Topological Phases of Matter Out of Equilibrium

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Solvay Workshop on Quantum Simulation
ULB, Brussels, 18 February 2019

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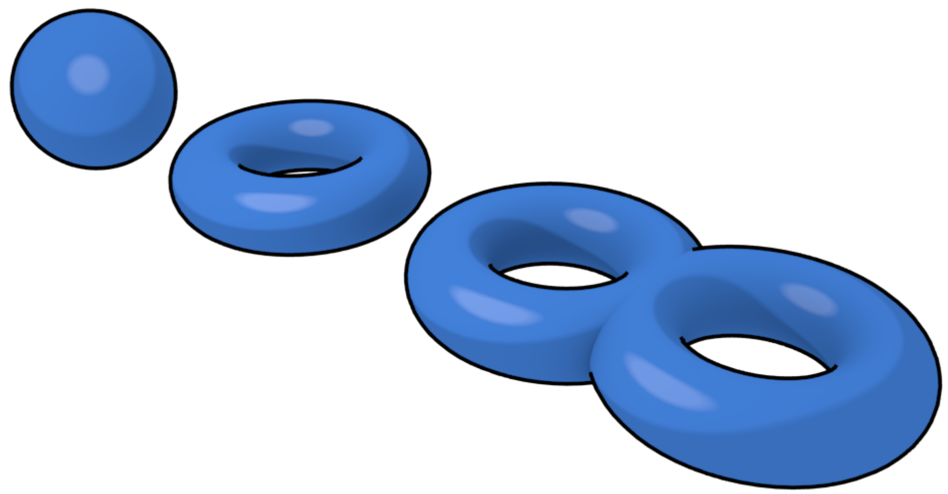
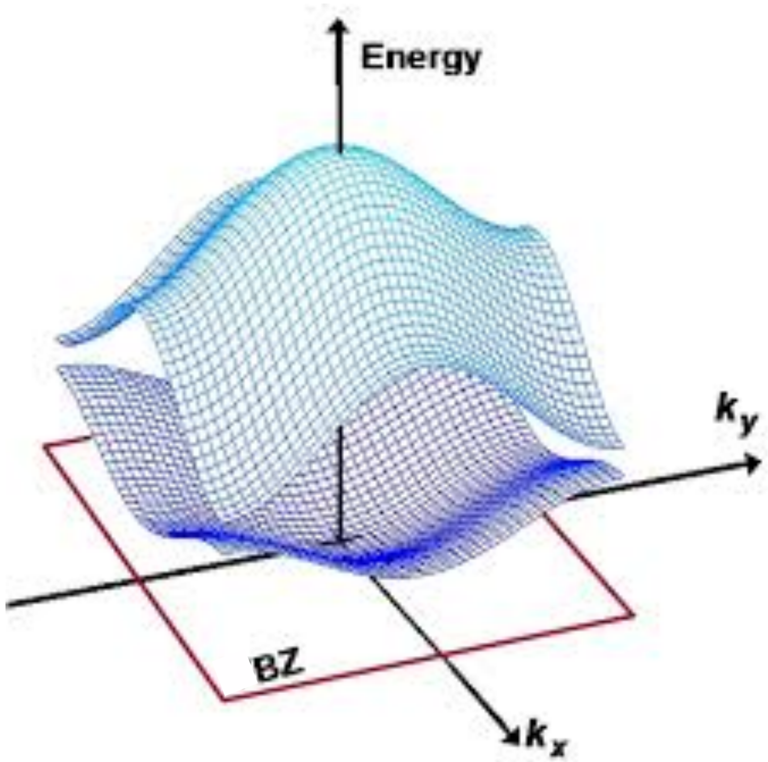
Topological Invariants



$$\frac{1}{2\pi}\int_{\text{closed surface}} \kappa \, dA \quad = \quad (2-2g)$$

Gaussian curvature $\quad \kappa = \dfrac{1}{R_1 R_2}$

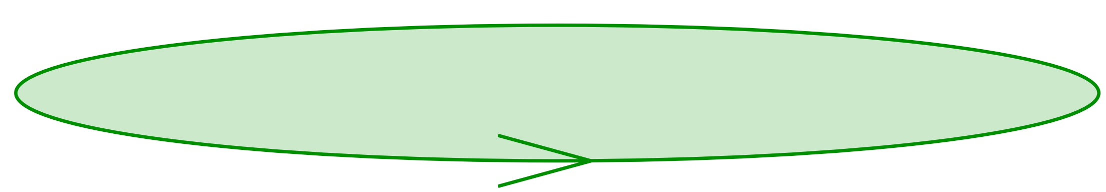
<u>2D Bloch Bands</u> [Thouless, Kohmoto, Nightingale & den Nijs, PRL 1982]



Chern number: $\quad \nu = \dfrac{1}{2\pi}\displaystyle\int_{\text{BZ}} d^2k \, \Omega_{\boldsymbol{k}} \leftarrow$

Berry curvature: $\quad \Omega_{\boldsymbol{k}} = -\, i \nabla_{\boldsymbol{k}} \times \langle u_{\boldsymbol{k}} | \nabla_{\boldsymbol{k}} u_{\boldsymbol{k}} \rangle \cdot \hat{z}$

- ν cannot change under smooth deformations
- Insulating bulk with ν gapless edge states

