The out of equilibrium birth of a (holographic) superfluid

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http://www.tcm.phy.cam.ac.uk/~amg73/

Phys. Rev. X 5 021015 (2015)

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FCT
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MARIE CURIE ACTIONS

EPSRC
Engineering and Physical Sciences
Research Council

Dynamical phase transitions

$\tau_{eq} = \tau_0 |\epsilon|^{-\nu z}$

$\xi_{eq} = \xi_0 |\epsilon|^{-\nu}$

$\epsilon(t) = 1 - \frac{T(t)}{T_c} = t/\tau_Q$

T_c

Unbroken Phase

Broken phase

$\langle \psi \rangle = 0$

$\langle \psi \rangle \neq 0$

t

$\langle \psi \rangle = \Delta(x, t) e^{i\theta(x,t)}$?

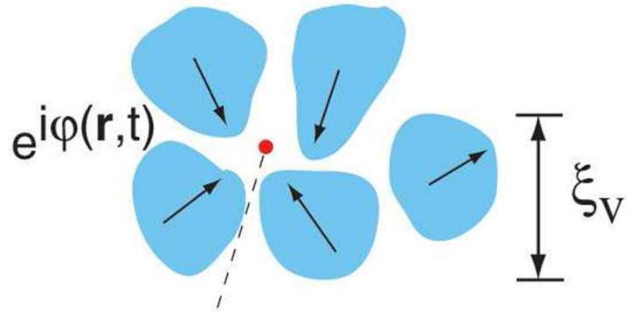
Kibble

J. Phys. A: Math. Gen. 9: 1387. (1976)

Causality

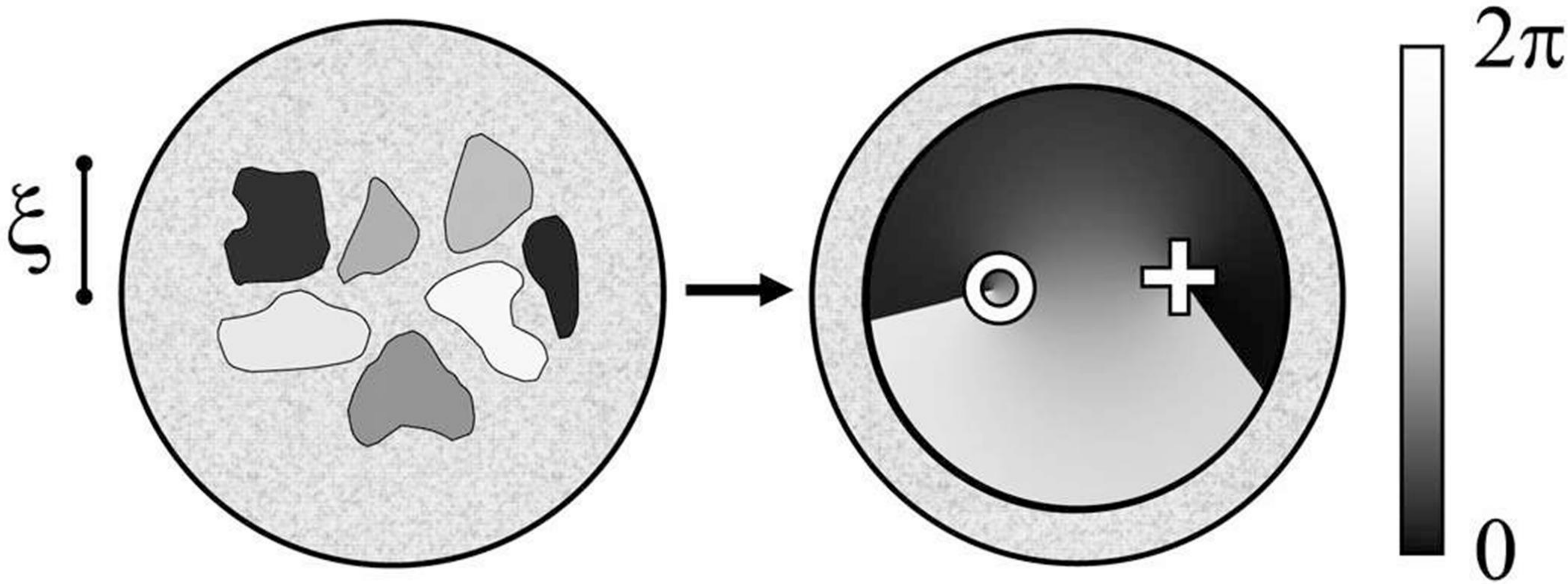
Vortices in the sky

Cosmic strings



Krusius, 2006

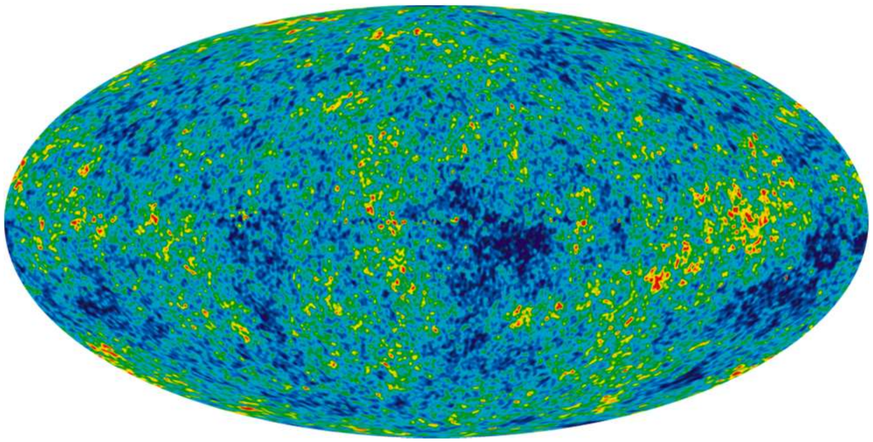
Generation of Structure



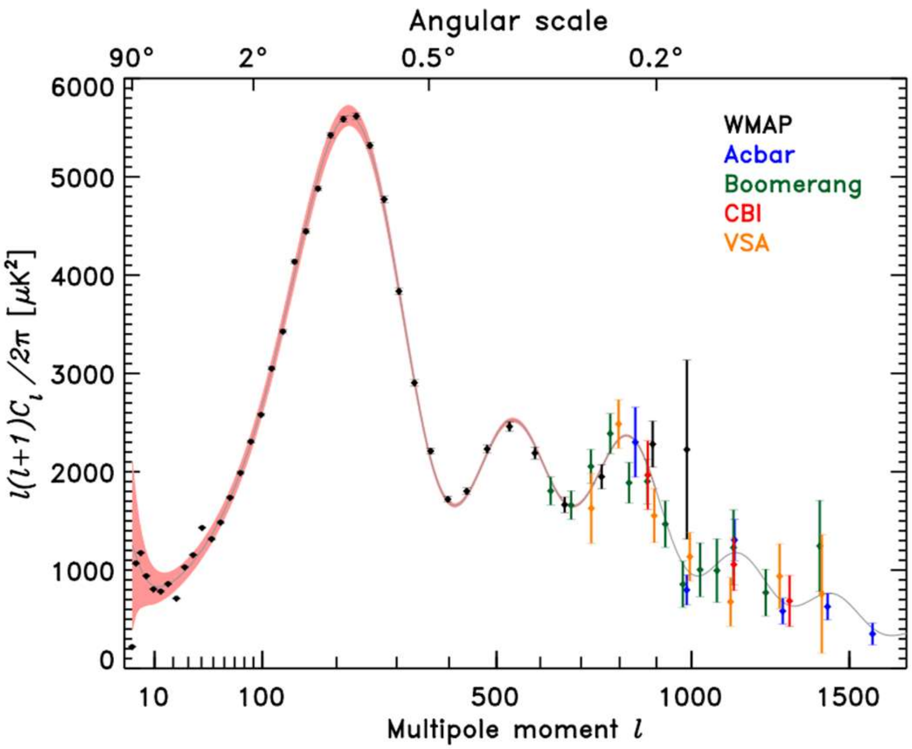
Weyler, Nature 2008

No evidence so far !

