLO in 3D atomic Fermi gas.

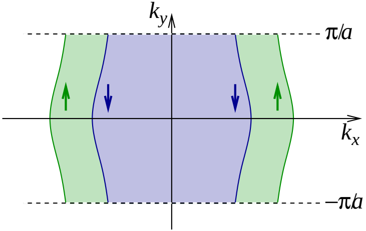
Quasi-1D Systems

+ 2D optical lattice
⇒ Weakly coupled 1D tubes

[Bloch, Dalibard & Zwerger, RMP (2008)]





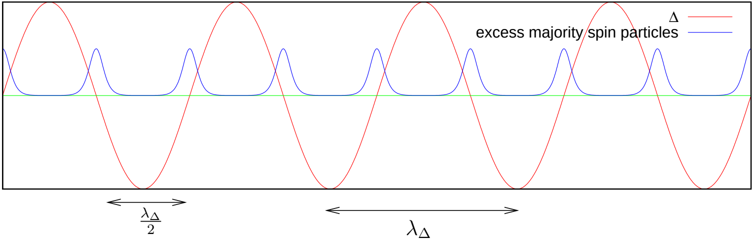


Fermi surface nesting
⇒ increased stability of FFLO

[Parish *et al.*, PRL **99**, 250403 (2007)]

Single 1D System

FFLO state has a simple interpretation



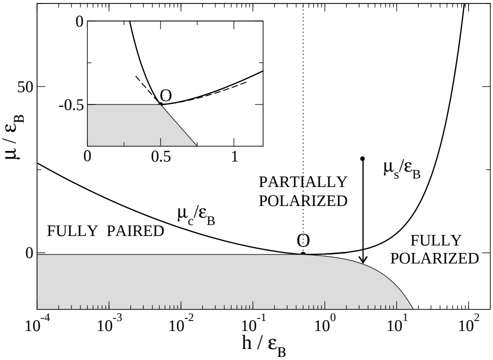
One majority spin particle at each node of $\Delta(x)$.

$$\frac{\lambda_\Delta}{2} = \frac{1}{|n_\uparrow - n_\downarrow|} \quad \Leftrightarrow \quad k_\Delta \equiv \frac{2\pi}{\lambda_\Delta} = |k_{F\uparrow} - k_{F\downarrow}|$$

1D System (Exact)

$$H = \sum_{\sigma=\uparrow,\downarrow} \sum_{i=1}^{N_\sigma} \frac{p_{i\sigma}^2}{2m} + g_{1d} \sum_{i=1}^{N_\uparrow} \sum_{j=1}^{N_\downarrow} \delta(x_{i\uparrow} - x_{j\downarrow})$$

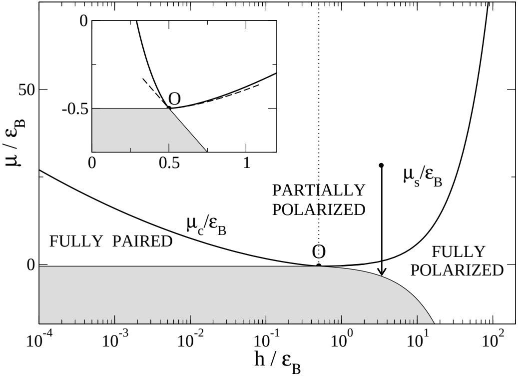
Gaudin model solvable by Bethe-Ansatz [Orso, PRL **98**, 070402 (2007)]



$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$$
$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$
$$\epsilon_B = \frac{g_{1D}^2 m^2}{4\hbar^2}$$

Partially-polarized phase associated with FFLO state [K. Yang, PRB (2001)]

1D System in a Trap



Bethe-Ansatz Phase Diagram + local density approximation:

centre partially polarized; [Orso, PRL **98**, 070402 (2007)]

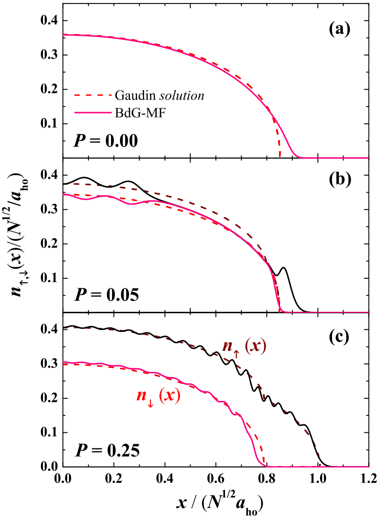
edge either fully paired or fully polarized.

