Topological superfluid phase of polar fermionic molecules

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"Topological Order" Workshop
MPIPKS, Dresden, 15 July 2009.

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Overview

- 2D $p_x + ip_y$ superfluid
– topological aspects
– atomic Fermi gas

- Dipolar interactions
– dressed polar molecules
– superfluid pairing
– stability

- Experimental consequences

2D $p_x + ip_y$ superfluid

BCS pairing of spinless fermions

$$\Delta_{\boldsymbol{k}} \sim \langle \hat{c}_{\boldsymbol{k}} \hat{c}_{-\boldsymbol{k}} \rangle = -\Delta_{-\boldsymbol{k}}$$

$\Delta_{\boldsymbol{k}} \sim k_x + ik_y$

Quasiparticle spectrum $\quad \epsilon_{\boldsymbol{k}} = \sqrt{\xi_k^2 + |\Delta_{\boldsymbol{k}}|^2} \qquad \xi_k \equiv \frac{\hbar^2 k^2}{2M} - \mu$

Fully gapped for $\mu \neq 0$

$\mu < 0$ and $\mu > 0$ separated by a *topological phase transition* [Volovik, JETP 1988]

$$\epsilon_{\boldsymbol{k}} = \sqrt{\xi_k^2 + |\Delta_{\boldsymbol{k}}|^2} \qquad \xi_k \equiv \frac{\hbar^2 k^2}{2M} - \mu$$

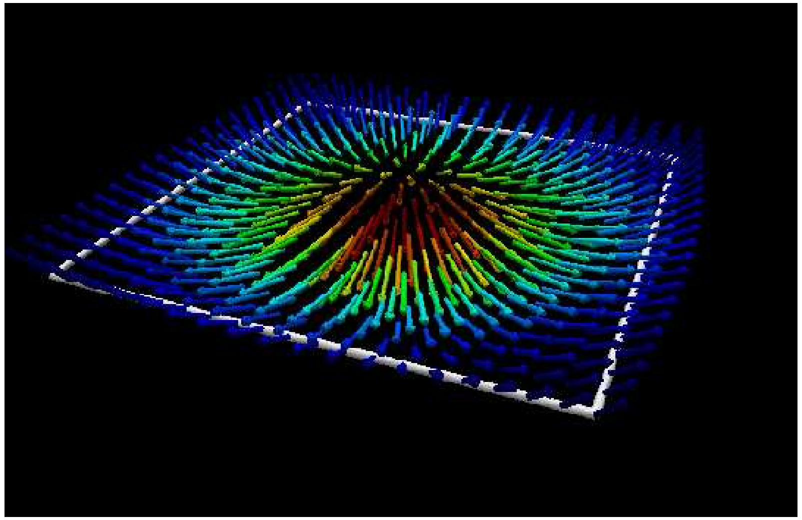
"pseudospin" $\qquad \vec{n}_{\boldsymbol{k}} \equiv \left(\mathrm{Re}\Delta_{\boldsymbol{k}}, \mathrm{Im}\Delta_{\boldsymbol{k}}, \xi_{\boldsymbol{k}}\right)/\epsilon_{\boldsymbol{k}} \qquad |\vec{n}_{\boldsymbol{k}}|^2 = 1$

Topological classification $\pi_2(S^2) = \mathbb{Z}$ [Volovik, JETP 1988]

$\Delta_{\boldsymbol{k}} \sim k_x + ik_y$

$\mu < 0$: topologically trivial (vacuum)

$\mu > 0$: "skyrmion" in $\boldsymbol{k}$-space



- Gapless mode between $\mu > 0$ phase and vacuum ($\mu < 0$).
- *Zero*-energy states on vortices of the $\mu > 0$ phase.

Non-abelian exchange statistics

$p_x + ip_y$ superfluid of composite fermions $\Leftrightarrow$ Moore-Read (Pfaffian) FQH state

[Read & Green, PRB **61**, 10267 (2000)]

quantized vortices $\Leftrightarrow$ FQH quasiparticles

One zero-energy Majorana mode per vortex $\Rightarrow$groundstate degeneracy, $2^{N_V/2-1}$.

Braiding of vortices $\Rightarrow$non-abelian exchange statistics [Ivanov, PRL **86**, 268 (2001)]

Can one realize this $p_x + ip_y$ phase in a ultracold Fermi gas?

For two-component atomic Fermi gas with "spin-orbit" interaction:

[Zhang, Tewari, Lutchyn and Das Sarma, PRL (2008); Sato, Takahashi, Fujimoto, PRL (2009).]

