PROBLEMS

Lectures 10-11

1. Prove the Anderson's theorem: elastic scattering does not change critical temperature of a superconductor.

2. Consider a superconductor with a small concentration of magnetic impurities. Interaction of an electron with these impurities is described by Hamiltonian with a random Zeemann field $\mathbf{h}(\mathbf{r})$:

   $$H_S = \int d\mathbf{r} \psi^+_\alpha(\mathbf{r})\mathbf{h}(\mathbf{r})\sigma_{\alpha\beta}\psi_\beta(\mathbf{r}), \quad \langle h_i(\mathbf{r})h_j(\mathbf{r}')\rangle = \frac{\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')}{3\pi\nu\tau_S}$$

   Find $T_c(\tau_S)$

3. Consider a layered superconductor. Introduce the order parameter $\psi_i(\rho)$ in each layer i. Assume that inralayer interction is described by extra term in free energy:

   $$F_{itra} = t\sum_i d\mathbf{r}[\psi_i(\rho)\psi^*_{i+1}(\rho) + c.c.]$$

   Find critical field in direction parallel to the layes

4. Consider a wide Josephson junction in magnetic field **H** parallel to plane of this junction. The phase difference $\phi(x,t)$ across the junction is a function of coordinate x along the junction in direction perpendicular to magnetic field. Derive equation $\phi(x,t)$ obeys to (Prange-Ferrell equation). Find out how critical current depends on magnetic field. Consider waves in broad junction.

5. Consider a thin film with a current j in it. Find maximal value of $j = j_c$ at which the film remains superconducting.

6. Little-Parks effect. Consider a hollow thin-well cylinder, made of a superconducting material, in magnetic field parallel to the cylinder's axis. Find dependence of crytical temperature $T_c(\Phi)$ on flux Φ.