Superradiance-induced particle flow via dynamical gauge coupling

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Wei Zheng & NRC, arXiv:1604.06630

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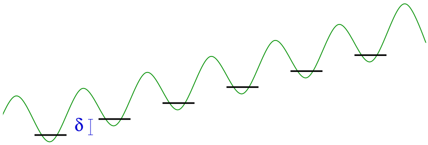
Static Gauge Fields from Photon-Assisted Tunneling

- Tight-binding lattice, tunnelling suppressed by energy offsets, δ

e.g. linear gradient



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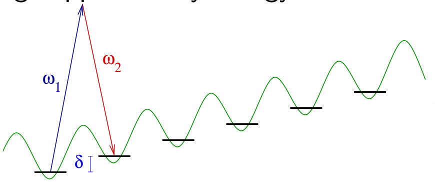


- Re-establish by resonant Raman coupling, $\omega_1 - \omega_2 = \delta$

[Jaksch & Zoller, NJP (2003); . . .]

$$H = -J \sum_x \left[e^{-i\theta} \hat{c}^\dagger_{x+1} \hat{c}_x + e^{i\theta} \hat{c}^\dagger_x \hat{c}_{x+1} \right]$$

$\Rightarrow$imprint phases on the tunneling matrix elements

Static Gauge Fields from Photon-Assisted Tunneling

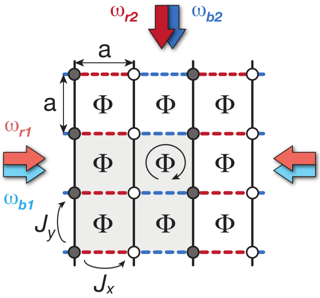
1D: Dispersion acquires a vector potential, $qA_x = \theta$

$$E_k = -2J\cos(k+\theta) \qquad \sim \frac{(\hbar k + qA_x)^2}{2m^*}$$

[can be removed by a gauge transformation]

2D: $\theta = \theta_y \Rightarrow A_x(y) \Rightarrow \vec{B} = \nabla \times \vec{A} \neq 0$

e.g. Harper-Hofstadter model
[LMU/MIT]

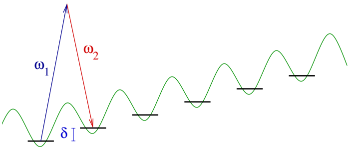


[Aidelsburger *et al.* [LMU], Nat. Phys. **11**, 162 (2015)]

Dynamical Gauge Field

Raman lasers → "pump" + cavity

$$Je^{-i\theta} \rightarrow \lambda \hat{a}^{\dagger}$$



Vector potential becomes a *quantum dynamical* variable

[cannot be removed by gauge transformation even in 1D]

In a full gauge theory all tunneling couplings, $J_{\mathbf{x},\mathbf{x}'}$, become quantum dynamical variables [e.g. extra bosons on links]

[e.g. Zohar, Cirac & Reznik, Rep. Prog. Phys. **79**, 014401 (2016)]

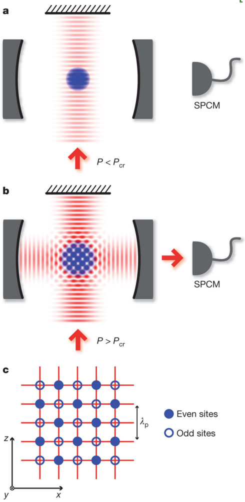
⇒extensive number of additional local bosonic degrees of freedom

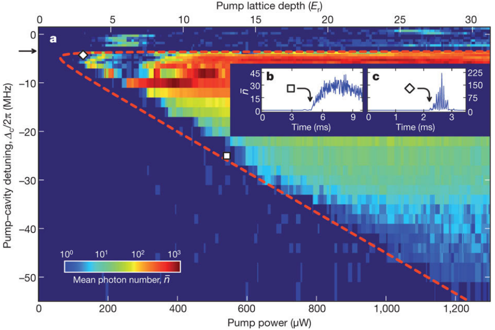
[for multimode cavities see [Ballatine, Keeling & Lev, arXiv:1608.07246]]

Here we have just one... ...but already interesting...

