Non-Hamiltonian Quantum Mechanics
(Some simple ideas)

Richmond F. Snyder Fund

(Chris Nex, James Annett, Matthew Foulkes)

Heisenberg's equation for interacting systems and its generalization

A simple Ising-like model

Does quantum mechanics make sense without a Hamiltonian?
Heisenberg's Equation (HE)

\[-i\hbar \frac{\partial}{\partial t} A(t) = [H, A(t)]\]

Where $H$ is the Hamiltonian and $A(t)$ is some observable.

Non-singular in macroscopic limit and local – unlike the Schrödinger equation.

Find examples of interacting $H$ and $A$ for which HE can be solved analytically.

No known orthogonal polynomial systems related to interacting HE.

Commutator makes HE complicated even if $H$ is simple.
Generalize Heisenberg's Equation

\[-i \hbar \frac{\partial}{\partial t} A(t) = \mathcal{L} A(t)\]

where $\mathcal{L}$ is a super-operator – linear operator on operators $\mathcal{L}$ often called Quantum Liouvillian

Properties of $\mathcal{L}$:

- Hermitian – unitary evolution of $A(t)$ ($tr\{A^\dagger A\}$ invariant)

- Acts like a time derivative $[\mathcal{L} A B] = [\mathcal{L} A] B + A [\mathcal{L} B]$

- Conserved quantities are stationary $\mathcal{L} S = 0$ where $S$ is some conserved quantity like energy or angular momentum

- Time-reversal symmetry: $(\mathcal{L} A)^\dagger = \mathcal{L} A^\dagger$
A Quantum Ising Model

Simplification of spin-1/2 Heisenberg model (Excitons?)

Sites – on a line, lattice, or some other structure

Neighbors – pairs of interacting sites

Site Operators – identity $i$ is no change, $a$ is a change (flip spin)

Total Operator – linear combination of products $o_1 o_2 \ldots o_N$

Operators of different sites commute

Rule – definition of $\mathcal{L} = \sum \mathcal{L}_{j,k}$ over pairs of neighbors $j$ and $k$

$\mathcal{L}_{j,k} o_{12} \ldots o_N = o_{12} \ldots [\mathcal{L}_{j,k} o_{jk}] \ldots o_N$

$\mathcal{L}_{j,k} i_i = 0, \mathcal{L}_{j,k} i_a = \mathcal{L}_{j,k} a_i = a_{aj}, \mathcal{L}_{j,k} a_i a_i = i_a + a_{ai}$
Quantum Ising Model Binding Energy

- Anti-Ferromagnetic (6 levels)
- Ferromagnetic (5 levels)
- Spin waves or spin diffusion
Classical Systems without Hamiltonians

Ball rolling on a plane – equations of motion not determined by the energy

Non-holonomic constraints

Mike Godfrey has studied a quantum version of the ball rolling on a plane
Is there a Hamiltonian?

Eigenvalues of $L$ are transition energies and eigen-operators are stationary transitions $A(t) = \exp\{i\omega t\} \ A(0)$

$L$ is the commutator of a Hamiltonian if Conservative in the sense that there are loops of transitions for which the product around a loop is the identity

Quantum Ising model violates this

Why should physical systems be conservative in this sense?

Wouldn't it be better to have equations of motion for observables directly, rather than trying to define states?
Summary

Heisenberg's equation is good for interacting systems.

Can generalize to equation of motion for observables.

Quantum Ising model is a simple interacting model without a Hamiltonian.

Perhaps some quantum systems don't have Hamiltonians.