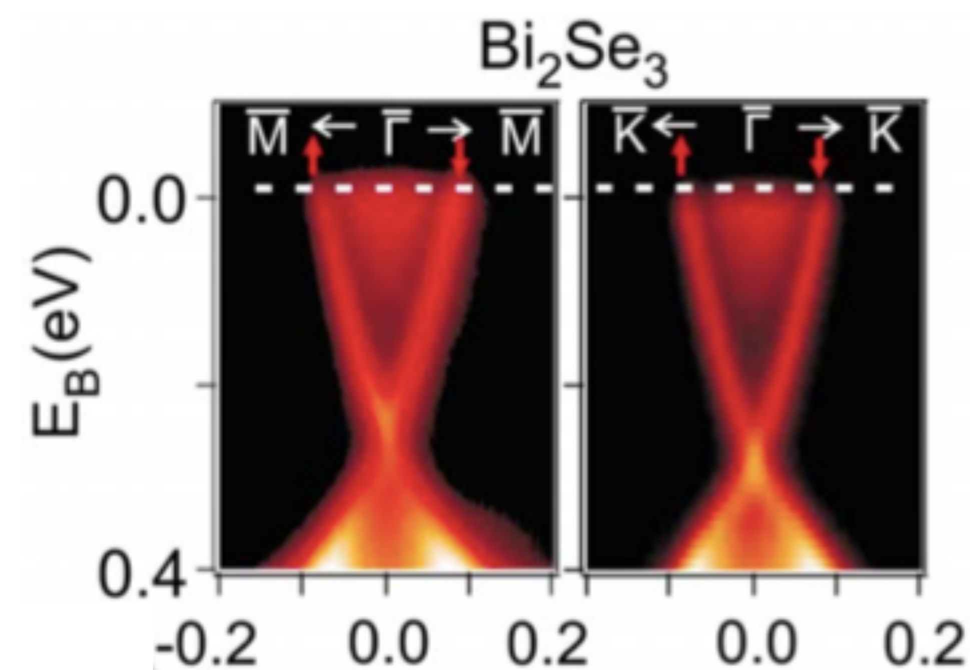


Topological Insulators

- Many generalisations when *symmetries* are included: [Hasan & Kane, RMP 2010]

topological insulators/superconductors in all spatial dimensions

$\Rightarrow$ bulk gap + gapless surface states



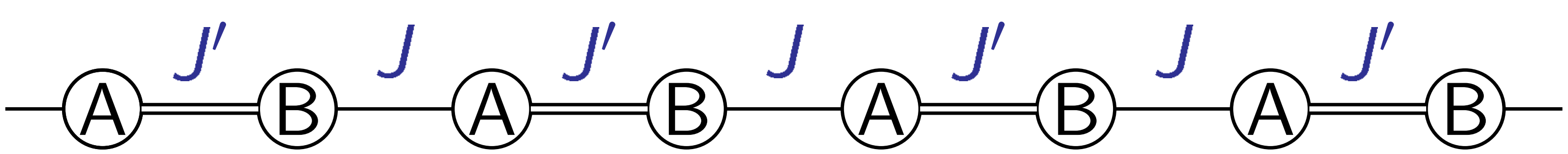
[ARPES: Xia *et al.*, 2008]

— Time reversal symmetry

(non-magnetic system in vanishing magnetic field)

— "Chiral" (sublattice) symmetry

e.g. Su-Schrieffer-Heeger model

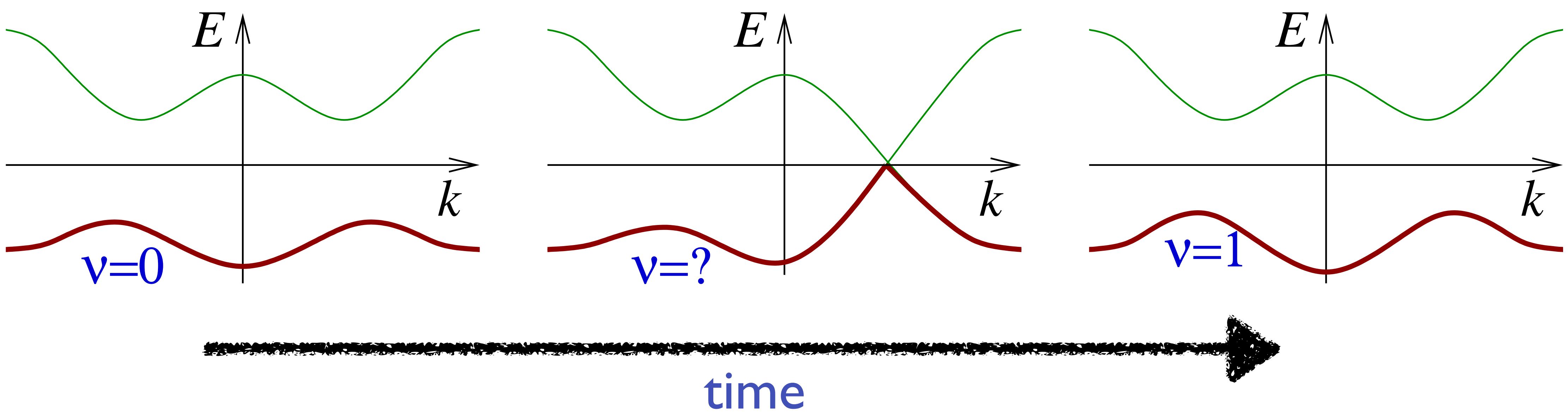


$\Rightarrow$ Detailed classification of topological matter at equilibrium

[Here for free fermions, but also for strongly interacting systems]

Non-Equilibrium Dynamics? [unitary evolution]

e.g. dynamical change in band topology



- Preparation of topological phases?