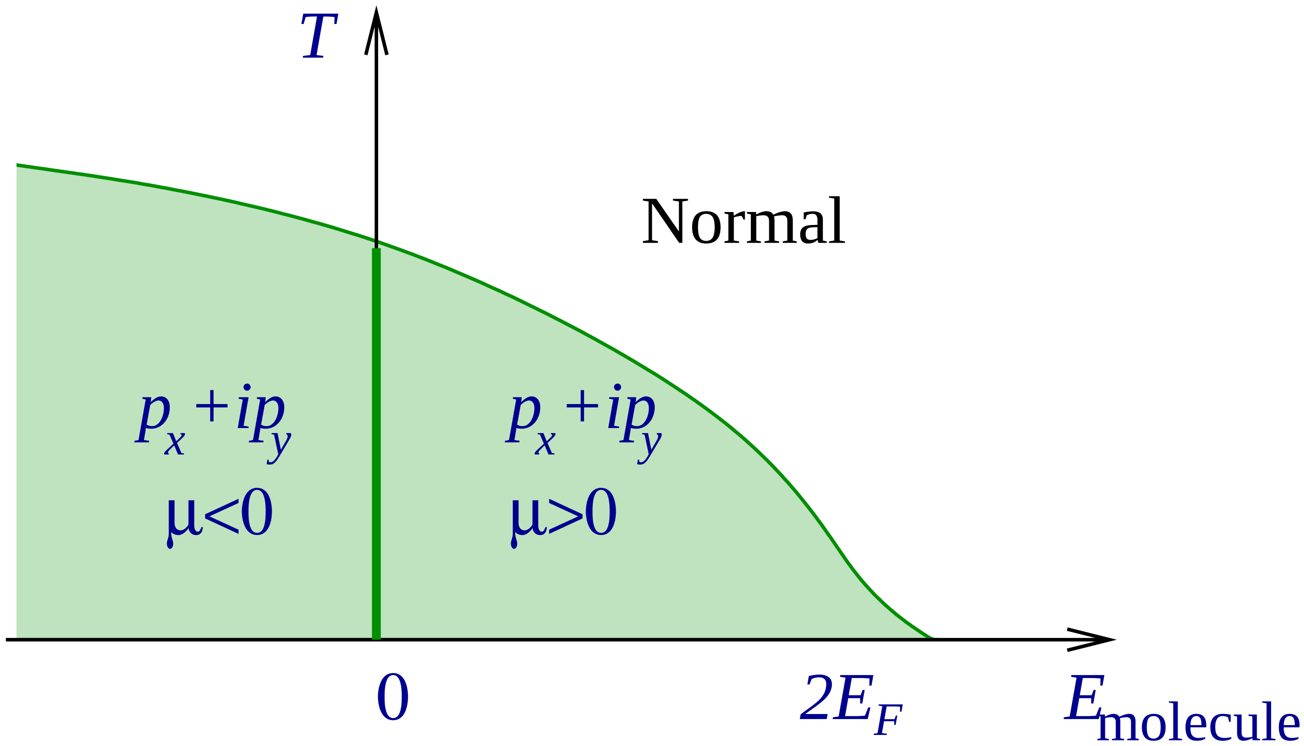
One-Component Atomic Fermi Gas

- Ultracold limit $k_F R_0 \ll 1 \Rightarrow p$-wave interactions very weak

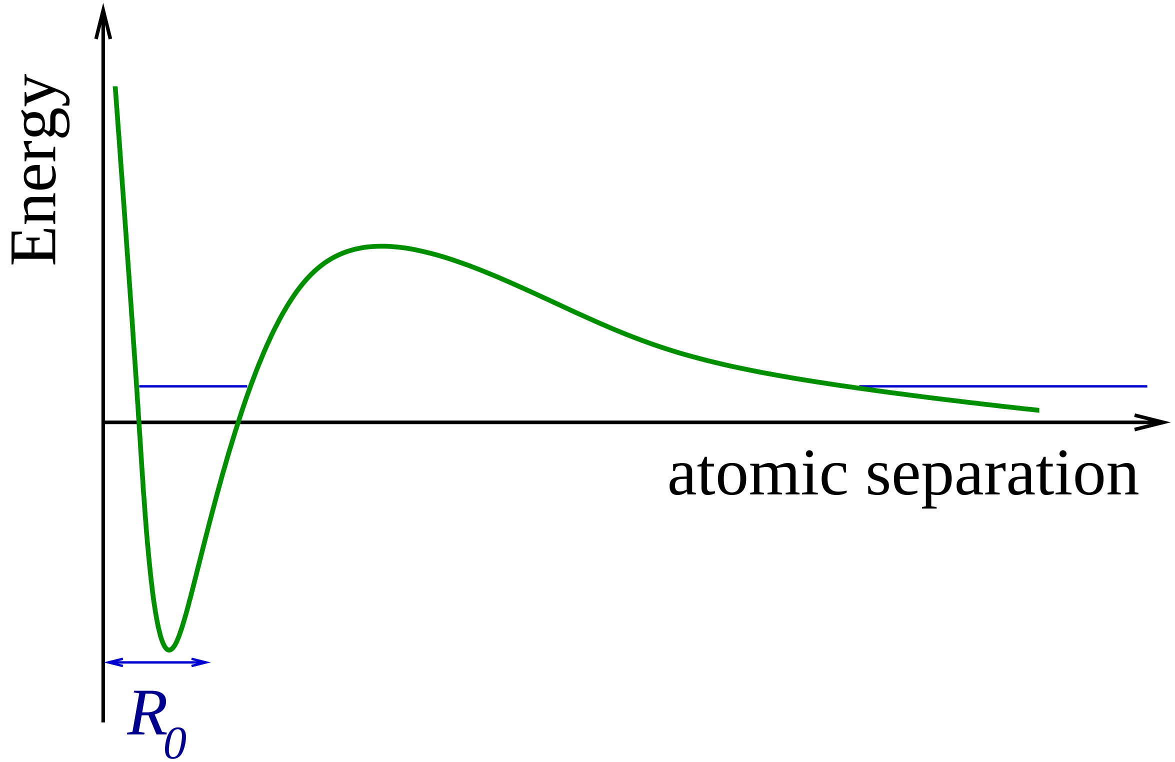
  BCS $\quad T_c \sim E_F e^{-\#/(k_F R_0)^2}$ is vanishingly small.

- Feshbach resonance: formation of p-wave boundstate