Mean field theory vs. exact solution

Bogoliubov de Gennes gives good agreement with exact solution, for $\gamma \equiv -\frac{mg_{1D}}{\hbar^2 n} \sim 1$

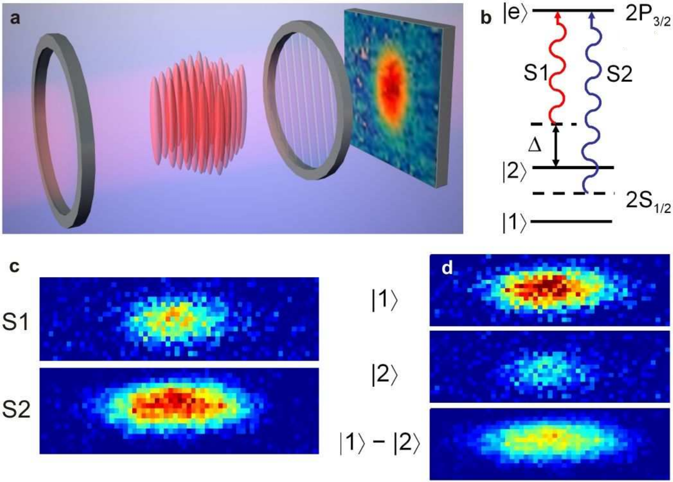
[Liu, Hu & Drummond, PRA **76**, 043605 (2007)]

$\gamma_{\text{centre}} = 1.6$

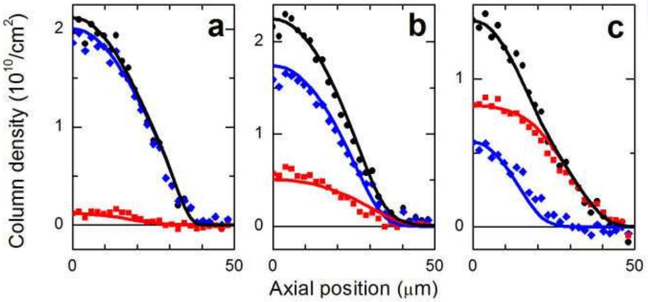


Experimental Results ^{6}Li

[Yean-an Liao *et al.* [Rice], arXiv:0912.0092]



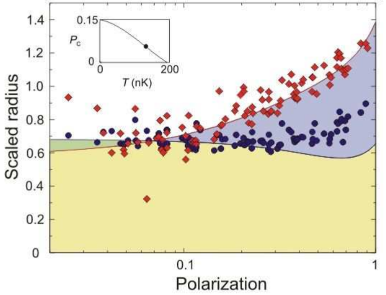
Author: Liao *et al.*
Figure S1



Author: Liao *et al.*
Figure 2

[Yean-an Liao *et al.* [Rice], arXiv:0912.0092]

cf. Bethe-Ansatz + LDA



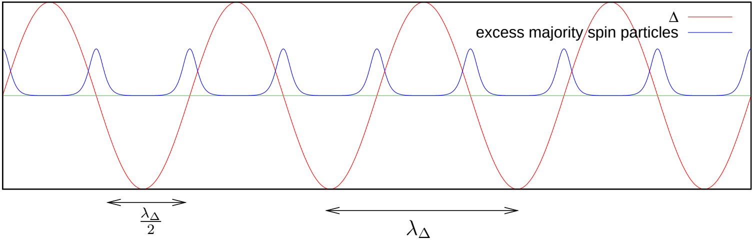
Author: Liao *et al.*
Figure 3

Detection of FFLO characteristics

- Expansion imaging
- RF excitation spectrum [Bakhtiari, Leskinen, & Törmä, PRL **101**, 120404 (2008)]

⇒ Collective Modes [J.M. Edge & NRC, PRL **103** 065301 (2009); arXiv:0912.5275]

The Key Idea



Look for *low-frequency* response to perturbations with *short* wavelengths ($\lambda \sim \lambda_\Delta/2$).

- Perturb by an optical lattice potential (short period)
- Observe response at long-wavelengths (size of the cloud)

Overview of Method

- Obtain equilibrium configuration within mean-field theory (self-consistent Bogoliubov de Gennes).