- Is there a topological classification of *non-equilibrium* many-body states?

Outline

- Dynamics of Chern Insulators (2D)
- Dynamics of Topological Phases in 1D
- Topological Classification Out of Equilibrium

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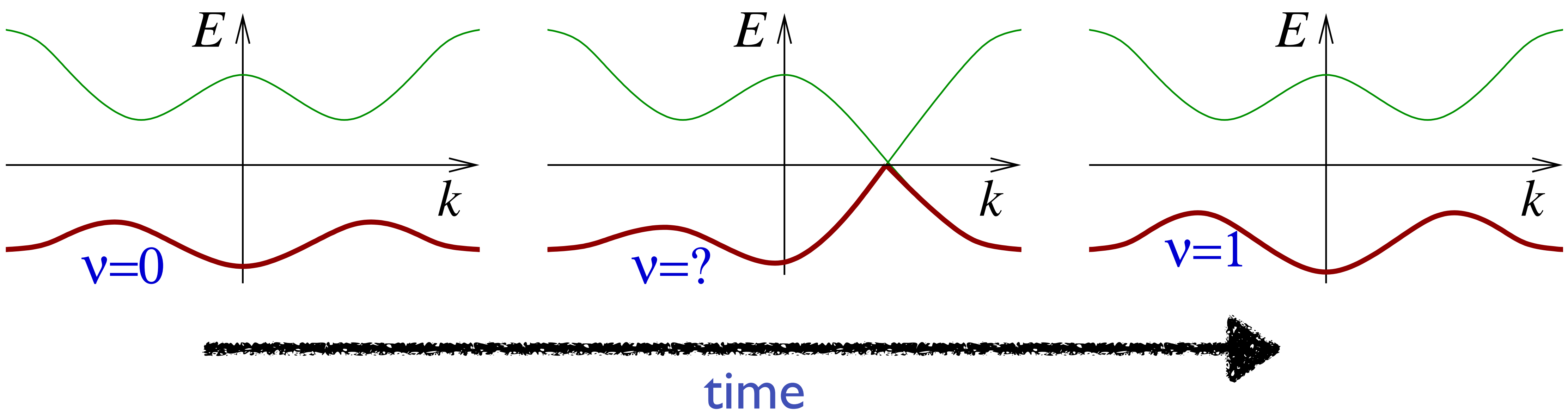
Dynamics of Chern Insulators (2D)

Quench: start in ground state of $\hat{H}^{\mathrm{i}}$ then time evolve under $\hat{H}^{\mathrm{f}}$



Time-evolving Bloch state of fermion at $\boldsymbol{k}$

$$|u_{\boldsymbol{k}}(t)\rangle = \exp(-i\hat{H}^{\mathrm{f}}_{\boldsymbol{k}}t)\,|u_{\boldsymbol{k}}(0)\rangle$$

$$\Omega_{\boldsymbol{k}}(t) = -\,i\nabla_{\boldsymbol{k}} \times \langle u_{\boldsymbol{k}}(t)\,|\,\nabla_{\boldsymbol{k}} u_{\boldsymbol{k}}(t)\rangle \cdot \hat{z}$$

⇒ Chern number of the many-body state is preserved

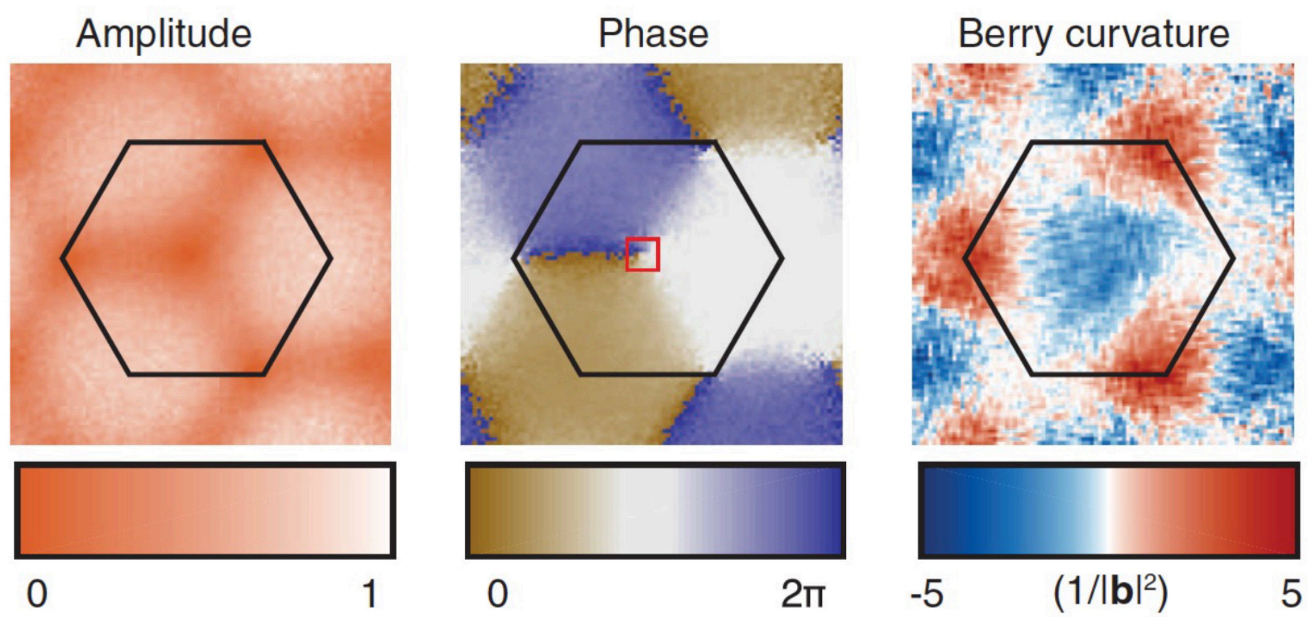
[D'Alessio & Rigol, Nat. Commun. 2015; Caio, NRC & Bhaseen, PRL 2015]

