

Contents

1 Wave mechanics and the Schrödinger equation	1
1.1 Historical foundations of quantum physics	1
1.1.1 Black-body radiation	1
1.1.2 Photoelectric effect	3
1.1.3 Compton Scattering	3
1.1.4 Atomic spectra	4
1.2 Wave mechanics	6
1.2.1 Maxwell's wave equation	6
1.2.2 Schrödinger's equation	8
1.2.3 Time-independent Schrödinger equation	8
1.2.4 Particle flux and conservation of probability	9
2 Quantum mechanics in one dimension	10
2.1 Wave mechanics of unbound particles	10
2.1.1 Free particle	10
2.1.2 Potential step	12
2.1.3 Potential barrier	13
2.1.4 The rectangular potential well	15
2.2 Wave mechanics of bound particles	15
2.2.1 The rectangular potential well (continued)	15
2.2.2 The δ -function potential well	16
2.2.3 INFO: The δ -function model of a crystal	17
3 Operator methods in quantum mechanics	19
3.1 Operators	20
3.1.1 Time-evolution operator	21
3.1.2 Uncertainty principle for non-commuting operators	22
3.1.3 Time-evolution of expectation values	23
3.2 Symmetry in quantum mechanics	24
3.2.1 Observables as generators of transformations	24
3.2.2 Consequences of symmetries: multiplets	26
3.3 The Heisenberg Picture	27
3.4 Quantum harmonic oscillator	27
3.5 Postulates of quantum theory	31
4 Quantum mechanics in more than one-dimension	33
4.1 Rigid diatomic molecule	33
4.2 Angular momentum	34
4.2.1 Commutation relations	34
4.2.2 Eigenvalues of angular momentum	34
4.2.3 Representation of the angular momentum states	36
4.3 The central potential	38
4.4 Atomic hydrogen	39

5 Motion in a magnetic field	44
5.1 Classical mechanics of a particle in a field	44
5.2 Quantum mechanics of a particle in a field	46
5.3 Atomic hydrogen: Normal Zeeman effect	47
5.4 Gauge invariance and the Aharonov-Bohm effect	48
5.5 Free electrons in a magnetic field: Landau levels	50
6 Spin	53
6.1 Spinors, spin operators, Pauli matrices	54
6.2 Relating the spinor to the spin direction	55
6.3 Spin precession in a magnetic field	56
6.3.1 Paramagnetic Resonance	56
6.4 Addition of angular momenta	58
6.4.1 Addition of two spin 1/2 degrees of freedom	59
6.4.2 Addition of angular momentum and spin	60
6.4.3 Addition of two angular momenta $J = 1$	61
7 Approximation methods for stationary states	62
7.1 Time-independent perturbation theory	62
7.1.1 The Perturbation Series	63
7.1.2 First order perturbation theory	64
7.1.3 Second order perturbation theory	65
7.2 Degenerate perturbation theory	67
7.3 Variational method	69
7.4 Wentzel, Kramers and Brillouin (WKB) method	72
7.4.1 Semi-classical approximation to leading order	73
7.4.2 Next to leading order correction	74
7.4.3 Connection formulae, boundary conditions and quantization rules	75
8 Identical Particles	78
8.1 Quantum statistics	78
8.2 Space and spin wavefunctions	80
8.3 Physical consequences of particle statistics	81
8.4 Ideal quantum gases	83
8.4.1 Non-interacting Fermi gas	84
8.4.2 Ideal Bose gas	86
9 Atomic structure	89
9.1 The “real” hydrogen atom	90
9.1.1 Relativistic correction to the kinetic energy	90
9.1.2 Spin-orbit coupling	91
9.1.3 Darwin term	93
9.1.4 Lamb shift	93
9.1.5 Hyperfine structure	95
9.2 Multi-electron atoms	96
9.2.1 Central field approximation	97
9.3 Coupling schemes	102
9.3.1 LS coupling scheme	102
9.3.2 jj coupling scheme	105
9.4 Atomic spectra	106
9.4.1 Single electron atoms	107
9.4.2 Helium and alkali earths	108
9.4.3 Multi-electron atoms	109

9.5 Zeeman effect	109
9.5.1 Single-electron atoms	109
9.5.2 Multi-electron atoms	110
10 From molecules to solids	112
10.1 The H_2^+ ion	113
10.2 The H_2 Molecule	116
10.3 From molecules to solids	118
10.4 Molecular spectra	122
10.4.1 Molecular rotation	124
10.4.2 Vibrational transitions	125
11 Field theory: from phonons to photons	127
11.1 Quantization of the classical atomic chain	127
11.1.1 Classical chain	127
11.1.2 Quantum chain	130
11.2 Quantum electrodynamics	134
11.2.1 Classical theory of the electromagnetic field	134
11.2.2 Quantum field theory of the electromagnetic field	137
12 Time-dependent perturbation theory	139
12.1 Time-dependent potentials: general formalism	139
12.2 Time-dependent perturbation theory	141
12.3 “Sudden” perturbation	143
12.3.1 Harmonic perturbations: Fermi’s Golden Rule	143
12.3.2 INFO: Harmonic perturbations: second-order transitions	144
13 Radiative transitions	146
13.1 Coupling of matter to the electromagnetic field	146
13.1.1 Spontaneous emission	147
13.1.2 Absorption and stimulated emission	148
13.2 Selection Rules	150
13.3 Lasers	152
13.3.1 Operating principles of a laser	153
13.3.2 Gain mechanism	153
13.4 Driven harmonic oscillator	155
14 Scattering theory	157
14.1 Basics	158
14.2 Method of partial waves	160
14.3 The Born approximation	162
14.4 INFO: Scattering of identical particles	164
14.5 Scattering by an atomic lattice	165
15 Relativistic Quantum Mechanics	167
15.1 Klein-Gordon equation	170
15.2 Dirac Equation	172
15.2.1 Density and Current	174
15.2.2 Relativistic Covariance	174
15.2.3 Angular momentum and spin	175
15.2.4 Parity	176
15.3 Free Particle Solution of the Dirac Equation	176
15.3.1 Klein paradox: anti-particles	177
15.4 Quantization of relativistic fields	180

15.4.1 INFO: Scalar field: Klein-Gordon equation revisited . . .	180
15.4.2 INFO: Charged Scalar Field	182
15.4.3 INFO: Dirac Field	183
15.5 The low energy limit of the Dirac equation	184
16 Problem sets	187
16.1 Problem Set I	187
16.2 Problem Set II	200
16.3 Problem Set III	207
16.4 Problem Set IV	216