- Collective modes via RPA.

[Mean-field theory in 1D!?]

Numerical Approach

- Discretization (1D Hubbard Hamiltonian)

$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^\dagger_{i+1,\sigma}\hat{c}_{i,\sigma} + h.c.\right) + \sum_{i,\sigma}(V_i^{\text{ext}} - \mu_\sigma)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma}$$
$$+ U\sum_i \hat{c}^\dagger_{i,\uparrow}\hat{c}^\dagger_{i,\downarrow}\hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}$$

Interaction parameter $\gamma \equiv -\frac{mg_{1D}}{\hbar^2 n} = -\frac{U}{2J(na_{\text{lattice}})}$

- Mean-field theory (BdG equations):

$$\hat{H}_{BdG} = -J\sum_{i}\left(\hat{c}^\dagger_{i+1,\sigma}\hat{c}_{i,\sigma} + h.c.\right) + \sum_{i,\sigma}W_{i,\sigma}\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma}$$
$$+ \sum_i\left(\Delta_i\hat{c}^\dagger_{i,\uparrow}\hat{c}^\dagger_{i,\downarrow} + \Delta_i^*\hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}\right)$$

$$W_{i,\sigma} \equiv V_i^{\text{ext}} + U\langle\hat{c}^\dagger_{i,\bar{\sigma}}\hat{c}_{i,\bar{\sigma}}\rangle - \mu_\sigma \qquad \Delta_i \equiv U\left\langle\hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}\right\rangle$$

Linear Response: Random Phase Approximation

$$\hat{H}' = \hat{H} + \sum_{j,\beta} \hat{A}_{j,\beta}\, \delta a_{j,\beta}\ \exp(i\omega t)$$

with $\hat{A}_{i,\alpha} = \left(\hat{c}^\dagger_{i,\uparrow}\hat{c}_{i,\uparrow}, \hat{c}^\dagger_{i,\uparrow}\hat{c}^\dagger_{i,\downarrow}, \hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}, \hat{c}^\dagger_{i,\downarrow}\hat{c}_{i,\downarrow}\right)$

- Linear response (Kubo formula) for $\hat{H} = \hat{H}_{BdG}$:

$$\delta\langle\hat{A}_{i,\alpha}\rangle = \chi^0_{i,\alpha;j,\beta}(\omega)\ \delta a_{j,\beta}$$

- Self-consistent perturbation (RPA):

$$\delta a^{\mathrm{tot}}_{i,\alpha} = \delta a^{\mathrm{ext}}_{i,\alpha} + M_{\alpha,\beta}\ \ \delta\langle\hat{A}_{i,\beta}\rangle$$

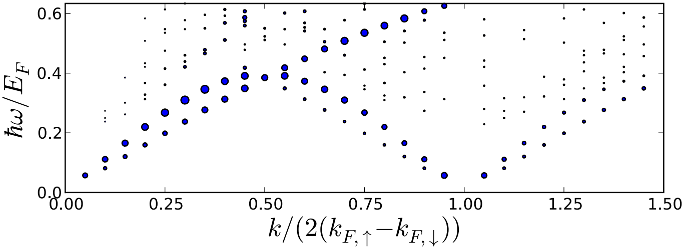
$$\Rightarrow \delta\langle\hat{A}\rangle = \frac{1}{1 - M\chi^0(\omega)}\ \chi^0(\omega)\ \delta a^{\mathrm{ext}}$$

Homogeneous System, $T = 0$

[J.M. Edge & NRC, PRL **103** 065301 (2009)]

$$\delta\hat{H}(t) = \sum_{i\sigma} V_\sigma \cos(kx_i)\exp(i\omega t)\hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma}$$

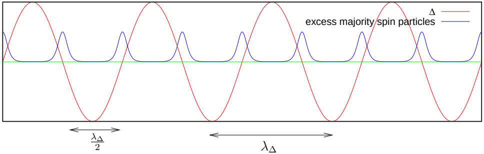
$V_\uparrow = V_\downarrow$: "spin-symmetric" perturbation
$V_\uparrow = -V_\downarrow$: "spin-asymmetric" perturbation



$P = \frac{n_\uparrow - n_\downarrow}{n} = 0.15 \qquad \gamma = 1.5$

Interpretation

- Broken gauge symmetry & translational invariance $\Rightarrow$ two Goldstone modes
- Symmetry under $x \to x + \lambda_\Delta/2 \quad \Delta(x) \to -\Delta(x)$