Superradiance in BEC

[K. Baumann, C. Guerlin, F. Brennecke & T. Esslinger, Nature **464**, 1301 (2010)]





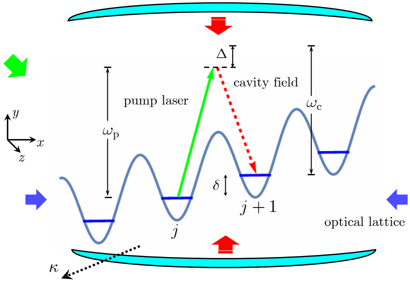
Cavity mode occupation + BEC density wave above critical pump power

Outline

Set-up: Cavity-Assisted Hopping

[Wei Zheng & NRC, arXiv:1604.06630]

- 1D lattice
- non-interacting fermions [or hardcore bosons]
- cavity-assisted hopping cavity loss rate κ



$$\hat{H}_{\text{lab}} = \omega_c \hat{a}^\dagger \hat{a} - \sum_{j=1}^{L-1} \left(\lambda e^{-i\omega_p t} \hat{a}^\dagger \hat{c}_{j+1}^\dagger \hat{c}_j + \lambda^* e^{+i\omega_p t} \hat{a} \hat{c}_j^\dagger \hat{c}_{j+1} \right) + \sum_{j=1}^{L} \delta j \hat{c}_j^\dagger \hat{c}_j$$

$[\hbar = 1]$

Set-up: Cavity-Assisted Hopping

Remove δ and $e^{\pm i\omega_p t}$ via a gauge transformation $[\Delta = \delta + \omega_c - \omega_p]$

$$\hat{H} = \Delta \hat{a}^\dagger \hat{a} - \sum_{j=1}^{L-1} \left(\lambda \hat{a}^\dagger \hat{c}_{j+1}^\dagger \hat{c}_j + \lambda^* \hat{a} \hat{c}_j^\dagger \hat{c}_{j+1} \right)$$

For a coherent state $\langle \hat{a} \rangle = \alpha = |\alpha| e^{i\theta}$ [choose $\lambda = \lambda^*$]

$$\hat{H}(\alpha) = \Delta |\alpha|^2 - \lambda |\alpha| \sum_j \left(e^{-i\theta} \hat{c}_{j+1}^\dagger \hat{c}_j + e^{i\theta} \hat{c}_j^\dagger \hat{c}_{j+1} \right)$$

Vector potential set by the phase of the cavity field

$$E_k = -2\lambda\,|\alpha| \cos\left(k + \theta\right)$$

[cf. cavity-assisted tunneling of fixed phase, but dynamical amplitude]

[C. Kollath *et al.*, PRL **116**, 060401 (2016); Sheikhan, Brennecke & Kollath, PRA **93**, 043609 (2016)]

Steady States

Master equation with cavity losses

$$\partial_t \rho = -i \left[\hat{H}, \rho\right] + \kappa \left(2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a}\right)$$

Mean-field theory, $\langle \hat{a}(t) \rangle = \alpha(t)$

$$\partial_t \alpha = -i \left(\Delta - i\kappa\right) \alpha + i\lambda K$$

$$K = \langle \hat{K} \rangle \text{ with } \hat{K} \equiv \sum_{j=1}^{L-1} \hat{c}_{j+1}^\dagger \hat{c}_j$$

Steady state: $\alpha = \dfrac{\lambda K}{\Delta - i\kappa}$

Mean-Field Steady States: Infinitely Long Lattice

$$K = \langle \hat{K} \rangle \equiv \sum_j \langle \hat{c}_{j+1}^\dagger \hat{c}_j \rangle \rightarrow K = \sum_k e^{-ik} \langle \hat{n}_k \rangle$$

Momentum occupation numbers, $\hat{n}_k$ are conserved by the dynamics

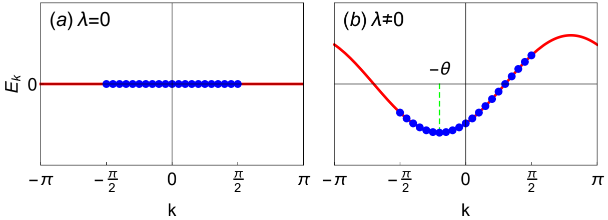
$\Rightarrow \lambda K$ behaves as a constant drive $\quad \alpha = \dfrac{\lambda K}{\Delta - i\kappa}$