[Gurarie, Radzihovsky & Andreev, PRL **94**, 230403 (2005); Cheng & Yip, PRL **95**, 070404 (2005).]

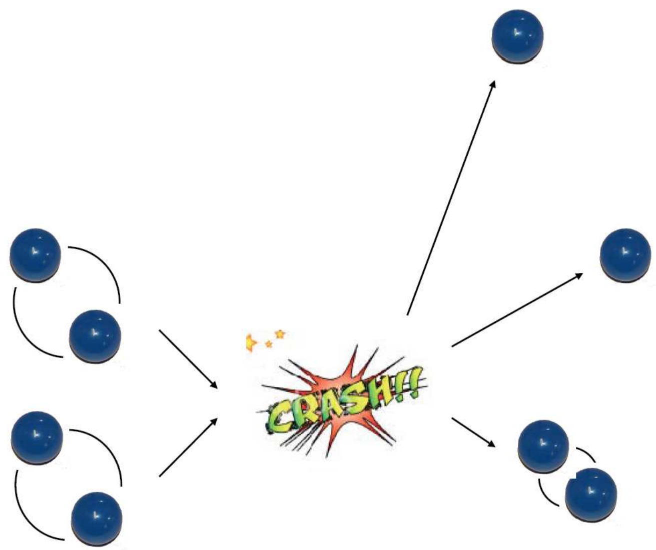


However... close to a p-wave Feshbach resonance



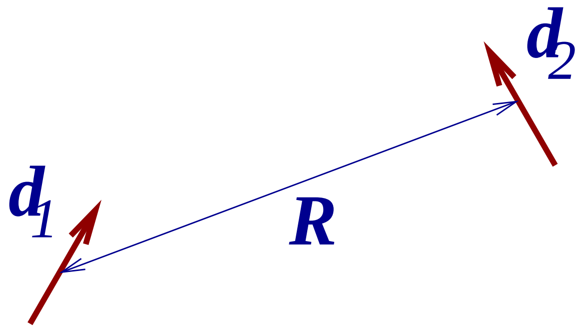
Dimer formation $\Rightarrow$collisional relaxation
[Levinsen, NRC & Gurarie, PRL **99**, 210402 (2007);

Jona-Lasinio, Pricoupenko & Castin, PRA **77**, 043611 (2008).]



