NATURAL SCIENCES TRIPOS Part II

Wednesday 18 January 2012

THEORETICAL PHYSICS I

Answers

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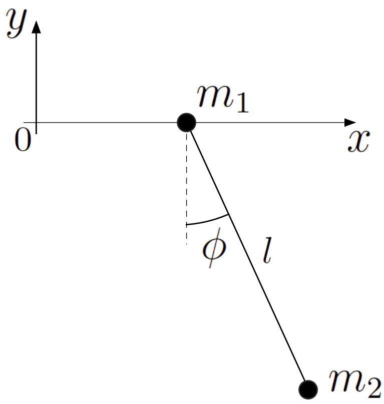

(a) The x position of mass m_2 is given by

$$\begin{aligned} x' &= x + l\sin\phi \\ \dot{x}' &= \dot{x} + l\dot{\phi}\cos\phi \\ \dot{x}'^2 &= \dot{x}^2 + 2l\dot{x}\dot{\phi}\cos\phi + l^2\dot{\phi}^2\cos^2\phi \end{aligned}$$

The y position of mass m_2 is given by

$$\begin{aligned} y' &= -l\cos\phi \\ \dot{y}' &= l\dot{\phi}\sin\phi \\ \dot{y}'^2 &= l^2\dot{\phi}^2\sin^2\phi \end{aligned}$$

The total kinetic energy is therefore

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\left(\dot{x}^2 + 2l\dot{x}\dot{\phi}\cos\phi + l^2\dot{\phi}^2\right)$$

and the potential energy is

$$V = -m_2 gl\cos\phi$$

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Hence the Lagrangian is given by

$$L = T - V = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\left(\dot{x}^2 + 2l\dot{x}\dot{\phi}\cos\phi + l^2\dot{\phi}^2\right) + m_2 g l\cos\phi$$

(b) The canonical momentum conjugate to x is

$$p_x = \frac{\partial L}{\partial \dot{x}} = (m_1 + m_2)\dot{x} + m_2 l\dot{\phi}\cos\phi$$

Using the associated Euler-Lagrange equation

$$\dot{p_x} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} = 0$$

so p_x is a conserved quantity.

The canonical momentum conjugate to ϕ is

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m_2 l^2\dot{\phi} + 2l\dot{x}\cos\phi$$

Using the associated Euler-Lagrange equation

$$\dot{p_\phi} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\phi}} = \frac{\partial L}{\partial \phi} = -m_2 l\dot{x}\dot{\phi}\sin\phi - m_2 g l\sin\phi$$

so p_ϕ is not a conserved quantity.

(c) Using conservation of p_x

$$0 = (m_1 + m_2)\dot{x} + m_2 l\dot{\phi}\cos\phi$$

Integrating this we find

$$\lambda = (m_1 + m_2)x + m_2 l\sin\phi$$

where λ is a constant. Using the expression for x' above we therefore find

$$\sin\phi = \frac{(m_1 + m_2)x' - \lambda}{m_1 l}$$

Re-arranging the expression for y' we have

$$\cos\phi = \frac{-y'}{l}$$

Squaring and summing these we find

$$\left(\frac{(m_1 + m_2)x' - \lambda}{m_1 l}\right)^2 + \left(\frac{y'}{l}\right)^2 = 1$$

which, as required, is an equation for an ellipse.

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(d) Energy is conserved so

$$E = T + V = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\left(\dot{x}^2 + 2l\dot{x}\dot{\phi}\cos\phi + l^2\dot{\phi}^2\right) - m_2gl\cos\phi \qquad (1)$$

Substituting for $\dot{x}$ from part (c) we find

$$E = \frac{1}{2}m_2l^2\dot{\phi}^2\left(\frac{m_1 + m_2\sin^2\phi}{m_1+m_2}\right) - m_2gl\cos\phi$$

Re-arranging this expression for $\dot{\phi}$ we find

$$l\frac{\mathrm{d}\phi}{\mathrm{d}t} = \sqrt{\frac{E + m_2gl\cos\phi}{\frac{1}{2}m_2l}.\frac{m_2+m_1}{m_1+m_2\sin^2\phi}}$$

Hence, integrating, we find

$$t = l\sqrt{\frac{m_2}{2(m_2+m_1)}}\int_{\phi_1}^{\phi_2}\mathrm{d}\phi\sqrt{\frac{m_1+m_2\sin^2\phi}{E+m_2gl\cos\phi}}$$