If the initial atom distribution has $|K| \neq 0$ – i.e. coherence – the steady state has $\alpha \neq 0$

$\Rightarrow$superradiance at vanishing pump threshold

[cf. Dicke-type models with non-zero threshold]

Mean-Field Steady States: Infinitely Long Lattice



$$E_k = -2\lambda\,|\alpha|\cos\left(k+\theta\right) \qquad \alpha = |\alpha|e^{i\theta} = \frac{\lambda K}{\Delta - i\kappa}$$

Phase θ adapts to the momentum distribution such that

$\Rightarrow$superradiance induces a steady particle current

Cavity losses balanced by photon scattering from pump to cavity

What is the fate of this flow in a finite lattice?

Mean-Field Steady States: Finite Lattice

$K = \langle \hat{K} \rangle \equiv \sum_j \langle \hat{c}^\dagger_{j+1} \hat{c}_j \rangle$ no longer conserved in time

[e.g. particles start to pile up at the right-hand boundary]

Mean-field steady-state for particle density matrix $\Rightarrow \alpha^* K = \alpha K^*$

Combining with $\alpha = \dfrac{\lambda K}{\Delta - i\kappa}$ requires $K = \alpha = 0$ for steady state

$\Rightarrow$In a finite lattice, mean-field dynamics drives the system to vanishing coherence, $K = 0$, and vanishing superradiance $\alpha = 0$

What is the long-time steady state beyond m.f.t.?

Non-Equilibrium Dynamics – Beyond Mean Field

Include fluctuation effects to order λ^2 [essential when $\alpha = 0$]

$$\partial_t \rho_{ij} = -i\lambda A_{ij}(t) + \frac{\kappa\lambda^2}{\Delta^2+\kappa^2} B_{ij}(t) \qquad [\rho_{ij} = \langle \hat{c}_i^\dagger \hat{c}_j \rangle]$$

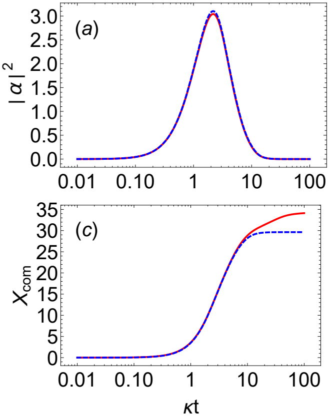
$A_{ij} = \alpha^* \rho_{i+1,j} + \alpha \rho_{i-1,j} - \alpha^* \rho_{i,j-1} - \alpha \rho_{i,j+1}$

$B_{ij} = 2\rho_{i-1,j-1} - 2\rho_{i,j} - \sum_l \left(\rho_{i-1,l-1}\rho_{l,j} + \rho_{i,l}\rho_{l-1,j-1}\right) + \sum_l \left(\rho_{i+1,l+1}\rho_{l,j} + \rho_{i,l}\rho_{l+1,j+1}\right)$

Total particle current, $J = J^{\mathrm{sr}} + J^{\mathrm{cl}} + J^{\mathrm{qu}}$

- $J^{\mathrm{sr}} = -\lambda \sum_i \mathrm{Im}\left(\alpha^* \rho_{i+1,i}\right)$ mean-field superradiant current
- $J^{\mathrm{cl}} = \frac{2\kappa\lambda^2}{\Delta^2+\kappa^2} \sum_i \left(1 - \rho_{i+1}\right)\rho_i$ semiclassical hopping [ASEP]
- $J^{\mathrm{qu}} = -\frac{2\kappa\lambda^2}{\Delta^2+\kappa^2} \sum_{i\neq l} \mathrm{Re}\left(\rho_{i+1,l+1}\rho_{l,i}\right)$ quantum correction from long-range coherent cavity field

Non-Equilibrium Dynamics: Numerical Results



blue dashed – m.f.t.
red solid – full theory
$L = 20,\ N = 5$
$\Delta/\kappa = \lambda/\kappa = 0.5$

