PROBLEMS

Lecture 3

1. A crystal has a soft optic mode with the spectrum

$$\omega^2(k) = \omega^2(0) + sk^2$$

   How does the coefficient of thermal expansion depend on temperature T for the temperature range $T << \omega_D$?

2. Find imaginary part of dielectric function of a crystal with optic phonons near infra-red adsorption line. Restore the whole dielectric function at $\mathbf{q} = 0$ and make a guess about its space dispersion.

3. Find fluctuation correction to the Curie-Weiss law in uni-axial ferroelectric near the transition.

4. Uni-axial crystal with pieso-effect in its high-temperature phase has a ferroelectric transition. Show that specific heat in such a crystals remains finite. Find derivative $(\partial C/\partial T)_p$

5. Solution of isotopes of liquid helium $He^3_x He^4_{1-x}$ has transition into superfluid phase at all concentrations x. In pure He^4 $(x = 0)$ this transition occurs at $T = T_\lambda = 2.2K$, while in pure He^3 $(x = 1)$ the critical temperature tends to zero. The order parameter ψ corresponds to superfluidity, carried mostly by atoms of He^4. Inter-atomic interaction of He^4 atoms is formed by the sum of initial repulsion and attraction due to mediation by He^3 atoms. As a result, the

coefficient at the front of ψ^4 term in the Landau expansion changes its sign whilst x varies from $x = O$ to $x = 1$. That point at the line $T_\lambda(x)$, where this change occurs, is called the tricritical point. Assuming that the coefficient at the front of ψ^6 is positive, draw the phase diagram for solution of helium isotopes.

6. Consider critical theory at a tricritical point.

7. The frequencies of the phonon modes $\omega(\mathbf{q})$ have either minima or maxima at the center of Brilluin zone. Consider a displacement type phase transition in a crystal near that value of pressure and temperature at which the minimum turns into a maximum. Write Landau functional for free energy of these crystal and draw the phase diagram near this critical point (Lifshits point).