Measuring the Superfluid Fraction of Ultracold Atomic Gases

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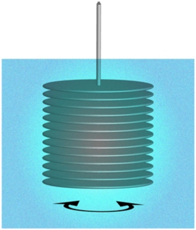
[NRC & Z. Hadzibabic, PRL **104**, 030401 (2010)]

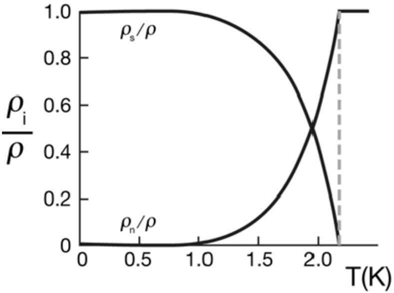
Outline

Superfluid vs. Condensate Fraction: ^{4}He

Two-fluid model: $\rho = \rho_s + \rho_n$ [Tisza (1940), Landau (1941)]

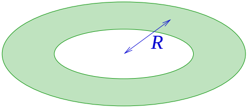
Andronikashvili experiment [E. L. Andronikashvili, J. Phys USSR **10**, 201 (1946)]





Superfluid Fraction

Ring Geometry, $R \gg \Delta R$ [A. J. Leggett, Phys. Rev. Lett. **25**, 1543 (1970)]



Classical moment of inertia $I_{\mathrm{cl}} = NMR^2$

Rotate walls with angular velocity ω

$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega \to 0} \left(\frac{\langle L \rangle}{I_{\mathrm{cl}}\omega} \right)$$

Condensate Fraction

Off-diagonal long range order [C. N. Yang, Rev. Mod. Phys. **34**, 694 (1962)]

$$\langle \hat{\psi}^\dagger(\mathbf{r}')\hat{\psi}(\mathbf{r})\rangle \xrightarrow{|\mathbf{r}'-\mathbf{r}|\to\infty} \rho_c/M$$

Ideal BEC ($T=0$): $\hat{\psi}(\mathbf{r}) = \sqrt{\rho/M}\, e^{i\phi} \Rightarrow \rho_c/\rho = 1$.

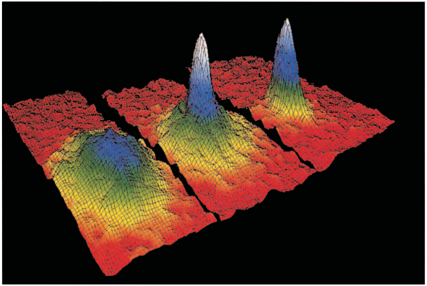
Neutron scattering [1979–]: $\rho_c/\rho \sim 0.1$ at low temperatures.
Condensate depletion by strong interactions.

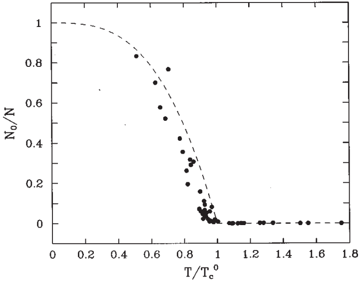
In 2D, $\rho_c = 0$ with $\rho_s \neq 0$.

Ultracold Atomic Gases: Condensate Fraction

Expansion Imaging

[M. H. Anderson *et al.*, Science **269**, 198 (1995)]



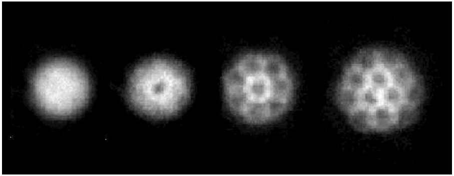


Condensate fraction as a function of T/T_c^0.

[Ensher *et. al.* [JILA], PRL **77**, 4984 (1996).]