CMB, galaxy distributions…



NASA/WMAP



Cosmological experiments in superfluid helium?

W. H. Zurek

Theoretical Astrophysics, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Nature 317, 505 (1985)

Doable for ^{4}He!!

Zurek

$T \approx T_c$

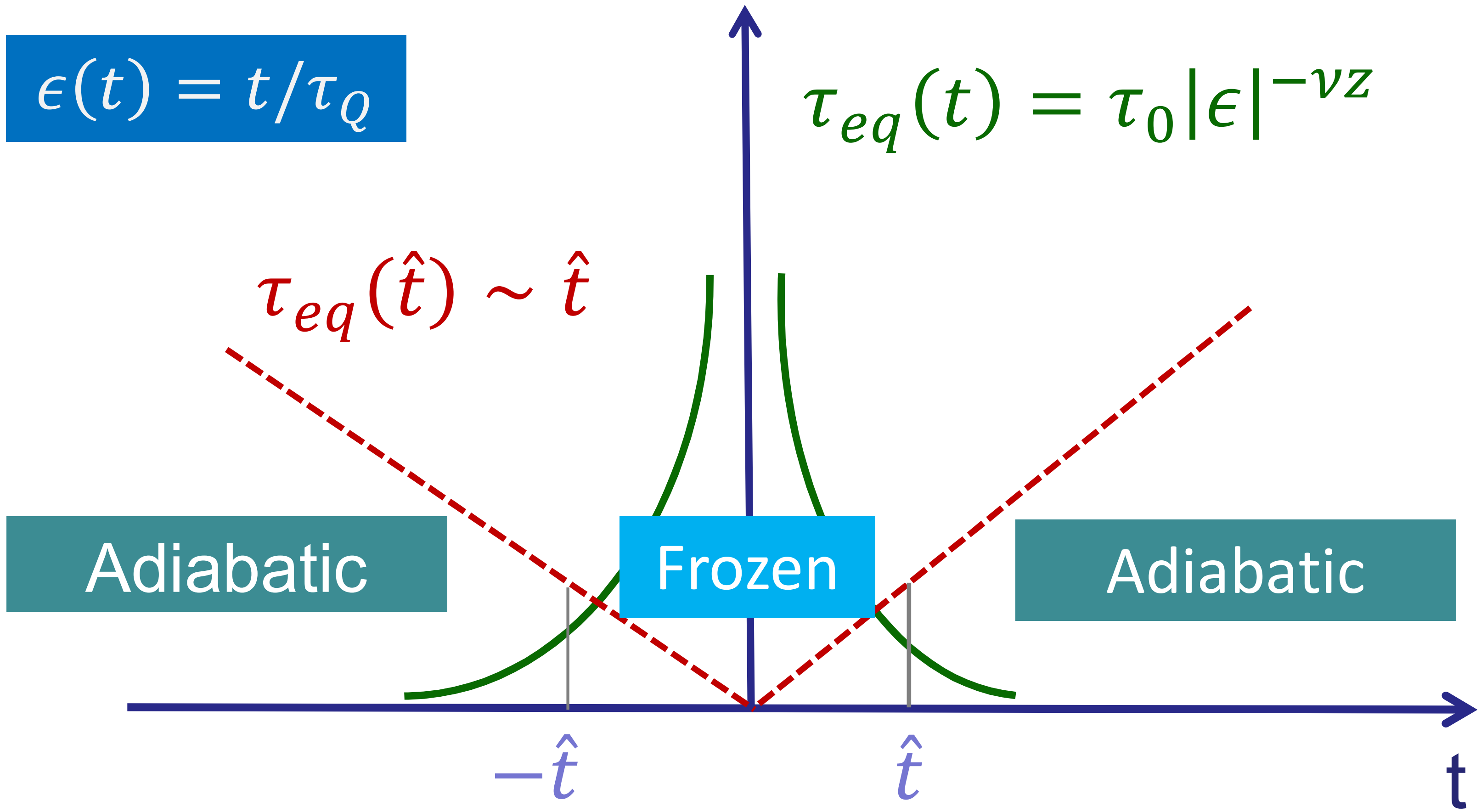
2nd order

Scaling

$\tau(T_c) = \infty$

$$\epsilon(t) = 1 - \frac{T(t)}{T_c} = t/\tau_Q$$

$\mathrm{t} = -\hat{t} \equiv -t_{freeze}$	Non adiabatic evolution
$\mathrm{t} = \hat{t} \equiv t_{freeze}$	Defect generation!



$$\hat{\xi} = \xi_0|\hat{\epsilon}|^{-\nu} = \xi_0\left(\tau_Q/\tau_0\right)^{\nu/(1+\nu z)}$$

Kibble-Zurek mechanism

$$\rho \sim \hat{\xi}^{-d} \sim \tau_Q{}^{-d\nu/(1+\nu z)}$$

Generation of defects in superfluid ^4He as an analogue of the formation of cosmic strings

P. C. Hendry*, N. S. Lawson*, R. A. M. Lee*, P. V. E. McClintock* & C. D. H. Williams†

* School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, UK
† Department of Physics, University of Exeter, Exeter EX4 4QL, UK