["topological invariant" under smooth changes of the Bloch states]

Dynamics of Chern Insulators: Physical Consequences

- Obstruction to preparation of a state with differing Chern number
  [For slow ramps, $\tau \gg L/\mathrm{v}$, deviations can be small]

- Chern number can be obtained by tomography of Bloch states
  [Two-band model: $|u_{\boldsymbol{k}}\rangle = \cos(\theta_{\boldsymbol{k}}/2)|A,\boldsymbol{k}\rangle + \sin(\theta_{\boldsymbol{k}}/2)e^{i\phi_{\boldsymbol{k}}}|B,\boldsymbol{k}\rangle$]



[Fläschner *et al.* [Hamburg], Science 2016]

- Topological of *final* Hamiltonian can be obtained by tracking the evolution of the Bloch states in time

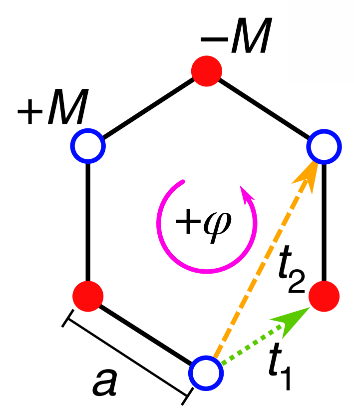
[Wang *et al.* [Tsinghua], PRL 2017; Tarnowski *et al.* [Hamburg], arXiv 2017]

Dynamics of Chern Insulators in Real Space

[Caio, Möller, NRC & Bhaseen, Nat. Phys. 2019]

Haldane model

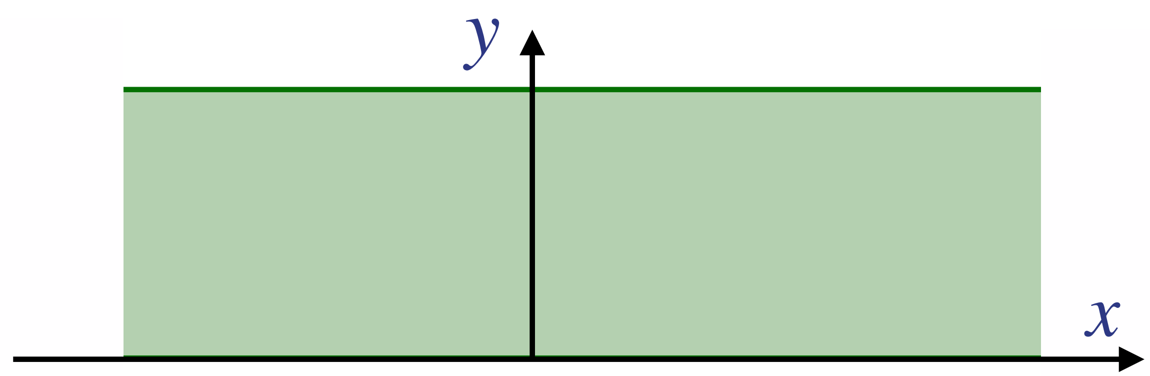
[Haldane, PRL 1988]