Lifetime of ^{6}Li dimers $\sim$ 6ms. [Gaebler *et al.* [JILA], PRL **98**, 200403 (2007)]

Dipolar Interactions



$$V = \frac{\boldsymbol{d}_1 \cdot \boldsymbol{d}_2 R^2 - 3(\boldsymbol{d}_1 \cdot \boldsymbol{R})(\boldsymbol{d}_2 \cdot \boldsymbol{R})}{R^5}$$

attraction 

repulsion 

3D $\quad \boldsymbol{d} = (0, 0, d) \qquad \Delta_{\boldsymbol{k}} \sim k_z$ [Baranov *et al.*, PRA **66**, 013606 (2002)]

2D $\quad \boldsymbol{d} = d(\sin\theta, 0, \cos\theta) \qquad \Delta_{\boldsymbol{k}} \sim k_x$ [Bruun & Taylor, PRL **101**, 245301 (2008)]

Can one engineer the 2D $k_x + ik_y$ phase?

Dressed Polar Molecules

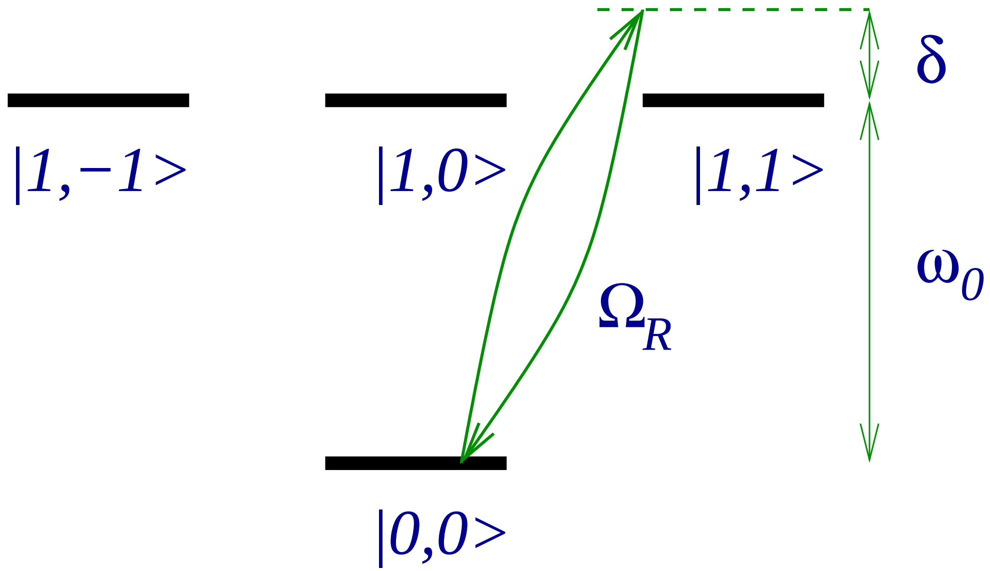
[NRC & Shlyapnikov, preprint]

^{40}K^{87}Rb [Ni *et al.*[JILA], Science (2008)]
^{7}Li^{133}Cs [Deiglmayer *et al.*[Heidelberg], PRL (2008)] $\Rightarrow$ Electric dipole moment

2D centre-of-mass motion, but 3D rotational levels:

$$|J, M_J\rangle \qquad \langle \boldsymbol{d} \rangle = 0$$

$+$ circularly polarized MW field [cf. Gorshkov *et al.*, PRL **101**, 073201 (2008)]

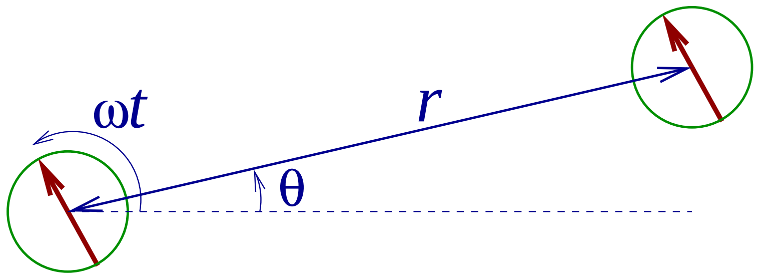


detuning $\delta \equiv \omega - \omega_0$
Rabi frequency Ω_R

Rotating wave approximation, $\delta, \Omega_R \ll \omega_0 \Rightarrow$ field-dressed states

$$|+\rangle = a|0,0\rangle + be^{-i\omega t}|1,1\rangle \qquad |-\rangle = b|0,0\rangle - ae^{-i\omega t}|1,1\rangle$$