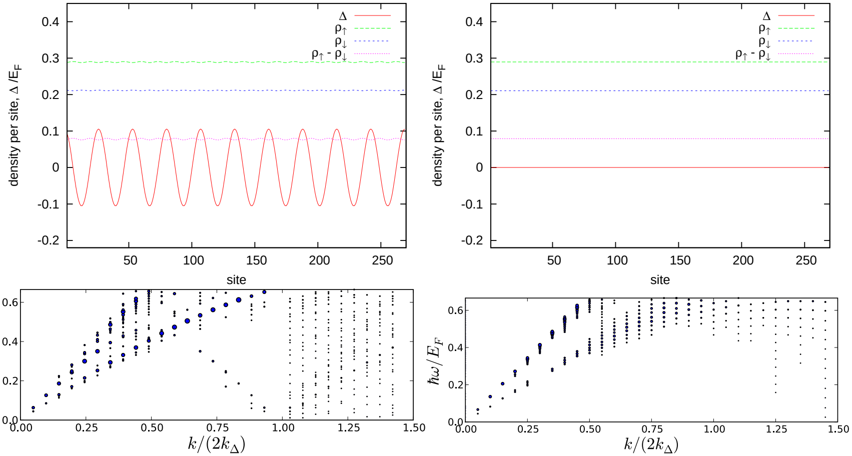
$\Rightarrow$ Wavevector is conserved modulo $2\pi/(\lambda_\Delta/2) = 2k_\Delta$.

Exact theory: Partially polarized phase has a second Luttinger liquid of excess particles, with density $n^{\text{excess}} = n_\uparrow - n_\downarrow$ [Yang, PRB (2001)].

- Two sound modes
- Gapless response at $k \simeq 2k_F^{\text{excess}} = 2(k_{F\uparrow} - k_{F\downarrow}) = 2k_\Delta$.

Homogeneous system, $T \neq 0$

[J.M. Edge & NRC, arXiv:0912.5275]



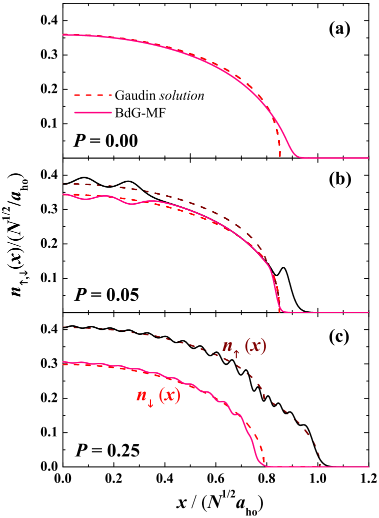
$$P = \frac{n_\uparrow - n_\downarrow}{n} = 0.15 \qquad \gamma = 1.5 \qquad T/T_F = 0.056, 0.076$$

Trapped System

Bogoliubov de Gennes gives good agreement with exact solution, for $\gamma \equiv -\frac{mg_{1D}}{\hbar^2 n} \sim 1$

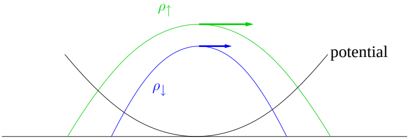
[Liu, Hu & Drummond, PRA **76**, 043605 (2007)]

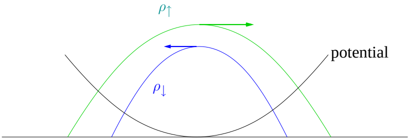
$\gamma_{\text{centre}} = 1.6$



Low Frequency Collective Modes

$$V_T(x) = \frac{1}{2} m\omega_T^2 x^2$$





density dipole (Kohn) mode

$\omega = \omega_T$

spin dipole mode

$\omega > \omega_T$

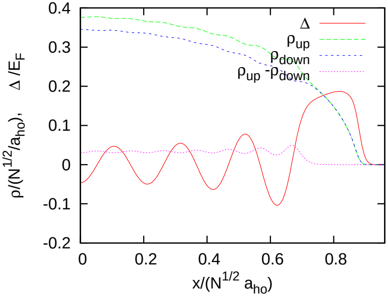
Well described by single mode approximation [J.M. Edge & NRC, arXiv:0912.5275]