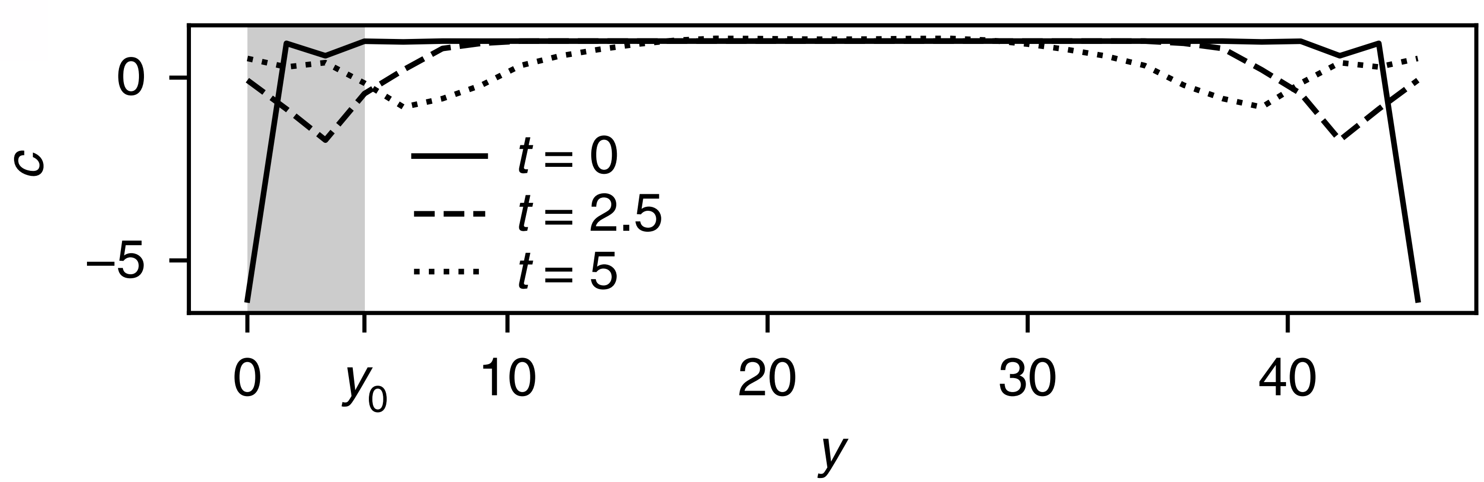


Local Chern Marker

[Bianco & Resta, PRB 2011]

$$c(\boldsymbol{r}_\alpha) = -\frac{4\pi}{A_c}\mathrm{Im}\sum_{s=A,B}\langle \boldsymbol{r}_{\alpha_s}|\hat{P}\hat{x}(\hat{1}-\hat{P})\hat{y}\hat{P}|\boldsymbol{r}_{\alpha_s}\rangle$$





global conservation

$$\int c(\boldsymbol{r})\, d^2 r = 0$$

$$\Rightarrow \quad \frac{\partial c}{\partial t} = -\nabla\cdot \boldsymbol{J}_c$$

Quench dynamics involves flow of the Chern marker

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Dynamics of Topological Phases in 1D (Free Fermions)

In 1D all topological invariants can be determined from

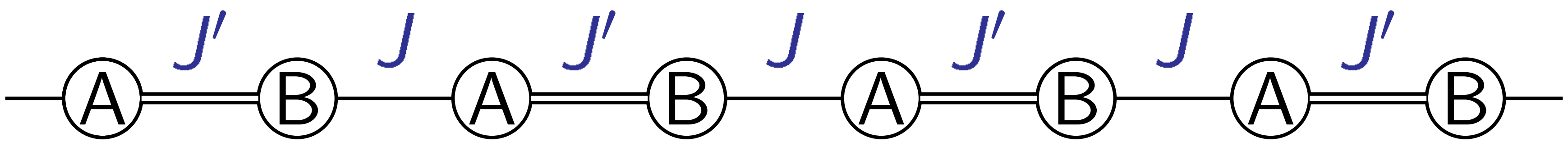
$$\mathrm{CS}_1 = \frac{1}{2\pi}\int_{\mathrm{BZ}} dk\,\langle u_k | \partial_k u_k \rangle$$

Equivalently: Berry phase around the Brillouin zone (Zak phase)

Only quantized in the presence of symmetries

In 1D, topology must be protected by symmetry

Example: Su-Schrieffer-Heeger Model

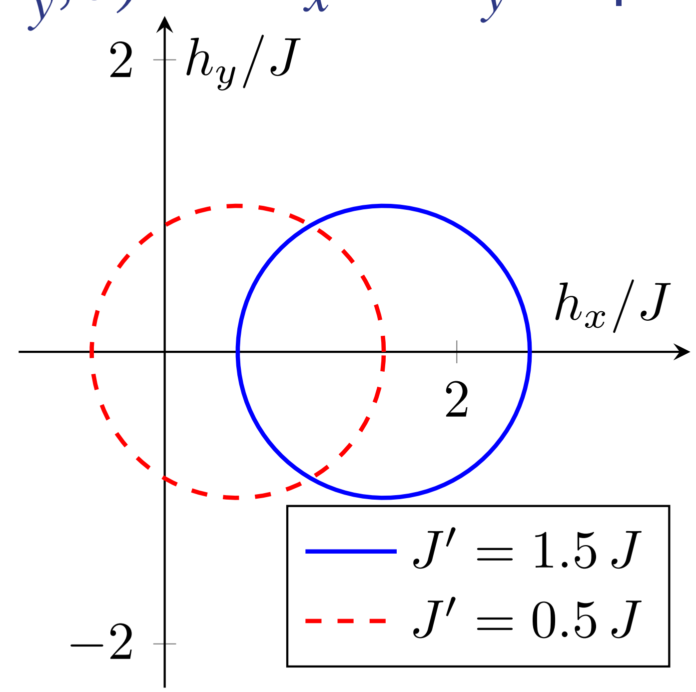


$$H_k = -\begin{pmatrix} 0 & J' + Je^{-ika} \\ J' + Je^{ika} & 0 \end{pmatrix} = -\boldsymbol{h}(k)\cdot\boldsymbol{\sigma}$$

"Chiral" (sublattice) symmetry $\Rightarrow \boldsymbol{h} = (h_x, h_y, 0) \Rightarrow h_x + ih_y = |\boldsymbol{h}(k)|\, e^{i\phi(k)}$

$\Rightarrow \mathrm{CS}_1 = N/2$

winding number $$N = \underbrace{\frac{1}{2\pi}\int_{\mathrm{BZ}} \frac{d\phi}{dk}dk}_{\text{integer}}$$



Is this topological invariant preserved out of equilibrium?

No… need to consider *symmetries!*

Symmetry-Protected Topology Out of Equilibrium

[Max McGinley & NRC, PRL 2018]

- Start in ground state of $\mathscr{H}_{\mathrm{i}}$ then time evolve under $\mathscr{H}_{\mathrm{f}}$
- $\mathscr{H}_{\mathrm{f}}$ breaks symmetry ⇒ topological "invariant" can vary

["explicit symmetry breaking"]

- What if $\mathscr{H}_{\mathrm{f}}$ respects the symmetry?