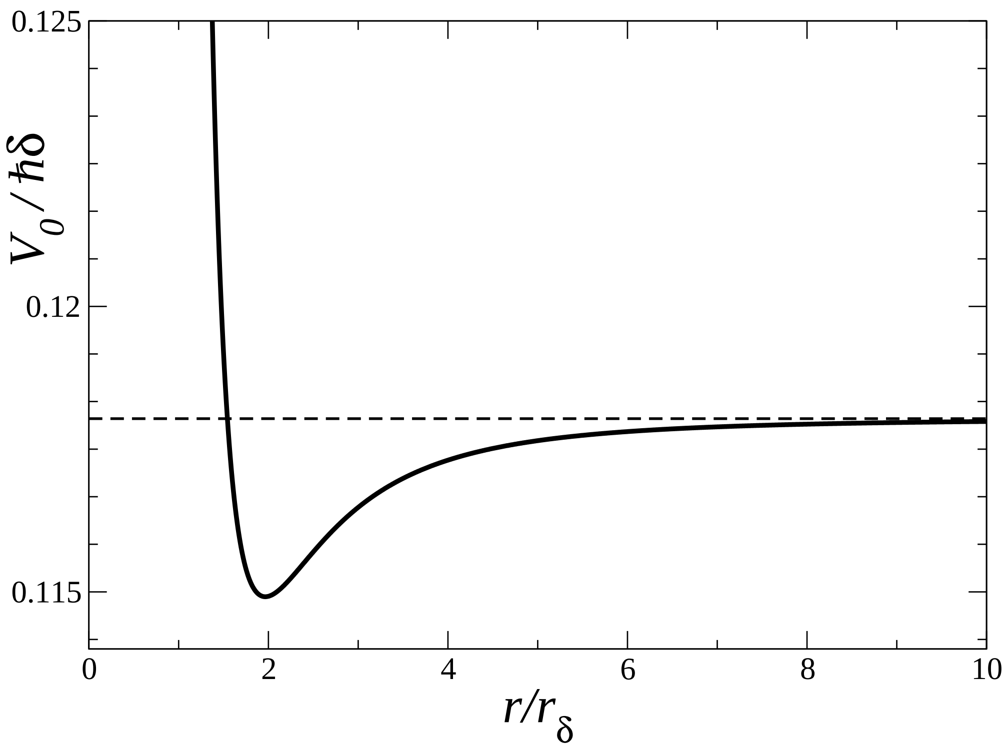
$$\langle +|\boldsymbol{d}|+\rangle = d_{\text{eff}}\,(\cos\omega t, \sin\omega t, 0)$$



$$V_0(r,\theta;t) = \frac{d_{\text{eff}}^2}{r^3}\left[1 - 3\cos^2(\omega t - \theta)\right]$$

$$\overline{V_0}(r) = -\frac{d_{\text{eff}}^2}{2r^3}$$

$$r^* \equiv \frac{M d_{\text{eff}}^2}{2\hbar^2}$$



Superfluid Transition

Vertex function ($E = 0$)

$$\Gamma(\boldsymbol{k},\boldsymbol{q}) = V_0(\boldsymbol{k}-\boldsymbol{q}) - \int \frac{d^2\boldsymbol{q}'}{(2\pi)^2}\frac{\Gamma(\boldsymbol{k},\boldsymbol{q}')V_0(\boldsymbol{q}-\boldsymbol{q}')}{E_{q'}}$$

