Quantum critical itinerant ferromagnetism

Gareth Conduit

Belitz et al., PRL 2005

Gareth Conduit
Cavendish Laboratory
University of Cambridge
Two types of ferromagnetism

- **Localised ferromagnetism**: moments localised in real space
  - Ferromagnet
  - Antiferromagnet

- **Itinerant ferromagnetism**: moments localised in reciprocal space

**Not magnetised**

**Partially magnetised**
Stoner model for itinerant ferromagnetism

- Repulsive interaction energy $U = gn \uparrow \downarrow$
- A $\Delta E$ shift in the Fermi surface causes:
  
  (i) Kinetic energy increase of $\frac{1}{2}v\Delta E^2$
  
  (ii) Reduction of repulsion of $-\frac{1}{2}gv^2\Delta E^2$

- Total energy shift is $\frac{1}{2}v\Delta E^2(1-gv)$ so a ferromagnetic transition occurs if $gv > 1$
The Stoner model has a *second order* transition of e.g. iron and nickel which is characterised by:

- Smoothly varying magnetisation
- A divergence of length-scales (peaked heat capacity and susceptibility)
At low temperature and high pressure $\text{ZrZn}_2$ has a first order transition.
Breakdown of Stoner criterion -- MnSi

- MnSi also displays a first order phase transition
• At low temperature $\text{UGe}_2$, $\text{ZrZn}_2$, $\text{MnSi}$, and others are first order

• Here I describe two projects that investigate the first order behaviour:

(i) Probe the first order transition without the lattice

(ii) Motivated by the FFLO phase, apply the formalism to search for a putative textured phase
To describe the transition we expand the total energy in the magnetisation

\[ F = r m^2 + um^4 + vm^6 \]

- Increase \( r \), second order transition
- Change sign of \( r \), first order transition
- Change sign of \( u \)
System free energy $F = -k_B T \ln Z$ is found via the partition function

$$Z = \sum_{\{m(x,t)\}} \exp\left(-\frac{E[m(x,t)]}{k_B T}\right)$$

the summation includes spatial and temporal fluctuations of the magnetisation

Using only the average magnetisation:

$$m(x,t) = \bar{m}$$

gives

$$F \propto (1 - g \nu) \bar{m}^2$$

i.e. the Stoner criterion
Consequences of fluctuations

\[
Z = \sum_{\{m(x,t)\}} \exp\left(- E[m] / k_B T\right)
\]

- We expand the energy to second order in fluctuations: \[ m \rightarrow \bar{m} + \phi \]

\[
Z = \sum_{\{\phi(x,t)\}} \exp\left(\frac{-1}{k_B T} \left( E[\bar{m}] + \phi^2 (x, t) E''[\bar{m}] \right) \right)
\]

- Larkin & Pikin [Zh. Eksp. Teor. Fiz. 1969] included auxiliary fluctuations of the lattice which introduced a negative magnetisation term, driving the transition first order

\[
\begin{align*}
= \int \exp\left(\frac{-1}{k_B T} \left( r m^2 + u m^4 + a \phi^2 \right) \right) d\phi \\
= \int \exp\left(\frac{-1}{k_B T} \left( r m^2 + (u-a) m^4 + a (\phi \pm m^2)^2 \right) \right) d\phi \\
\sim \exp\left(\frac{-1}{k_B T} \left( r m^2 + (u-a) m^4 \right) \right)
\end{align*}
\]

- Similarly here considering the soft transverse magnetic fluctuations drives the transition of the longitudinal first order
The results give the following phase diagram

Uhlarz et al., PRL 2004
QMC calculations

- Fluctuation corrections are not exact and higher order terms might destroy the first order phase transition
- Exact (except for the fixed node approximation) Quantum Monte Carlo calculations confirmed a first order phase transition
Summary of uniform work

- Consideration of corrections due to fluctuations in magnetisation and density revealed a first order phase transition
- Nature of transition confirmed by Quantum Monte Carlo calculations
- Motivated by Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) and experiment now examine a putative textured ferromagnetic phase
- Textured phase already considered in terms of a consequence of the lattice
\( \text{NbFe}_2 \) displays antiferromagnetic order where it is expected to be ferromagnetic -- could this be a textured ferromagnetic phase?

Crook & Cywinski, JMMM 1995
Sr$_3$Ru$_2$O$_7$

- Resistance anomaly

Scattering of $M$ fluctuations

Scattering off $M$ crystal?

- Consistent with a new crystalline phase

Grigera et al., Science 2004
In analogy to FFLO\textsuperscript{1}, we can look at a Ginzburg-Landau analysis.

\[ F = rm^2 + um^4 + vm^6 + \frac{2}{3} u (\nabla m)^2 + \frac{3}{5} v (\nabla^2 m)^2 - hm \]

The first order transition is accompanied by a textured phase.

Consider the lowest order term in a Ginzburg-Landau expansion, which is a function of the wave vector \( q \) of the textured phase.

\[ F = \sum_q \alpha_q m_q^2 \]

When \( \alpha_q > 0 \) zero magnetisation is favourable, if \( \alpha_q < 0 \) a textured phase preempts the first order ferromagnetic transition.

\textsuperscript{1}Saint-James \textit{et al.} 1969, \textsuperscript{2}Buzdin & Kachkachi 1996
The phase diagram of the uniform system is
Analytical results

- Textured phase preempted transition with $q=0.1k_F$
QMC results

- Textured phase preempted transition and penetrated uniform phase
Summary

- Developed a field theoretic construction to understand the first order transition
- Ginzburg-Landau analysis of spin spiral textured ferromagnetic phase
- Confirmed the phases with QMC calculations
- Acknowledgements: Ben Simons & Andrew Green, EPSRC