Symmetry can still be broken!

- Anti-unitary symmetries [$\langle \mathscr{O}\Psi, \mathscr{O}\Phi\rangle = \langle \Phi, \Psi\rangle^*$]

$$\mathscr{O}e^{-i\mathscr{H}t}\mathscr{O}^{-1} = e^{+i\mathscr{H}t}$$

Symmetry broken in the non-equilibrium state

["dynamically induced symmetry breaking"]

Topological invariant time-varying even if symmetries respected!

Time-Varying $\mathrm{CS}_1(t)$: Physical Consequences

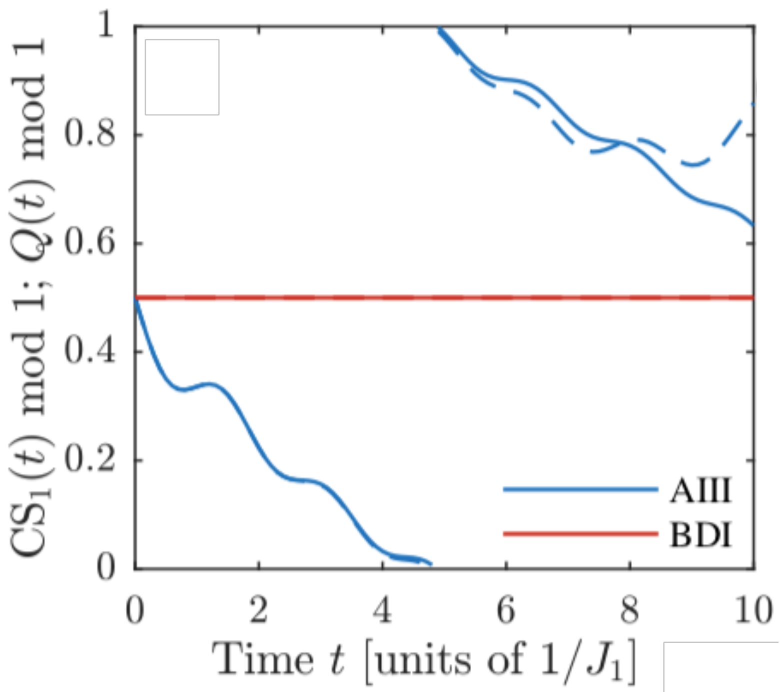
[Max McGinley & NRC, PRL 2018]

- Could be observed in Bloch state tomography [cf. Chern number]

- Directly measure via $\frac{d}{dt}\mathrm{CS}_1(t) = j(t) = \dot{Q}(t)$

[cf. Chern number $\neq$ Hall conductance out of equilibrium]

Example: quenches in a generalised SSH model



AIII: chiral symmetry only

BDI: time-reversal, particle-hole & chiral

How can we define topology out of equilibrium?

Outline

- Dynamics of Chern Insulators (2D)
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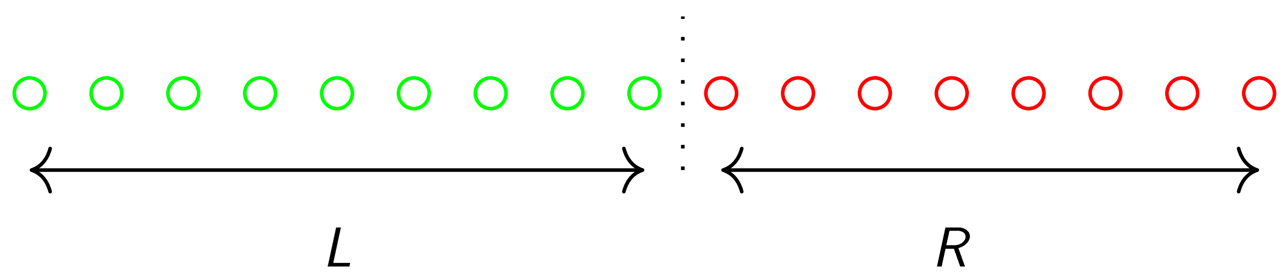
Topological Classification Out Of Equilibrium

[Max McGinley & NRC, PRL 2018]

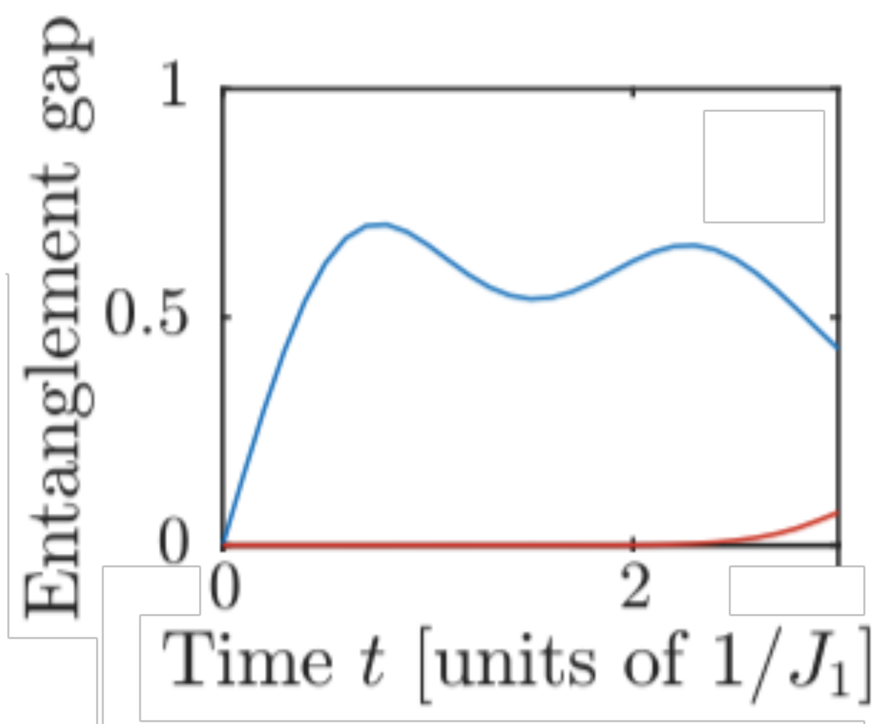
- Equilibrium topological phase ⇒ gapless surface states
- Non-equilibrium topological state ⇒ gapless *entanglement spectrum*



