Advanced Quantum Physics

Lecture Handout

Preface

Quantum mechanics underpins a variety of broad subject areas within physics and the physical sciences from high energy particle physics, solid state and atomic physics through to chemistry. As such, the subject resides at the core of every physics programme. By building upon the conceptual foundations introduced in the IB Quantum Physics course, the aim of Part II Advanced Quantum Physics is to develop further conceptual insights and technical fluency in the subject.

Although 24 lectures is a long course to prepare(!), it is still insufficient to cover all of the topics that different physicists will describe as "core". For example, some will argue that concepts of quantum computation should already be included as an established component of the core. Simiilarly, in the field of quantum optics, some will say that a detailed knowledge of atomic and molecular spectroscopy is key. In the field of solid state physics, the concept of second quantization in many-body physics is also considered central. In all of these cases, we will be able to touch only the surface of the subject. However, the material included in this course has been chosen to cover the key conceptual foundations that provide access to these more advanced subjects, the majority of which will be covered in subequent optional courses in Part II and Part III.

In the following, we list an approximate "lecture by lecture" synopsis of the different topics treated in this course. Those topics marked by a † will be covered if time permits.

1 **Foundations of quantum physics:** Overview of course structure and organization; brief revision of historical background: from wave mechanics to the Schrödinger equation.

2 **Quantum mechanics in one dimension:** Wave mechanics of unbound particles; potential step; potential barrier and quantum tunneling; bound states; rectangular well; δ-function potential well; †Kronig-Penney model of a crystal.

3 **Operator methods in quantum mechanics:** Operator methods; uncertainty principle for non-commuting operators; Ehrenfest theorem and the time-dependence of operators; symmetry in quantum mechanics; Heisenberg representation; postulates of quantum theory; quantum harmonic oscillator.

4 **Quantum mechanics in more than one dimension:** Rigid diatomic molecule; angular momentum; commutation relations; raising and lowering operators; representation of angular momentum states.

5 **Quantum mechanics in more than one dimension:** Central potential; atomic hydrogen; radial wavefunction.

6 **Motion of charged particle in an electromagnetic field:** Classical mechanics of a particle in a field; quantum mechanics of particle in a field; atomic hydrogen – normal Zeeman effect; †diamagnetic hydrogen and quantum chaos; gauge invariance and the Aharonov-Bohm effect; free electrons in a magnetic field – Landau levels.

7-8 **Quantum mechanical spin:** History and the Stern-Gerlach experiment; spinors, spin operators and Pauli matrices; relating the spinor to spin direction; spin precession in a magnetic field; parametric resonance; addition of angular momenta.

9 **Time-independent perturbation theory:** Perturbation series; first and second order expansion; degenerate perturbation theory; Stark effect; nearly free electron model.

10 **Variational and †WKB method:** Ground state energy and eigenfunctions; application to helium; excited states; †Wentzel-Kramers-Brillouin method.

11 **Identical particles:** Particle indistinguishability and quantum statistics; space and spin wavefunctions; consequences of particle statistics; ideal quantum gases; degeneracy pressure in neutron stars; Bose-Einstein condensation in ultracold atomic gases.

12-13 **Atomic structure:** Relativistic corrections; spin-orbit coupling; Darwin structure; Lamb shift; hyperfine structure; Multi-electron atoms; Helium; Hartree approximation and beyond; Hund's rule; periodic table; coupling schemes LS and jj; atomic spectra; Zeeman effect.

14-15 **Molecular structure:** Born-Oppenheimer approximation; H_2+ion; H_2 molecule; ionic and covalent bonding; molecular spectra; rotation; nuclear statistics; vibrational transitions.

16 **Field theory of atomic chain:** From particles to fields: classical field theory of the harmonic atomic chain; quantization of the atomic chain; phonons.

17 **Quantum electrodynamics:** Classical theory of the electromagnetic field; theory of waveguide; quantization of the electromagnetic field and photons.

18 **Time-independent perturbation theory:** Time-evolution operator; Rabi oscillations in two level systems; time-dependent potentials – general formalism; perturbation theory; sudden approximation; harmonic perturbations and Fermi's Golden rule; second order transitions.