NATURE · VOL 368 · 24 MARCH 1994

Transient attenuation of second sound amplitude

But vortices induced by stirring up!

VOLUME 81, NUMBER 17 PHYSICAL REVIEW LETTERS 26 OCTOBER 1998

Nonappearance of Vortices in Fast Mechanical Expansions of Liquid ^4He through the Lambda Transition

M. E. Dodd,[1] P. C. Hendry,[1] N. S. Lawson,[1] P. V. E. McClintock,[1] and C. D. H. Williams[2]
[1]*Department of Physics, Lancaster University, Lancaster, LA1 4YB, United Kingdom*
[2]*Department of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom*

No vortices in ^4He!!

G. Karra, R. J. Rivers, PRL. 81, 3707 (1998)

Vortex formation in neutron-irradiated superfluid ^{3}He as an analogue of cosmological defect formation

Ruutu, Nature 382, 334-336 (1996)

OK

Laboratory simulation of cosmic string formation in the early Universe using superfluid ^{3}He

C. Bäuerle et al. Nature 382, 332 (1996)

Thin SC films, nematic liquid crystal..

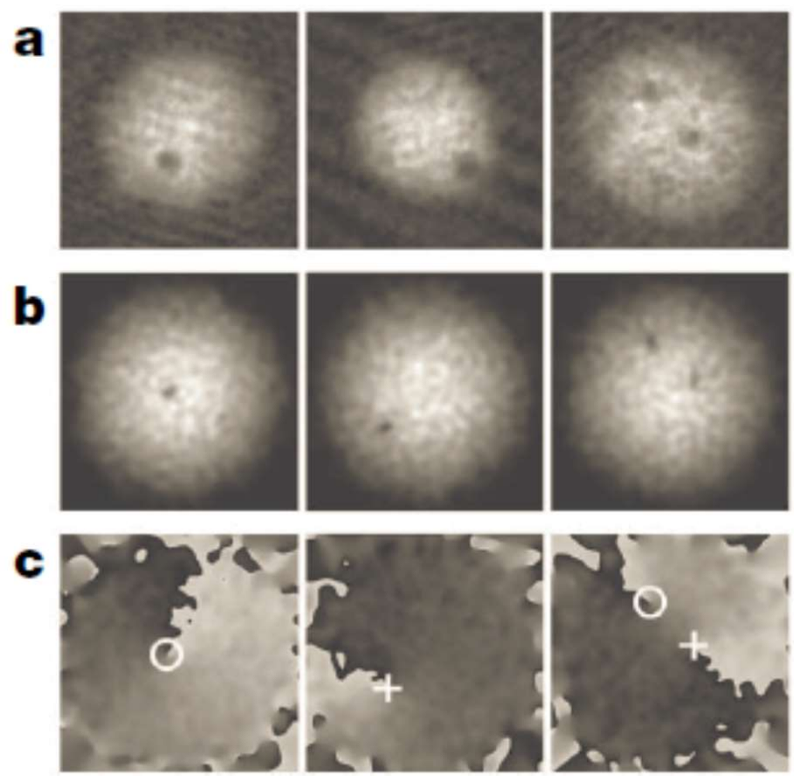
?