Ultracold Atomic Gases: Superfluidity

- Quantized vortices



[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

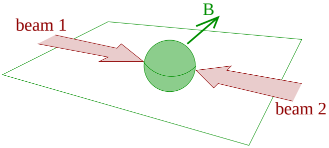
- Critical velocity [C. Raman *et al.*, Phys. Rev. Lett. **83**, 2502 (1999)]

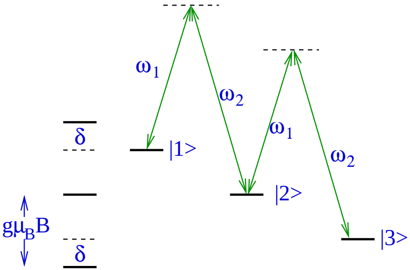
- Persistent currents [C. Ryu *et al.*, Phys. Rev. Lett. **99**, 260401 (2007)]

Superfluid density? [T.-L. Ho and Q. Zhou, Nature Phys. **6**, 131 (2010)]

Optically Induced Gauge Potentials

[I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)]



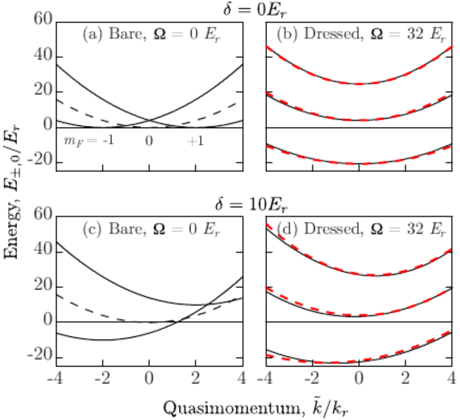


$$\delta = g\mu_B B/\hbar - (\omega_2 - \omega_1) \qquad \Delta k = k_1 - k_2 \simeq 2k_r$$

Three-level system

$$\begin{pmatrix} \frac{\hbar}{2M}(k+\Delta k)^2 - \delta & \Omega_R/2 & 0 \\ \Omega_R/2 & \frac{\hbar}{2M}k^2 - \epsilon & \Omega_R/2 \\ 0 & \Omega_R/2 & \frac{\hbar}{2M}(k-\Delta k)^2 + \delta \end{pmatrix}$$

[I. B. Spielman, Phys. Rev. A **79**, 063613 (2009)]

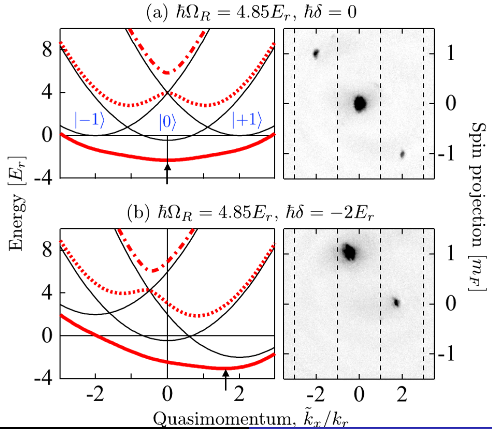


$$E \simeq E_0 + \frac{(p-qA)^2}{2M}$$

Experimental Implementation: Uniform Vector Potential

Implementation for $^{87}Rb\ F=1,\ m_F=-1,0,1$

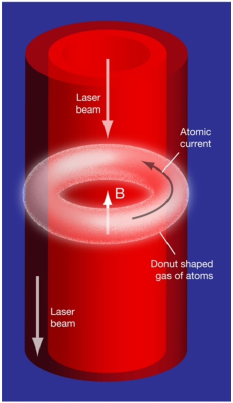
[Y.-J. Lin *et al.*, Phys. Rev. Lett. **102**, 130401 (2009)]

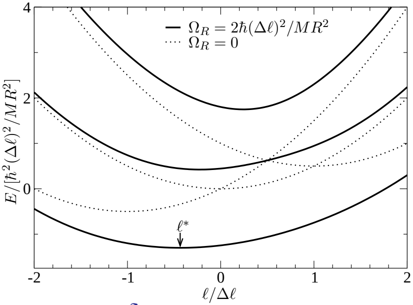


Superfluid Fraction: Ring Geometry

$R \gg \Delta R$

[NRC & Zoran Hadzibabic, PRL **104**, 030401 (2010)]





$$\ell^* \simeq -\sqrt{2}\frac{\delta}{\Omega_R}\Delta\ell + \mathcal{O}(1/\Omega_R^2)$$

Orbital angular momentum $\Delta\ell \equiv \ell_2 - \ell_1$

$$E \simeq E_0 + \frac{\hbar^2}{M^* R^2}\left(\frac{\ell^2}{2} - \ell\,\ell^*\right)$$

With light on, the lab. behaves as a rotating frame

(i) Hamiltonian in a rotating frame

$$H_{\rm rot} = H - \omega L \quad \Rightarrow \quad \omega_{\rm eff} = \frac{\hbar\ell^*}{M^* R^2}$$