$$|\Psi(t)\rangle = \sum_i e^{-\lambda_i} |\psi_L^i\rangle \otimes |\psi_R^i\rangle$$

[Li & Haldane, PRL 2008]

Example: quenches in a generalised SSH model



AIII: chiral symmetry only

BDI: time-reversal, particle-hole & chiral

⇒ meaningful topological classification out of equilibrium

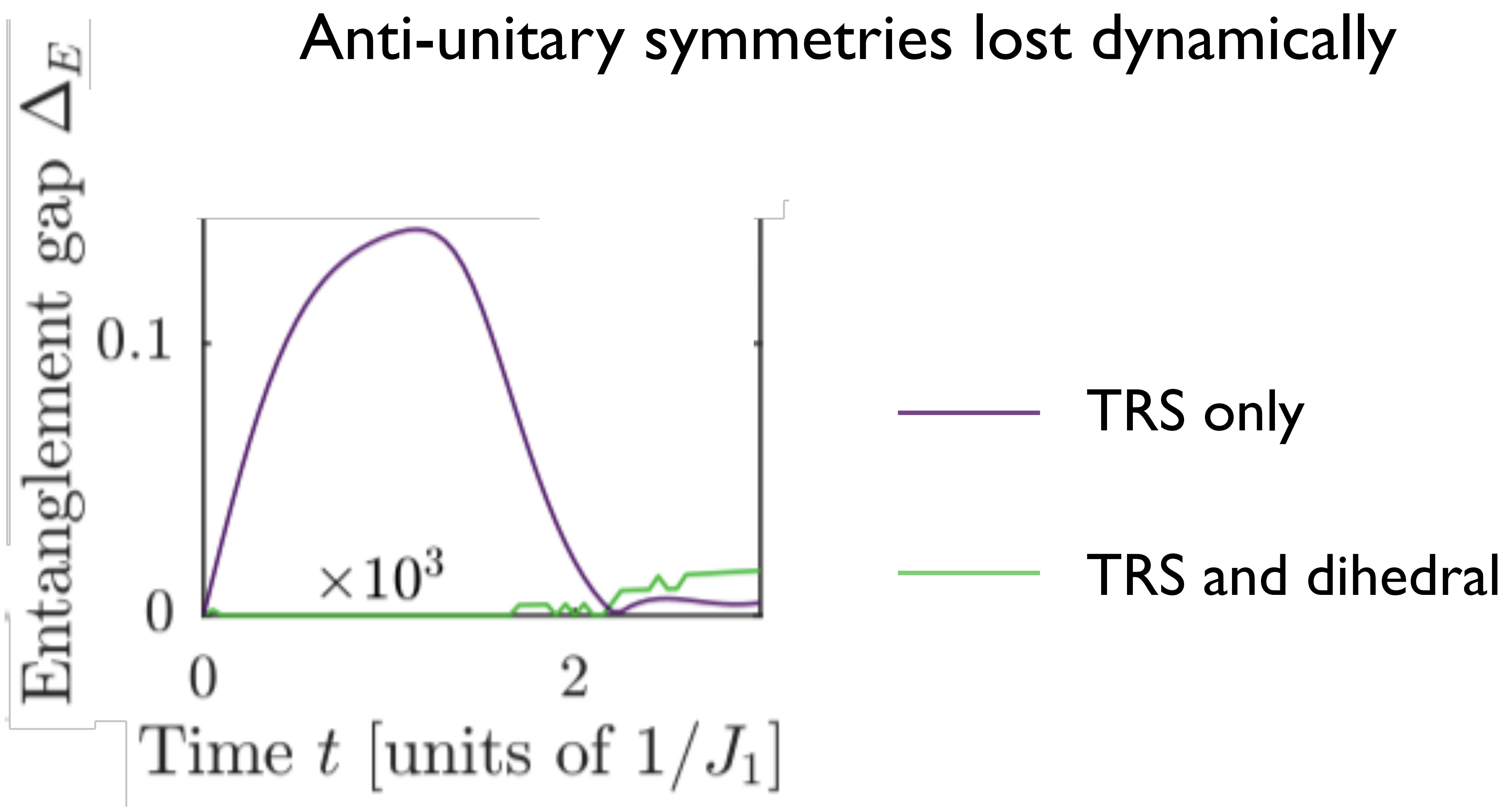
Generalization to Interacting SPT Phases