2 (a) The transformation

$$\begin{aligned} x &= X + \alpha_1X^2 + 2\alpha_2XP + \alpha_3P^2 \\ p &= P + \beta_1X^2 + 2\beta_2XP + \beta_3P^2 \end{aligned}$$

will be canonical if the Poisson bracket

$$\begin{aligned} \{x,p\}_{X,P} &= \frac{\partial x}{\partial X}\frac{\partial p}{\partial P} - \frac{\partial x}{\partial P}\frac{\partial p}{\partial X} = 1 \\ &= (1 + 2\alpha_1X + 2\alpha_2P)(1 + 2\beta_2X = 2\beta_3P) + \textit{higher order terms} \\ &= 1 + 2(\alpha_1+\beta_2)X + 2(\alpha_2+\beta_3)P + \textit{higher order terms} \end{aligned}$$

Therefore we must have $\beta_2 = -\alpha_1$ and $\beta_3 = -\alpha_2$.

(b)

$$\begin{aligned} K(X,P) &= \frac{(X + \beta_1X^2 - 2\alpha_1XP + \alpha_2P^2)^2}{2m} + \frac{1}{2}m\omega^2(X + \alpha_1X^2 + 2\alpha_2XP + \alpha_3P^2)^2 \\ &+ \lambda(X + \alpha_1X^2 + 2\alpha_2XP + \alpha_3P^2)^3 \\ &= \frac{P^2}{2m} + \frac{1}{2}m\omega^2X^2 + X^3(\alpha_1m\omega^2 + \lambda) + P^3(-\frac{\alpha_2}{m}) \\ &+ XP^2(-\frac{2\alpha_1}{m} + \alpha_3m\omega^2) + PX^2(\frac{\beta_1}{m} + 2\alpha_2m\omega^2) \end{aligned}$$

Hence we must have

$$\begin{aligned} \alpha_1 &= -\frac{\lambda}{m\omega^2},\ \alpha_2 = 0,\ \alpha_3 = -\frac{2}{m}\frac{\lambda}{m\omega^2} \\ \beta_1 &= 0,\ \beta_2 = \frac{\lambda}{m\omega^2},\ \beta_3 = 0 \end{aligned}$$

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hence

$$
\begin{aligned}
x &= X - \frac{\lambda}{m\omega}(X^2 + \frac{2}{m}\frac{P^2}{m\omega^2}) \\
p &= P + \frac{2\lambda}{m\omega^2} XP
\end{aligned}
$$

(c)

$$
\begin{aligned}
\frac{\mathrm{dX}}{\mathrm{d}t} &= \frac{\partial K}{\partial P} = \frac{P}{m} \\
\frac{\mathrm{dP}}{\mathrm{d}t} &= -\frac{\partial K}{\partial X} = -m\omega^2 X
\end{aligned}
$$

Hence

$$
\begin{aligned}
P &= A\cos(\omega t + \phi) \\
X &= \frac{A}{m\omega}\sin(\omega t + \phi)
\end{aligned}
$$

(d) Using definitions for X and P from part (c)

$$
\begin{aligned}
x &= X - \frac{\lambda}{m\omega}(X^2 + \frac{2}{m}\frac{P^2}{m\omega^2}) \\
p &= P + \frac{2\lambda}{m\omega^2} XP
\end{aligned}
$$

Substituting for X and P from above we find that x and p now have components oscillating at 2ω .

3 The Euler-Lagrange equation is:

$$
\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{v}}\right) = \frac{\partial L}{\partial \boldsymbol{x}} = e\,\nabla(\boldsymbol{v}\cdot\boldsymbol{A}) - e\,\nabla\phi = e(\boldsymbol{v}\cdot\nabla)\boldsymbol{A} + e[\boldsymbol{v}\times(\nabla\times\boldsymbol{A})] - e\,\nabla\phi
$$

Then using

$$
\frac{d\boldsymbol{A}}{dt} = \frac{\partial \boldsymbol{A}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{A},
$$

we obtain the equation of motion

$$
\frac{d(m\boldsymbol{v})}{dt} = -e\frac{\partial \boldsymbol{A}}{\partial t} - e\,\nabla\phi + e[\boldsymbol{v}\times(\nabla\times\boldsymbol{A})] = e\boldsymbol{E} + e\,\boldsymbol{v}\times\boldsymbol{B},
$$

as expected.