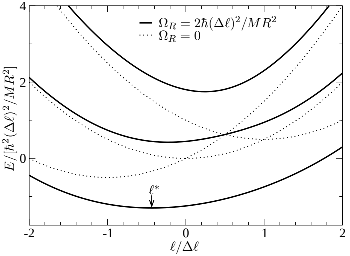
(ii) Angular group velocity

$$\omega_{\rm light} \equiv \frac{1}{\hbar}\frac{dE}{d\ell} = \frac{\hbar}{M^* R^2}\left(\ell - \ell^*\right)$$

with $\omega_{\rm eff} \equiv \dfrac{\hbar\ell^*}{M^* R^2}$

Lab. behaves as rotating frame

$$\omega_{\rm eff} \equiv \frac{\hbar\ell^*}{M^* R^2}$$



- Normal fluid: $\langle L\rangle/(\hbar N) = \ell^*$ (at rest in the lab. frame)
- Superfluid: $\langle L\rangle = 0$ (rotating in the lab. frame)

$$\frac{\rho_s}{\rho} \equiv 1 - \lim_{\omega_{\rm eff}\to 0}\left(\frac{\langle L\rangle}{I_{\rm cl}\omega_{\rm eff}}\right) \qquad [I_{\rm cl}\omega_{\rm eff} = NM^* R^2\omega_{\rm eff} = N\hbar\ell^*]$$

Measuring $\langle L \rangle$

Andronikashvili, $\langle L \rangle = I\omega$:

$$\langle H_{\mathrm{rot}} \rangle = -\frac{1}{2} I \omega^2$$

$\Rightarrow$shift in resonance frequency of torsional oscillator.

Here:

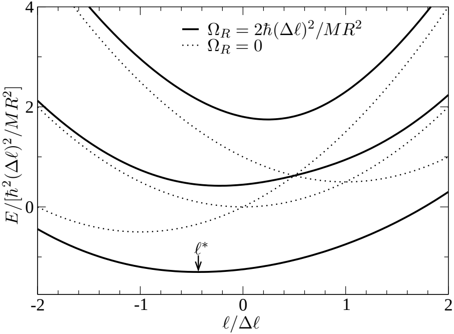
$$\langle H_{\mathrm{light}} \rangle = -\frac{1}{2} I \omega_{\mathrm{eff}}^2$$

Couple ω_{eff} to an oscillator, e.g. $\omega_{\mathrm{eff}} \propto \ell^* \propto B$.

But, *total* energy available $\frac{1}{2} I (\hbar \Delta \ell / M^* R^2)^2 \simeq 0.1 \mu\mathrm{eV}$.

Measuring $\langle L \rangle$: Spectroscopy

[NRC & Zoran Hadzibabic, PRL **104**, 030401 (2010)]



$$|\psi_{-1}|^2 - |\psi_1|^2 \equiv \Delta p_0 + \Delta p' \ell + \mathcal{O}(\ell^2)$$

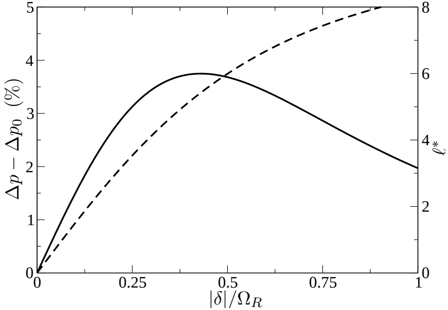
Measurement of hyperfine population imbalance

$$\begin{aligned}\Delta p \equiv \frac{N_{-1} - N_1}{N} &= \frac{\sum_\ell \langle n_\ell \rangle \left[|\psi_{-1}|^2 - |\psi_1|^2 \right]}{\sum_\ell \langle n_\ell \rangle} \\ &= \frac{\sum_\ell \langle n_\ell \rangle \left[\Delta p_0 + \Delta p' \ell \right]}{\sum_\ell \langle n_\ell \rangle} + \mathcal{O}(\mu/\hbar\Omega_R) \\ &= \Delta p_0 + \Delta p' \frac{\langle L \rangle}{\hbar N} + \mathcal{O}(\mu/\hbar\Omega_R)\end{aligned}$$

$$\frac{\rho_s}{\rho} = 1 - \lim_{\ell^* \to 0} \left(\frac{\Delta p - \Delta p_0}{\ell^* \Delta p'} \right) + \mathcal{O}(\mu/\hbar\Omega_R)$$