Signature of FFLO State

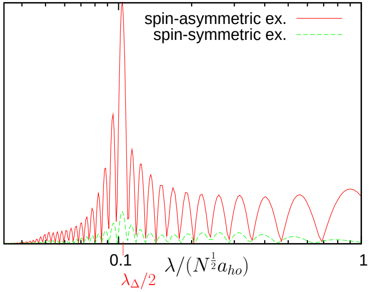
[J.M. Edge & NRC, PRL **103** 065301 (2009)]

- Perturb with periodic potential $V_\sigma(x) = V_\sigma \sin\left[\frac{2\pi}{\lambda}(x - x_0)\right]$
- Look at response of low frequency modes: spin dipole mode

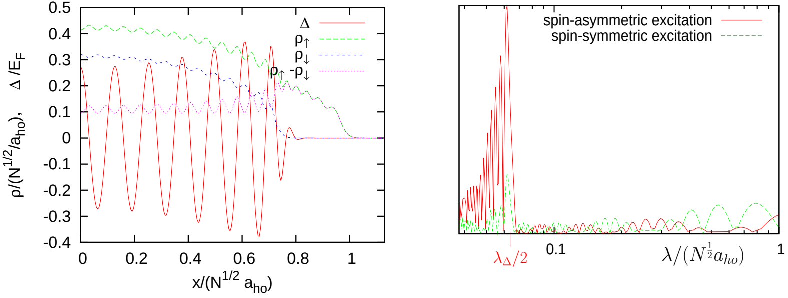
Unpolarized Edge



$[P = 0.048, \gamma_{\mathrm{centre}} = 0.93, N = 290]$



Fully polarized Edge $[P = 0.25, \gamma_{\text{centre}} = 1.5, N = 143]$



- Large response of spin-dipole mode at $\lambda = \frac{\lambda_\Delta}{2}$ indicates the presence of the FFLO state, independent of the edge configuration.

Experimental Considerations

- Requires tunable wavelength optical lattice potential.

  (Stronger signal for spin-asymmetric coupling.)

- Modulate amplitude at $\omega_{\mathrm{s.d.}}$, or switch off abruptly and watch oscillations.

- 2D optical lattice $\Rightarrow$ many 1D tubes, with inhomogeneous density and λ_{Δ}.

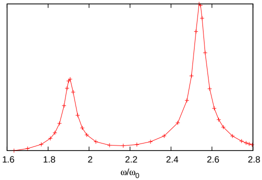
  Effects of inhomogeneity reduced by in situ spatial resolution.

Modulation of the Scattering Length

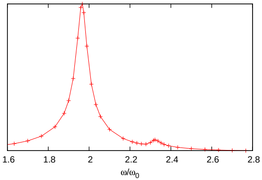
[J.M. Edge & NRC, arXiv:0912.5275. Thanks to Carlos Lobo.]

$$\delta\hat{H}(t) = \delta U(t) \sum_i \hat{c}^\dagger_{i,\uparrow}\hat{c}^\dagger_{i,\downarrow}\hat{c}_{i,\downarrow}\hat{c}_{i,\uparrow}$$

- Relative amplitudes of the response for the two lowest frequency modes ($\omega \sim 2\omega_T$) depends on whether edge is fully paired or fully polarized.



Edge fully paired

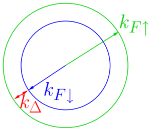


Edge fully polarized

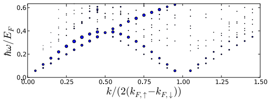
- No clear correlation with presence/absence of FFLO state.

Summary

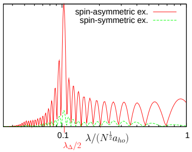
- The FFLO state is expected to form in ultracold atomic Fermi gases, with a wide range of stability in (quasi)-1D.



- The FFLO phase has distinctive low-frequency features in its collective mode spectrum.



- These could be probed in experiments on cold gases: a short-wavelength perturbation can excite long-wavelength oscillations.



Open Questions & Outlook

- Strong coupling limit.
- Damping of FFLO collective modes at non-zero T.
- Other experimental signatures of FFLO state.
- FFLO in 3D?
- FFLO in 3D to 1D crossover (multiple subbands)?
- Probing short-range correlations by long-wavelength oscillations in other situations (AFM/SDW)?