Phase diagram of the symmetric electron-hole bilayer

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In preparation

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The model system

- **Experimental setup:** double-well heterojunction + electric field

- **Model defined by:**
  - two parallel infinite layers of infinitesimal width
  - separation $d$
  - in-layer density parameter $r_s$
  - electron-hole mass ratio $m_h/m_e = 1$
Expected phases

- \( d \gg r_s \): recovers the 2D HEG phase diagram (fluid at low \( r_s \), Wigner crystal at high \( r_s \))

- \( d \lesssim r_s \): exciton formation

- \( d \lesssim 0.38 \) a.u.: biexciton formation possible \(^1\)

(NB, we ignore the Wigner crystal phase at this stage of the study, and concentrate on \( r_s \leq 10 \))

De Palo et al. (2002)

Phase determined comparing energies obtained with different wave functions, each representing a phase\(^2\)

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A single wave function

Form of $\Psi_S$

$$\Psi_S = \det \left[ \phi (e_i^\uparrow - h_j^\downarrow) \right] \det \left[ \phi (e_i^\downarrow - h_j^\uparrow) \right]$$

Form of pairing orbital

$$\phi (r) = \sum_{l=1}^{n_{PW}} p_l \cos (k_l \cdot r) + f(r; L) \sum_{m=0}^{n_{poly}} c_m r^m$$

- Describes pure fluid phase when $n_{PW} = N$, $p_l \neq 0$ and $c_m = 0$
- Describes pure excitonic phase when $p_l = 0$
- Phase determined by computing e–h condensate fraction (from two-body density matrix) and pair-correlation functions
Why use a single wave function?

- Associating phases and wave functions involves either:
  - limiting the flexibility of the wave function, or
  - risking obtaining a wrong result
    (the “magic backflow” argument)

- Describes region near boundary

- Single calculation per point in \((r_s, d)\)
At $d = 0$ we impose e–h Kato cusp condition ($\Gamma_{eh} = -1$)

At $d > 0$ we impose e–h cuspleness ($\Gamma_{eh} = 0$)

Not a smooth change! Hard to study $d \to 0$

The quasi-cusp Jastrow term ($d > 0$)

$$Q(r) = \Gamma_{eh} \left[ \sqrt{r^2 + z^2} - \sqrt{L^2 + z^2} \right] g(r/L)$$

$$g(x) = 1 - 6x^2 + 8x^3 - 3x^4$$

(Plane-cone intersection, if you’re wondering)
Behaviour of the energy

- Behaviour at $r_s = 1$ is correct, uninteresting, also in HEG
- Four odd-looking points are human error (now corrected)
Behaviour of the condensate fraction

Ryo’s e-h bilayer calculations
Condensate fraction versus $r_s$ for different layer separations

- What’s with the condensate fraction at $d = 0$?
Pair-correlation function in the excitonic phase

- Parallel-spin same-particle pairs
- Antiparallel-spin same-particle pairs
- Parallel-spin e-h pairs
- Antiparallel-spin e-h pairs

PCF for $r_s = 4$ a.u., $d = 0.4$ a.u.
Pair-correlation function in the “problematic” region

- Biexcitonic phase!
- Jastrow factor is likely responsible for biexciton description
- Phase onset at $d \sim 0.2$, consistent with Lee et al.  

Summary

- More robust scheme than associating wave functions to phases
- Excellent results, unsought appearance of biexcitonic phase
- In progress:
  - Pretty plots
  - Investigation of finite size effects
- Future work:
  - Include Wigner crystal: geminal-type wave function (thanks Pascal!), static structure factor detects phase
  - More experimentally-relevant mass ratios
  - Similar scheme for HEG?
The symmetric electron-hole bilayer
The electron-hole bilayer
Results
Summary

End