$$E_q = \hbar^2 q^2/2M$$

BCS gap equation

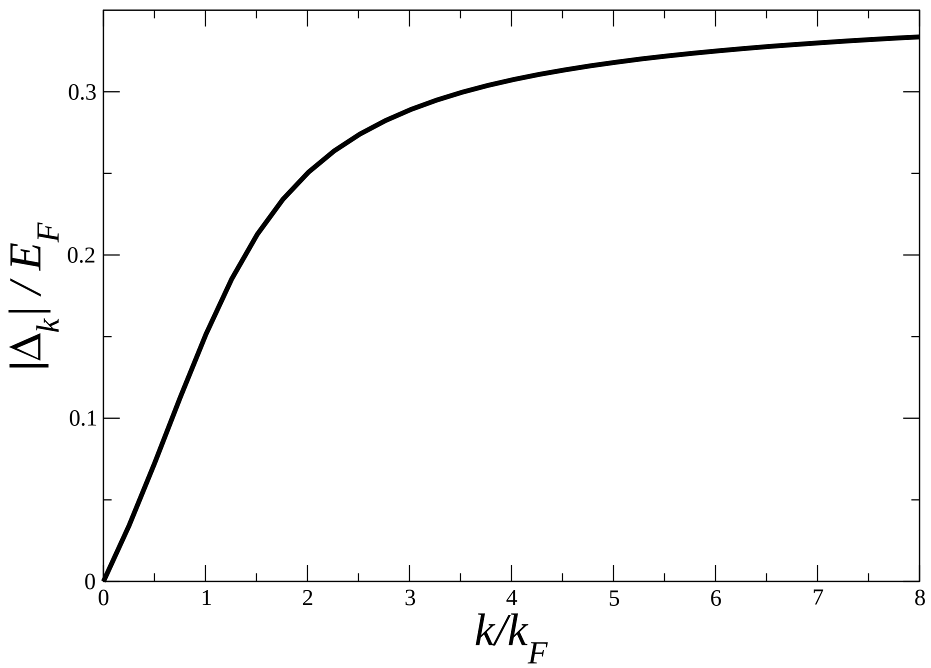
$$\Delta_{\boldsymbol{k}} = -\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\Gamma(\boldsymbol{k},\boldsymbol{q})\frac{\Delta_{\boldsymbol{q}}}{2}\left[\frac{\tanh(\epsilon_{\boldsymbol{q}}/2T)}{\epsilon_{\boldsymbol{q}}} - \frac{1}{E_q}\right]$$

$$\epsilon_{\boldsymbol{q}} = \sqrt{(E_q-\mu)^2 + |\Delta_{\boldsymbol{q}}|^2}$$

Born approx. $\Gamma(\boldsymbol{k},\boldsymbol{q}) \simeq V_0(\boldsymbol{k}-\boldsymbol{q})$

- $\underline{T \to 0}$

Most stable phase has $\Delta_{\boldsymbol{k}} \sim (k_x \pm i k_y)$



- $\underline{\text{BCS limit, } k_F r^* \lesssim 1}$

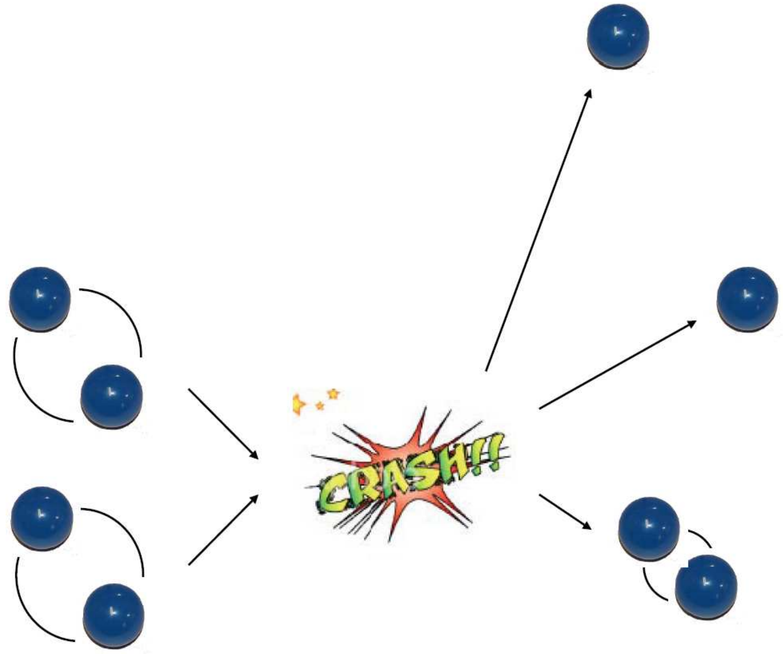
$$\boxed{T_c \simeq E_F\ \exp\left(-\frac{3\pi}{4k_F r^*}\right)}$$

(K-T transition temperature similar [Miyake, Prog. Theor. Phys. (1983)])

cf. short-range potential, $T_c \simeq E_F \exp\left(-\frac{\#}{k_F^2 R_0^2}\right)$

$1/r^3 \Rightarrow$ "anomalous" scattering from $r \sim 1/k$

Feasibility?