Required sensitivity $\ell^* \Delta p' \sim \dfrac{2\hbar(\Delta\ell)^2\,\delta}{MR^2\,\Omega_R^2}$ $[\delta/\Omega_R \ll 1]$

Parameters for ^{23}Na:
$R = 10\,\mu\text{m}$
$\Omega_R \simeq 2\pi \times 4.4\,\text{kHz}$
$\Delta\ell = 10$

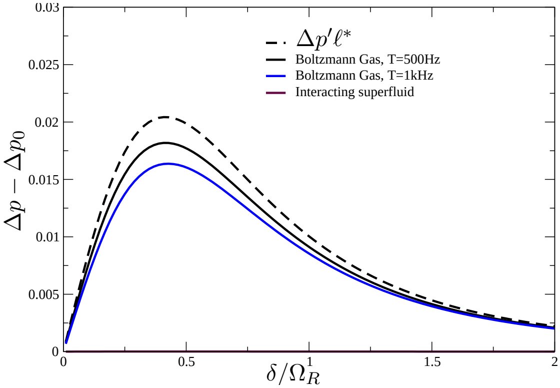


Effects of non-parabolicity

$$|\psi_{-1}|^2 - |\psi_1|^2 \equiv \Delta p_0 + \Delta p' \ell + \mathcal{O}(\ell^2)$$

[Sebastian John, Zoran Hadzibabic & NRC, (unpublished)]

Parameters for ^{23}Na:
$R = 10\,\mu$m
$\Omega_R \simeq 2\pi \times 10$ kHz
$\Delta\ell = 10$
$\bar{n} = 10^{14}$cm^{-3}
$g\bar{n} = 5$kHz



Superfluid Fraction: Disk Geometry

Local (Born-Oppenheimer) optically dressed states, $|n_{\mathbf{r}}\rangle$, $E_n(\mathbf{r})$

$$\Psi(\mathbf{r}) = \sum_n \psi_n(\mathbf{r})\,|n_{\mathbf{r}}\rangle$$

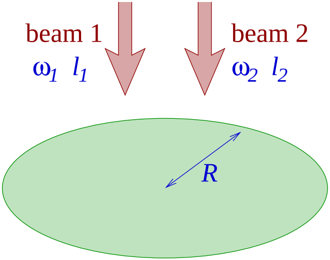
Full Hamiltonian: $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2M} + \sum_n E_n(\mathbf{r})|n_{\mathbf{r}}\rangle\langle n_{\mathbf{r}}|$

Slow (adiabatic) motion $\Rightarrow$ $\hat{H}_n = \frac{(\hat{\mathbf{p}} - q\mathbf{A})^2}{2M} + V_n(\mathbf{r})$

$$q\mathbf{A} = i\hbar\langle n_{\mathbf{r}}|\boldsymbol{\nabla} n_{\mathbf{r}}\rangle$$

Two-level system

$$|0_{\mathbf{r}}\rangle = \begin{pmatrix} e^{-i\chi}\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}$$



$$\chi(\mathbf{r}) = \Delta\ell\phi \;\Rightarrow\; q\mathbf{A} = \hbar\Delta\ell\sin^2(\theta/2)\frac{\hat{\mathbf{e}}_\phi}{r}$$

$\sin(\theta/2) = \alpha r \;\; (\theta = 2\alpha r \ll 1) \;\Rightarrow$ uniform rotation, $\omega_{\rm eff} = \frac{\hbar}{M}\Delta\ell\alpha^2$

$$|\Psi_\uparrow|^2 - |\Psi_\downarrow|^2 \simeq \Delta p_0 + \Delta p' r\langle p_\phi(\mathbf{r})\rangle + \ldots$$

Population imbalance (over all sample) probes angular momentum.

Summary

- Quantum liquid phases of bosons are characterized by both superfluid and condensate fractions.
- The use of optically induced gauge potentials allows a direct (spectroscopic) determination of the superfluid fraction.
- The method applies to both ring and disk geometries.
- The approach is readily generalized to other situations.

Aside: Simulated Magnetic Field/Rotation

$A_x \propto \delta \propto B \Rightarrow$ field gradient $B \propto y$

$\Rightarrow \vec{\nabla} \times \vec{A} \neq 0 \Rightarrow$ quantized vortices

[Y.-J. Lin *et al.*, Nature **462**, 628 (2009)]