nature Vol 455|16 October 2008|doi:10.1038/nature07334

LETTERS

Spontaneous vortices in the formation of Bose–Einstein condensates

Chad N. Weiler[1], Tyler W. Neely[1], David R. Scherer[1], Ashton S. Bradley[2]†, Matthew J. Davis[2] & Brian P. Anderson[1]



nature COMMUNICATIONS

ARTICLE

Received 25 Mar 2013 | Accepted 11 Jul 2013 | Published 7 Aug 2013

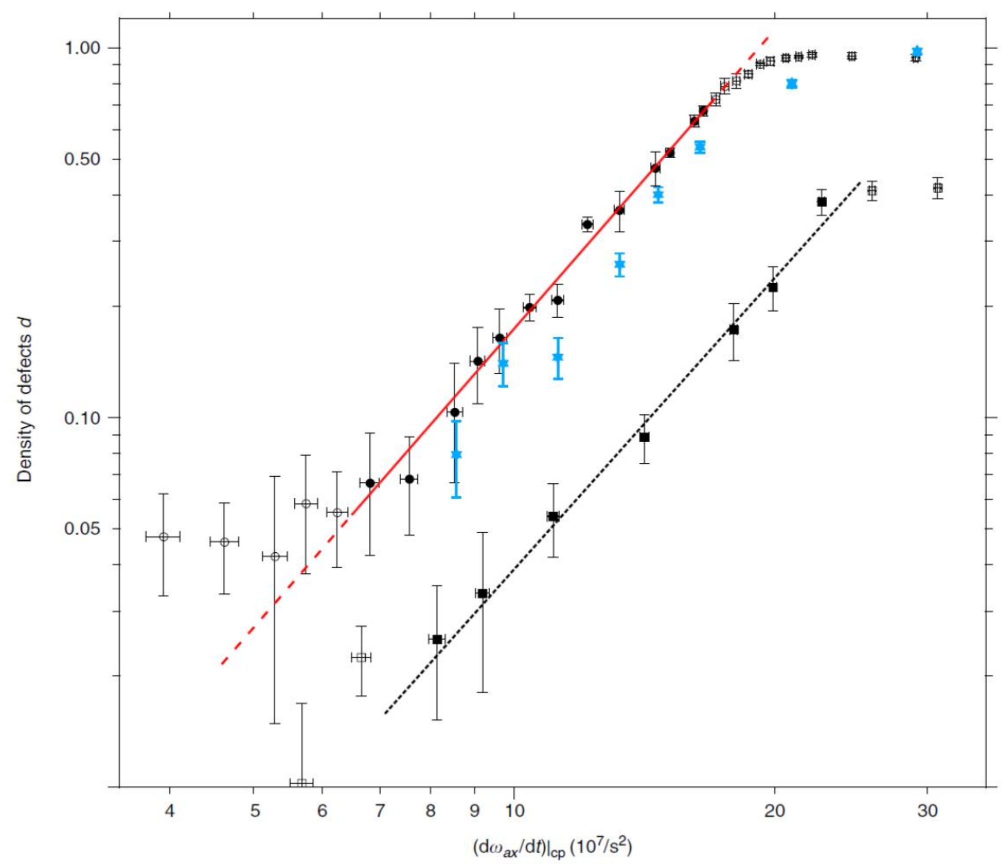
DOI: 10.1038/ncomms3290

Observation of the Kibble–Zurek scaling law for defect formation in ion crystals

S. Ulm[1], J. Roßnagel[1], G. Jacob[1], C. Degünther[1], S.T. Dawkins[1], U.G. Poschinger[1], R. Nigmatullin[2,3], A. Retzker[4], M.B. Plenio[2,3], F. Schmidt-Kaler[1] & K. Singer[1]

KZ scaling with the quench speed

Too few defects



Extension to quantum phase transitions