The equation of motion of a physical particle is determined by the physically observable fields $\boldsymbol{E}$ and $\boldsymbol{B}$. However the potentials ϕ and $\boldsymbol{A}$ which determine these

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fields and contribute to the Lagrangian function are not unique. If we add the gradient of an arbitrary scalar function $f(\boldsymbol{x}, t)$ to the vector potential $\boldsymbol{A}$, i.e.

$$A'_i = A_i + \frac{\partial f}{\partial x_i},$$

the magnetic flux density $\boldsymbol{B}$ will not change, because $\text{curl}\, \nabla f \equiv 0$. To have the electric field unchanged as well, we must simultaneously subtract the time-derivative of f from the scalar potential:

$$\phi' = \phi - \frac{\partial f}{\partial t}.$$

The invariance of all electromagnetic processes with respect to the above transformation of the potentials by an arbitrary function f is called *gauge invariance.*

(a) Using the given expressions for $\boldsymbol{A}$ and ϕ, the Lagrangian becomes

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - e\lambda z^2 + e\mu r^2\dot{\theta}.$$

The E-L equation corresponding to coordinate r is then:

$$\frac{d(m\dot{r})}{dt} = mr\dot{\theta}^2 + 2e\mu r\dot{\theta} = r\dot{\theta}[m\dot{\theta} + 2e\mu],$$

and for θ:

$$\frac{d}{dt}\left[mr^2\dot{\theta} + e\mu r^2\right] = 0,$$

and for z:

$$\frac{d(m\dot{z})}{dt} = -2e\lambda z \quad \Rightarrow \ddot{z} + \kappa^2 z = 0, \quad \kappa^2 = 2e\lambda/m.$$

(b) Because L does not depend explicitly on t, the total energy as given by the Hamiltonian of the system is conserved, $dH/dt = 0$. In general

$$H = \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L.$$

with here $q_i = (r, \theta, z)$. Hence the total energy of the particle

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) + e\lambda z^2$$

is a constant of the motion.

(c) From the E-L equation for $\theta(t)$ above, we have immediately that

$$J = mr^2\dot{\theta} + e\mu r^2 = r^2[m\dot{\theta} + e\mu]$$

is another constant of the motion (generalised angular momentum).

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(d) If $r = R$, then $\dot{r} = \ddot{r} = 0$ and from the above equation for r we obtain $\dot{\theta} = -2e\mu/m$ =constant, i.e. circular motion around the z axis with constant angular velocity. In terms of the z coordinate, the particle undergoes simple harmonic motion, $z(t) = a \sin \kappa t + b \cos \kappa t$, with average value $z = 0$.

(e) The time for one rotation around the z axis is $T = m/(2e\mu)$. Suppose $\kappa T = 2\pi n$, i.e. $\lambda = (2e\mu^2/m)n^2$. Then the period of rotation around the z axis is an integer multiple of the simple harmonic oscillation in the z direction, i.e. the two motions are *in phase.*

4 We start from the Euler-Lagrange equations for ϕ:

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \frac{\partial}{\partial x^\mu} \left(\frac{\partial \mathcal{L}}{\partial(\partial \varphi / \partial x^\mu)} \right) \equiv \partial_\mu \frac{\partial \mathcal{L}}{\partial[\partial_\mu \varphi]},$$

which immediately gives

$$-m^2 \varphi = \partial^\mu \partial_\mu \varphi \quad \Rightarrow \quad \partial^\mu \partial_\mu \varphi + m^2 \varphi = 0 \quad \Rightarrow \quad \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi + m^2 \varphi = 0.$$

The Fourier transformed field $\tilde{\varphi}(\boldsymbol{k}, t)$ is defined by

$$\varphi(\boldsymbol{x}, t) = \int d^3\boldsymbol{k}\, \tilde{\varphi}(\boldsymbol{k}, t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}.$$

Substituting into the equation of motion gives

$$\frac{\partial^2 \tilde{\varphi}}{\partial t^2} + (m^2 + k^2)\tilde{\varphi} = 0.$$

Define $\omega = +\sqrt{m^2 + k^2}$. Then

$$\tilde{\varphi}(\boldsymbol{k}, t) = a(\boldsymbol{k})e^{-i\omega t} + b(\boldsymbol{k})e^{i\omega t}.$$

The reality of φ requires $b(\boldsymbol{k}) = a^*(-\boldsymbol{k})$. With

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \varphi_1)(\partial_\mu \varphi_1) - \frac{1}{2}m^2\varphi_1^2 + \frac{1}{2}(\partial^\mu \varphi_2)(\partial_\mu \varphi_2) - \frac{1}{2}m^2\varphi_2^2 + g\varphi_1\varphi_2$$

we now have two equations of motion, corresponding to the E-L equations corresponding to φ_1 and φ_2 respectively:

$$\partial^\mu \partial_\mu \varphi_1 + m^2 \varphi_1 - g\varphi_2 = 0, \quad \partial^\mu \partial_\mu \varphi_2 + m^2 \varphi_2 - g\varphi_1 = 0.$$