19 **Radiative transitions:** Light-matter interaction; spontaneous emission; absorption and stimulated emission; Einstein's A and B coefficents; dipole approximation; selection rules; lasers.

20-21 **Scattering theory I:** Basics; elastic and inelastic scattering; method of particle waves; Born approximation; scattering of identical particles.

22-24 **Relativistic quantum mechanics:** History; Klein-Gordon equation; Dirac equation; relativistic covariance and spin; free relativistic particles and the Klein paradox; antiparticles and the positron; Coupling to EM field: gauge invariance, minimal coupling and the connection to non-relativistic quantum mechanics; †field quantization.

Handout

To accompany the course, a substantial handout has been prepared.[1] In some cases, the handout contains material (usually listed as INFO blocks) that goes beyond the scope of the lectures. Needless to say, the examination will be

[1] I should note that, in preparing the handout, I have made use of some web-based material – particularly the excellent lecture notes by Fowler at Virginia – and notes prepared by David Ward in previous generations of the course. I have also included links to useful material on the course webpage.

limited to material that is covered in lectures and not the handout. Since this handout is substantially new, it is inevitable that there will be some typographical errors – some of them may even be important... I would be most grateful if you could e-mail the errors that you find to `bds10@cam`. I will try to maintain a "corrected" set of notes on the web.

The overheads used in lectures can also be recovered from the course website[2] along with other relevant and useful material.

Problem Sets

The problem sets are a vital and integral part of the course providing the means to reinforce key ideas as well as practice techniques. Problems indicated by a † symbol are regarded as challenging. Throughout these notes, I have included a number of simpler exercises which may be completed "along the way", and aim to reinforce some of the ideas developed in the text.

Books

As a core of every undergraduate and graduate physics programme, there is a wealth of excellent textbooks on the subject. Choosing ones that suit is a subjective exercise. Apart from the handout, I am not aware of a text that addresses all of the material covered in this course: Most are of course more dense and far-reaching, and others are simply more advanced or imbalanced towards specialist topics. At the same time, I would not recommend relying solely on the handout. Apart from the range of additional examples they offer, the textbooks will often provide a more erudite and engaging discussion of the material. Although there are too many texts on the subject to discuss every one, I have included below some of the books that I believe to be particularly useful.

[2] `http://www.tcm.phy.cam.ac.uk/~bds10/aqp.html`

Bibliography

[1] B. H. Bransden and C. J. Joachain, **Quantum Mechanics**, (2nd edition, Pearson, 2000). This is a classic text which covers core elements of advanced quantum mechanics. It is particularly strong in the area of atomic physics, but weaker on many-particle physics.

[2] S. Gasiorowicz, **Quantum Physics**, (2nd edn. Wiley 1996, 3rd edition, Wiley, 2003). This is an excellent textbook that covers material at approximately the right level for the course. However, the published text (as opposed to the supplementary material available online) omits some topics which are addressed in this course.

[3] H. Haken and H. C. Wolf, **The Physics of Atoms and Quanta**, (6th edn Springer, 2000). This is a more advanced text which addresses many aspects of atomic physics and quantum optics in a readable manner.

[4] K. Konishi and G. Paffuti, **Quantum Mechanics: A New Introduction**, (OUP, 2009). This is a new text which includes some entertaining new topics within an old field. It also has a useful set of mathematica-based examples if you can get hold of the disk!

[5] L. D. Landau and L. M. Lifshitz, **Quantum Mechanics: Non-Relativistic Theory, Volume 3**, (Butterworth-Heinemann, 3rd edition, 1981). This is a rich and classic text which covers the many of the core topics in this course in most cases at a level that goes well-beyond our target.

[6] F. Schwabl, **Quantum Mechanics**, (Springer, 4th edition, 2007). This book provides a clear exposition of the core topics addressed at the same level as the current text.

[7] R. Shankar, **Principles of Quantum Mechanics**, (Springer; 2nd edition, 1994). This is a very interesting and idiosyncratic text that I particularly like. It is not comprehensive enough to cover every topic in this course. But where there is overlap, it provides an excellent and interesting commentary.