- Atoms: BCS $T_c \simeq E_F \, e^{-\frac{\#}{k_F^2 R_0^2}}$

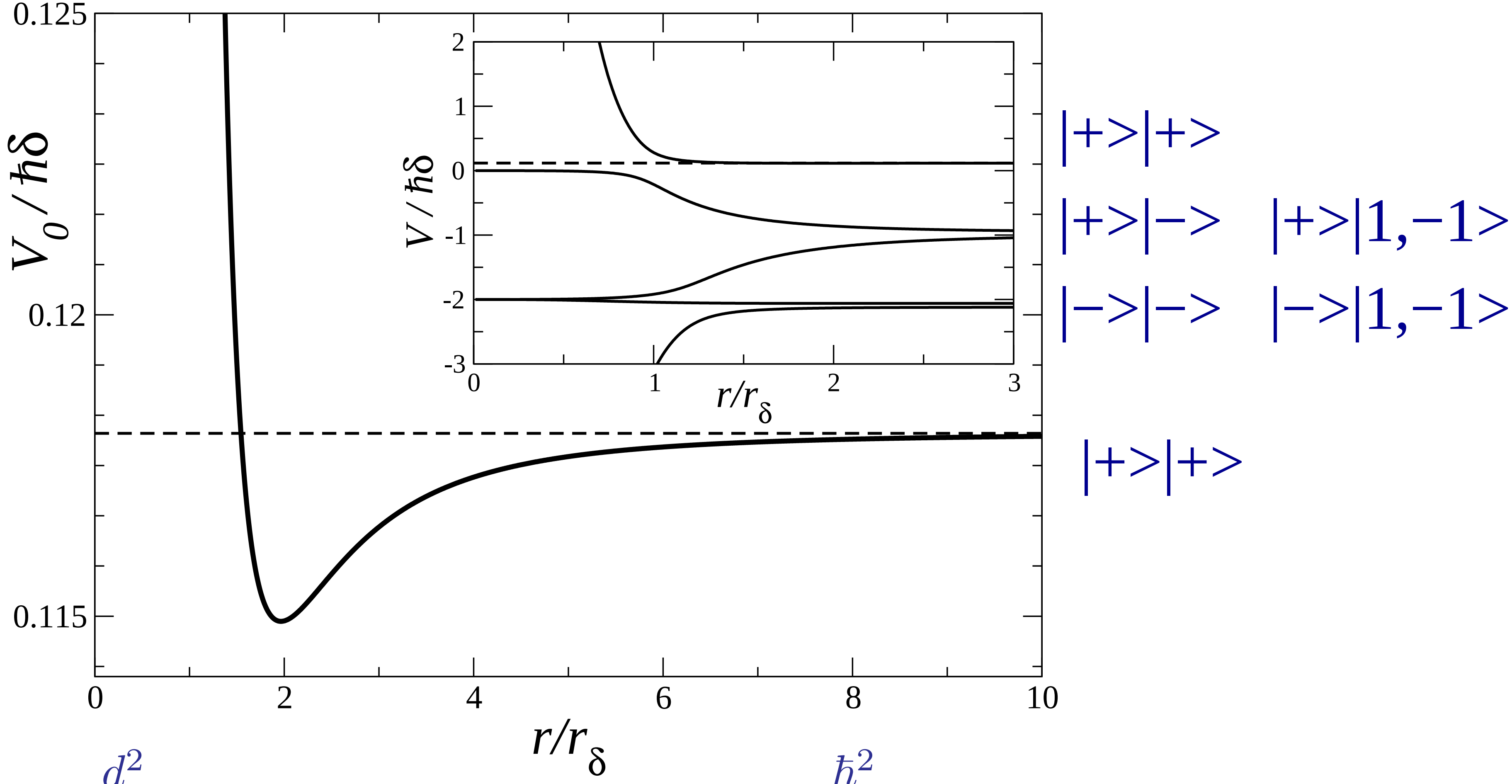
Feshbach resonance
$\Rightarrow$dimer formation
$\Rightarrow$collisional relaxation
[Levinsen, NRC & Gurarie, PRL (2007); Jona-Lasinio, Pricoupenko & Castin, PRA (2008).]

- Polar mols: BCS $T_c \simeq E_F \, e^{-\frac{3\pi}{4k_F r^*}}$

$$r^* \equiv \frac{M d_{\text{eff}}^2}{2\hbar^2}$$

T_c can be large without dimer formation

2-body losses



$$\frac{d^2}{r_\delta^3} \equiv \hbar\delta \qquad \frac{\hbar^2}{M\lambda_\delta^2} \equiv \hbar\delta$$

suppression of inelastic losses for $r_\delta/\lambda_\delta \gg 1$

Example: ^{7}Li^{40}K

dipole moment $d = 3.5$ D [M. Aymar and O. Dulieu, J. Chem. Phys. **122**, 204302 (2005)]