Now define two linear combinations of the φ_i fields: $\varphi_\pm = \varphi_1 \pm \varphi_2$. By adding and subtracting the above two equations of motion, we obtain two corresponding equations for the $\varphi_\pm$:

$$\partial^\mu \partial_\mu \varphi_+ + (m^2 - g)\varphi_+ = 0, \quad \partial^\mu \partial_\mu \varphi_- + (m^2 + g)\varphi_- = 0.$$

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Note that these are now decoupled, and so we can solve them as we do for the normal (massive) Klein-Gordon field. Thus

$$\varphi_\pm(\boldsymbol{x}, t) = \int d^3\boldsymbol{k}\, \left[a_\pm(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_\pm t} + a^*_\pm(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}+i\omega_\pm t}\right]$$

where the frequencies $\omega_\pm$ are given by

$$\omega_\pm^2 = k^2 + m^2 \mp g > 0.$$

It is now straightforward to recover the solutions for φ_1 and φ_2:

$$\varphi_1(\boldsymbol{x}, t) = \int d^3\boldsymbol{k}\, N(\boldsymbol{k})\frac{1}{2}\left[a_+(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_+ t} + a_-(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_- t} + \text{c.c.}\right],$$

$$\varphi_2(\boldsymbol{x}, t) = \int d^3\boldsymbol{k}\, N(\boldsymbol{k})\frac{1}{2}\left[a_+(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_+ t} - a_-(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_- t} + \text{c.c.}\right].$$

To solve for the fields for the given boundary conditions at $t = 0$, we first transform these into boundary conditions on $\varphi_\pm$:

$$\varphi_+(\boldsymbol{x}, 0) = \varphi_-(\boldsymbol{x}, 0) = A\sin(\boldsymbol{q}\cdot\boldsymbol{x}), \qquad \dot\varphi_+(\boldsymbol{x}, 0) = \dot\varphi_-(\boldsymbol{x}, 0) = 0.$$

This suggests looking for real solutions of the form:

$$\varphi_\pm = \sin(\boldsymbol{q}\cdot\boldsymbol{x})[\alpha\cos(\omega_\pm t) + \beta\sin(\omega_\pm t)]$$

where now $\omega_\pm = \sqrt{q^2 + m^2 \mp g}$. Evidently the boundary conditions are satisfied for $\alpha = A$ and $\beta = 0$. Hence

$$\varphi_1(\boldsymbol{x}, t) = \frac{A}{2}\sin(\boldsymbol{q}\cdot\boldsymbol{x})[\cos(\omega_+ t) + \cos(\omega_- t)],$$

$$\varphi_2(\boldsymbol{x}, t) = \frac{A}{2}\sin(\boldsymbol{q}\cdot\boldsymbol{x})[\cos(\omega_+ t) - \cos(\omega_- t)].$$

Note that in the limit $g \to 0$, $\varphi_2 \to 0$.

5 The relationship between symmetries and conserved quantities, and the effects of symmetry breaking, are amongst the most important in theoretical physics. *Noether's theorem* is an important general result, which tells us that there is a *conserved current* associated with every continuous *symmetry* of the Lagrangian, i.e. with symmetry under a transformation of the form $\varphi \to \varphi + \delta\varphi$ where $\delta\varphi$ is infinitesimal. Symmetry means that $\mathcal{L}$ does not change under this field transformation.

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi}\delta\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\delta\varphi' + \frac{\partial\mathcal{L}}{\partial\dot\varphi}\delta\dot\varphi = 0$$

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where

$$\delta\varphi' = \delta\left(\frac{\partial\varphi}{\partial x}\right) = \frac{\partial}{\partial x}\delta\varphi$$

$$\delta\dot{\varphi} = \delta\left(\frac{\partial\varphi}{\partial t}\right) = \frac{\partial}{\partial t}\delta\varphi$$

(easily generalized to 3 spatial dimensions).