Zurek, Zoller, et al, "Dynamics of a quantum phase transition." , PRL 95.10 (2005): 105701.

Demonstration of KZ scaling in a 1d Ising chain in transverse field

Dziarmaga "Dynamics of a quantum phase transition: Exact solution of the quantum Ising model." PRL 95.24 (2005): 245701.

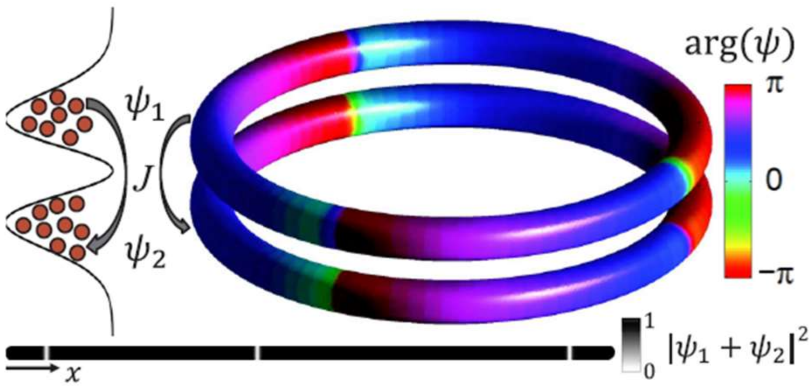
Calculation of correlation functions

Kibble-Zurek problem: Universality and the scaling limit
PRB 86, 064304, (2012), Gubser, Sondhi et al.

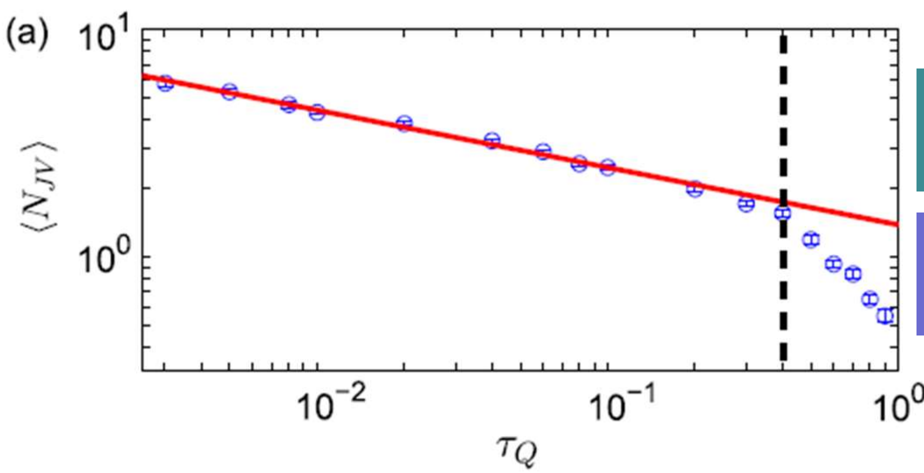
PRL 110, 215302 (2013) PHYSICAL REVIEW LETTERS week ending 24 MAY 2013

Kibble-Zurek Scaling and its Breakdown for Spontaneous Generation of Josephson Vortices in Bose-Einstein Condensates

Shih-Wei Su,[1] Shih-Chuan Gou,[2] Ashton Bradley,[3] Oleksandr Fialko,[4] and Joachim Brand[4]



Stochastic Gross-Pitaesvkii



Breaking of KZ scaling

Too few vortices !