$\Omega_R/\delta = 0.25 \Rightarrow r^* \simeq 200$ nm

$n = 2 \times 10^8 \text{cm}^{-2} \Rightarrow k_F r^* \simeq 1$

- $E_F \simeq 120$nK
- $T_c \simeq 10$nK

For $r_\delta \simeq 30$ nm $\Rightarrow \alpha_{\text{loss}} \simeq 4 \times 10^{-4} \frac{\hbar}{M}$

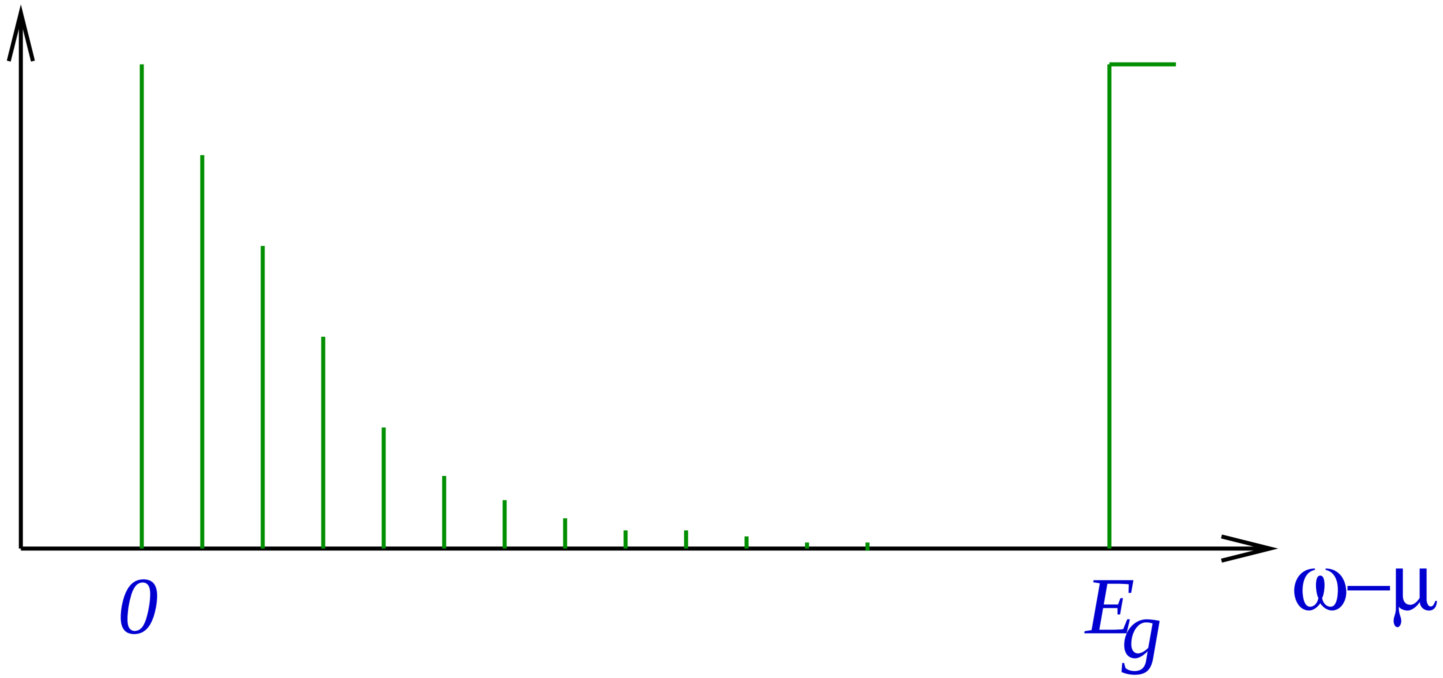
- $\tau_{\text{loss}} = 1/(\alpha_{\text{loss}} n) \sim 1$s

Experimental Consequences

Field-dressed polar molecules are in the topological phase $p_x + ip_y$, $\mu > 0$.

Vortices carry zero energy Majorana modes. [Volovik; Read & Green]

- RF absorption [Grosfeld, NRC, Stern & Ilan, PRB **76**, 104516 (2007)]



- Non-abelian statistics [Tewari, Das Sarma, Nayak, Zhang & Zoller, PRL **98**, 010506 (2007)]

Summary

- Field-dressed polar molecules in 2D can have strong attractive interactions.

- For a one-component gas of fermionic molecules, the groundstate is the $p_x + ip_y$ superfluid phase with interesting topological properties.

- Energy scales and inelastic lifetimes appear favourable.

- These systems could allow the detection of Majorana modes on vortices, and non-abelian exchange statistics.