The Euler-Lagrange equation of motion

$$\frac{\partial\mathcal{L}}{\partial\varphi} - \frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\right) - \frac{\partial}{\partial t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\right) = 0$$

then implies that

$$\delta\mathcal{L} = \frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\right)\delta\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\frac{\partial}{\partial x}(\delta\varphi) + \frac{\partial}{\partial t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\right)\delta\varphi + \frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\frac{\partial}{\partial t}(\delta\varphi) = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\delta\varphi\right) + \frac{\partial}{\partial t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\delta\varphi\right) = 0$$

Comparing with the conservation/continuity equation (in 1 spatial dimension)

$$\frac{\partial}{\partial x}(J_x) + \frac{\partial\rho}{\partial t} = 0$$

we see that the conserved density and current are (proportional to)

$$\rho = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}}\delta\varphi\ , \qquad J_x = \frac{\partial\mathcal{L}}{\partial\varphi'}\delta\varphi$$

In more than 1 spatial dimension

$$J_x = \frac{\partial\mathcal{L}}{\partial(\partial\varphi/\partial x)}\delta\varphi\ ,\quad J_y = \frac{\partial\mathcal{L}}{\partial(\partial\varphi/\partial y)}\delta\varphi\ ,\ldots$$

and hence in covariant notation

$$J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi.$$

The Lagrangian density for a scalar field in n space-time dimensions, $\varphi(t, x_1, x_2, ..., x_{n-1})$, is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \lambda\varphi^4.$$

We use the E-L equation in the form

$$\frac{\partial\mathcal{L}}{\partial\varphi} = \partial_\mu\frac{\partial\mathcal{L}}{\partial[\partial_\mu\varphi]},$$

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to immediately obtain the equation of motion

$$\partial^\mu \partial_\mu \varphi + 4\lambda\phi^3 = 0.$$

A current J^μ is defined by

$$J^\mu = (\varphi + x^\nu \partial_\nu \varphi)\partial^\mu \varphi - x^\mu \mathcal{L}.$$

Splitting this into two pieces, we first have

$$\partial_\mu \left[(\varphi + x^\nu \partial_\nu \varphi)(\partial^\mu \varphi)\right] = 2(\partial_\mu \varphi)(\partial^\mu \varphi) + x^\nu(\partial_\mu \partial_\nu \varphi)(\partial^\mu \varphi) + (\varphi + x^\nu \partial_\nu \varphi)(-4\lambda\varphi^3),$$

where we have used the equation of motion in the last term. Also

$$\partial_\mu(x^\mu \mathcal{L}) = n\mathcal{L} - x^\mu \left[(\partial_\mu \partial_\nu \varphi)(\partial^\mu \varphi) - 4\lambda\varphi^3(\partial_\mu \varphi)\right].$$

Subtracting these and cancelling terms then gives

$$\partial_\mu J^\mu = \left(2 - \frac{n}{2}\right)(\partial_\mu \varphi)(\partial^\mu \varphi) + \lambda\varphi^4(-4+n) = (4-n)\mathcal{L}.$$

For $n = 4$ the right-hand side vanishes and the current is conserved.

6 (a) Taking the F.T in x we have

$$\left(k^2 + 2\alpha\frac{\partial}{\partial t} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(k, t-t') = \delta(t-t')$$

This can be solved either using the 'jump' condition method from 1B maths or by taking a further F.T in t and using contour integration. The equation is identical in form to the damped harmonic oscillator for which the full solution is given in the lecture notes on pages 52,52, question 4 in the examples and Q6 in the 2010 paper.

$$\begin{aligned} G(k, t-t') &= 0 \qquad t < t' \\ &= \frac{1}{\sqrt{\alpha^2 - k^2/c^2}} e^{-\alpha c^2(t-t')} \sinh \sqrt{\alpha^2 c^4 - k^2 c^2}(t-t') \end{aligned}$$

(b) From the inverse Fourier transform we have

$$G(x, x'; t, t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik(x-x')} G(k, t-t') dk$$

and with $s(x,t) = \cos(px)\delta(t-t_0)$

$$\begin{aligned} T(x,t) &= \int_{-\infty}^{t^+} \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}x' s(x', t') G(x, x'; t, t') \\ &= \int_{-\infty}^{\infty} \mathrm{d}x' \cos(px').\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k e^{ik(x-x')} G(k, t-t_0) \\ &= \int_{-\infty}^{\infty} \mathrm{d}k \frac{1}{2.2\pi}\int_{-\infty}^{\infty} \mathrm{d}x' \left[e^{i(k-p)(x-x')} e^{ipx} G(k, t-t_0) + e^{i(k+p)(x-x')} e^{-ipx} G(k, t-t_0)\right] \\ &= \frac{1}{2}\left(e^{ipx} G(p, t-t_0) + e^{-ipx} G(-p, t-t_0)\right) = \cos(px) G(p, t-t_0) \end{aligned}$$

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Hence, for $\alpha c > p$ the oscillating temperature distribution decays without oscillating and for $\alpha c < p$ it executes damped harmonic motion.

END OF PAPER