PROBLEMS

Introduction

1. A free electron has a very bad affinity to helium atoms. This leads to to a very high potential energy of an electron while it is in liquid helium. So, the electron forms a "bubble", an empty cavity, which contains the electron. The wave function of the electron vanishes at the boundary of this cavity. Estimate the radius R of the "bubble" at the ground state of electron. Discuss the changes of the shape of the "bubble" if the electron is at its first excited state.

2. Consider following "1D model of a quasicrystal" : The initial set of points are the vertices of a square lattice in two dimensions with the lattice constant a. A straight line, called "the axis" goes through the origin under angle α with $\tan \alpha$ being an irrational number. All vertices of initial lattice, which belongs to the stripe of width w and parallel to the axis, projected on the axes, form the a one-dimensional array of the points (See Fig 1).
   Show that:
   The form-factor $S(k) = \sum_i e^{ikx_i}$ with x_i being the points of the array is a sum of δ-functions in k; This set of δ-peaks has no periodicity Discuss the shape of diffractogrammes of "1D model of a quasicrystal". the character of vibration modes, the spectrum of electrons in "quasicrystals" and magnetism in "quasicrystals".

3. Consider vibrations in a two-dimensional crystal. Show

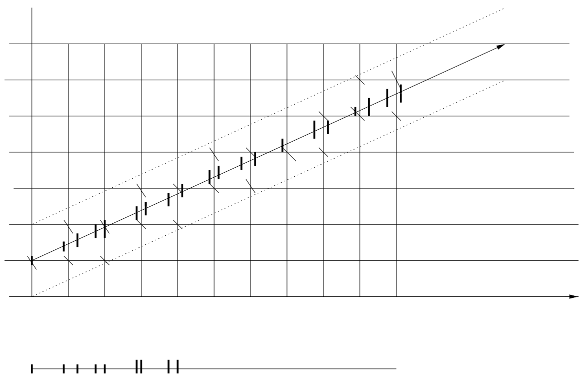

Figure 1: One-dimensional model of quasi-crystal

that the mean square of displacements diverges at long distances.

4. The lattice of Abrikosov vortices in a superconductor is characterised by the density of the order parameter, which is periodic in x, y plane and invariant with respect to translation along direction z of magnetic field. Consider thermal fluctuations of deformations $\mathbf{u}$ of this lattice and make a conclusion about its stability.