- Superradiant pulse (mean-field dominated)
- Fluctuations control the long time behaviour, $\alpha = 0$

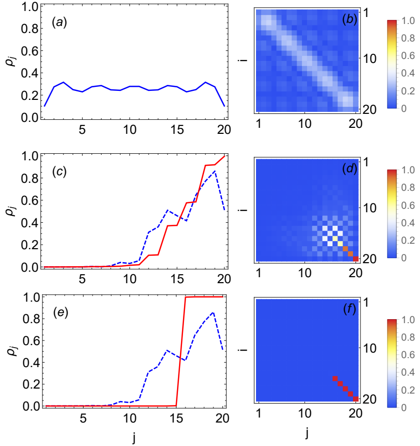
Steady State(s): Numerical Results

$$\rho_j = \langle \hat{c}_j^\dagger \hat{c}_j \rangle \qquad \rho_{ij} = \langle \hat{c}_i^\dagger \hat{c}_j \rangle$$

initial density

final density (fermions)

final density (h.-c. bosons)



Steady State(s): Analytic Description

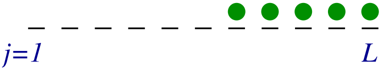
No superradiance, $\alpha = \langle \hat{a} \rangle = 0$

$$\partial_t \rho_\mathrm{f} = -i \left[\hat{H}_\mathrm{eff}, \rho_\mathrm{f} \right] + \frac{\kappa \lambda^2}{\Delta^2 + \kappa^2} \left(2 \hat{L}_\mathrm{eff} \rho_\mathrm{f} \hat{L}_\mathrm{eff}^\dagger - \hat{L}_\mathrm{eff}^\dagger \hat{L}_\mathrm{eff} \rho_\mathrm{f} - \rho_\mathrm{f} \hat{L}_\mathrm{eff}^\dagger \hat{L}_\mathrm{eff} \right)$$

for $\hat{L}_\mathrm{eff} = \hat{K}$ and $\hat{H}_\mathrm{eff} = -\frac{\kappa \lambda^2}{\Delta^2 + \kappa^2} \hat{K}^\dagger \hat{K}$, with $\hat{K} \equiv \sum_{j=1}^{L-1} \hat{c}_{j+1}^\dagger \hat{c}_j$

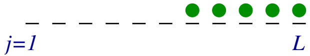
$$\hat{K} \left| D \right\rangle = 0 \quad \Rightarrow \quad \text{steady state, } \rho_\mathrm{f} = \left| D \right\rangle \left\langle D \right|$$

e.g. $\left| \mathrm{step} \right\rangle \equiv \prod_{j=1}^{N} \hat{c}_{L-N+j}^\dagger \left| 0 \right\rangle$



Steady State(s): Analytic Description

$|\mathrm{step}\rangle \equiv \prod_{j=1}^{N} \hat{c}^\dagger_{L-N+j} |0\rangle \sim$ Fermi sea in position space



Define bosonic operators $\hat{b}^\dagger_s = \sum_{j=s+1}^{L} \hat{c}^\dagger_{j-s}\hat{c}_j$ (cf. Tomonaga-Luttinger)

$\hat{K} \equiv \sum_{j=1}^{L-1} \hat{c}^\dagger_{j+1}\hat{c}_j$ behaves as the annihilation operator $\hat{b}_1$

cavity damps only this mode

$\Rightarrow$multiple steady states: $\prod_{\alpha=1}^{n_\mathrm{b}} \hat{b}^\dagger_{s_\alpha} |\mathrm{step}\rangle$ for $s_\alpha \neq 1$

[cf. models with many dissipation channels [S. Diehl *et al.*, Nature Phys. **4**, 878 (2008)]]

Summary

- Photon-assisted hopping via pump + cavity causes atoms to experience a dynamical gauge field.
- For an atomic gas with initial coherence, mean-field theory predicts a transition to a superradiant state at infinitesimal pump threshold, with superradiance-induced current.
- Particle accumulation at the boundary causes collapse of superradiance and a transition to a regime in which particle motion is governed by cavity fluctuations.
- At long times there are multiple steady states: collective excitations decouple and only one is damped by cavity losses.