[Max McGinley & NRC, PRL 2018]

Example: Haldane phase of a S=1 spin chain is an SPT phase that can be protected by a variety of symmetries:

- Time-reversal symmetry (anti-unitary)
- Dihedral symmetry (unitary)

Anti-unitary symmetries lost dynamically



Generalization to other Spatial Dimensions

"Ten-fold way" for free fermions [Chiu, Teo, Schnyder & Ryu, RMP 2016]

[Time-reversal, particle-hole & chiral symmetries]

	symmetries			spatial dimension							
Class	T	C	S	0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	$+$	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	$+$	$+$	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	$+$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	$-$	$+$	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	$-$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	$-$	$-$	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	$-$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	$+$	$-$	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Topological Classification Out Of Equilibrium

"Ten-fold way" for free fermions

[Chiu, Teo, Schnyder & Ryu, RMP 2016]

[Time-reversal, particle-hole & chiral symmetries]

Non-Equilibrium Classification

[Max McGinley & NRC, arXiv:1811.00889]

Class	Symmetries			Spatial dimension d							
	T	C	S	0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z} \to 0$	0	$\mathbb{Z} \to 0$	0	$\mathbb{Z} \to 0$	0	$\mathbb{Z} \to 0$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2 \to 0$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z} \to \mathbb{Z}_2$	0	0	0	$2\mathbb{Z} \to 0$	0	$\mathbb{Z}_2 \to 0$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2 \to 0$	$\mathbb{Z} \to 0$	0	0	0	$2\mathbb{Z} \to 0$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z} \to 0$	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2$	$\mathbb{Z} \to \mathbb{Z}_2$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z} \to 0$	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2 \to 0$	$\mathbb{Z} \to 0$

Physical Consequences of Non-Equilibrium Classification

[Max McGinley & NRC, arXiv:1811.00889]

- Preparation of topological states

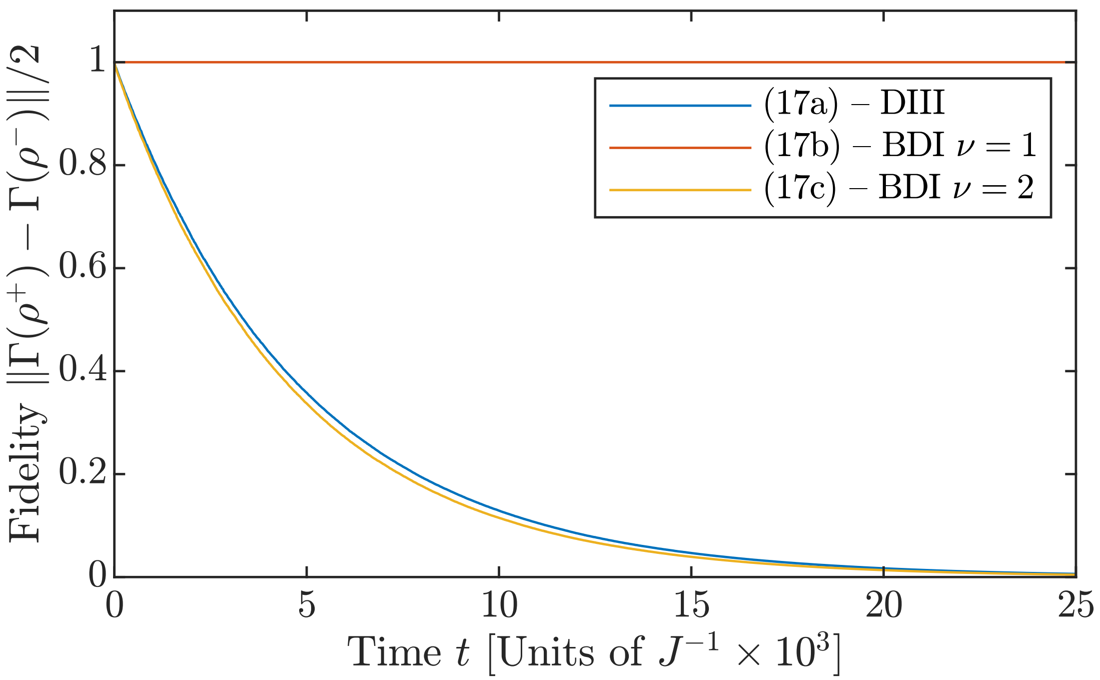
  ⇒ States with non-trivial index cannot be prepared on timescales short compared to the inverse level spacing $\sim L/v$

- Stability of "topologically protected" boundary modes

  ⇒ Adiabatic mixing of Kramers pairs under braiding [Wölms, Stern & Flensberg, PRL 2014]

  ⇒ Decoherence of Majorana qubit memories due to *noise*

Retrieval of quantum information via "recovery fidelity"
[Mazza, Rizzi, Lukin & Cirac, PRB 2013]



Summary

- There exists a topological classification of non-equilibrium quantum states, which differs from that at equilibrium:
  ⇒ bulk-boundary correspondence applies to the entanglement spectrum.
  [holds also for interacting and disordered systems]

- For 2D systems, the Chern number is preserved under unitary dynamics. However, there can be a spatial flow of the local Chern marker.

- More generally, topological "invariants" can vary in time
  - ⇒ in 1D such variations appear as a current
  - ⇒ no obstruction to changing the topology of the state dynamically
  - ⇒ sensitivity of "topologically protected" degrees of freedom to noise