Issues with KZ Too many defects

Adiabatic at t_{freeze}?
Defects without a condensate?

$t_{eq} > t > t_{freeze}$ is relevant

PRX 5, 021015 (2015)

Chesler, AGG, Liu

Slow Quenches	Linear response
$t > t_{freeze}$	Scaling

KZ	Frozen	Adiabatic	Adiabatic
US	Frozen	Coarsening	Adiabatic

t_{freeze} t_{eq}

$$\frac{t_{eq}}{t_{freeze}} \sim (\log R)^{\frac{1}{1+\nu z}}$$

$$|\psi|^2(t) \propto e^{a_2 \bar{t}^{1+z\nu}}$$

$$|\psi|^2(\epsilon) \propto \epsilon^{2\beta}$$

$$\Lambda = (d-z)\nu - 2\beta$$

$$R \sim \xi^{-1} \tau_Q{}^{\Lambda/1+\nu z}$$

$$\gamma = \frac{1+(z-2)\nu}{2(1+z\nu)}$$

$$\rho(t_{eq}) \sim [\log R]^{\gamma} \rho_{KZ}$$

Non adiabatic growth after t_{freeze}

$$C(t,\boldsymbol{r}) \equiv \langle \psi^*(t,\boldsymbol{x}+\boldsymbol{r})\psi(t,\boldsymbol{x})\rangle$$

$$\psi(t,\boldsymbol{q}) = \int dt' G_{\mathrm{R}}(t,t',q)\varphi(t,\boldsymbol{q})$$

$$\langle \varphi^*(t,\boldsymbol{x})\varphi(t',\boldsymbol{x}')\rangle = \zeta\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')$$

$$G_R(t,t',q) = \theta(t-t')H(q)e^{-i\int_{t'}^{t} dt''\mathfrak{w}_0(\epsilon(t''),q)}$$

$$C(t,q) = \int dt'\,\zeta|G_R(t,t',q)|^2$$

Linear response $t > t_{freeze}$

$|\partial_t \log \mathfrak{w}_0| < |\mathfrak{w}_0|$

$$C(t,q) = \int_{t_{\mathrm{freeze}}}^{t} dt' \zeta |H(q)|^2 e^{2\int_{t'}^{t} dt'' \mathrm{Im}\, \mathfrak{w}_0(\epsilon(t''),q)} + \cdots$$

$$\mathfrak{w}_0(\epsilon, q) = \epsilon^{z\nu} h(q\epsilon^{-\nu})$$

$$\mathrm{Im}\, \mathfrak{w}_0 = -a\epsilon^{(z-2)\nu} q^2 + b\epsilon^{z\nu} + \ldots,$$

$q_{max} \sim \epsilon(t)^{\nu}$

$\mathrm{Im}\, \mathfrak{w}_0 > 0$

Unstable Modes → Growth

$\langle \psi(t) \rangle \;\; t > t_{freeze}$

Protocol

$\epsilon(t) = t/\tau_Q$

$t \in (t_i, t_f)$

$t_i = (1 - T_i/T_c)\tau_Q < 0$

$t_f = (1 - T_f/T_c)\tau_Q > 0$

Slow quenches

$t_f \geq t_{eq}$

$t > t_{freeze}$

Correlation length increases

$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}} \qquad \bar{t} \equiv \frac{t}{t_{\rm freeze}}$$

$$\ell_{\rm co}(\bar{t}) = a_3 \xi_{\rm freeze} \bar{t}^{\frac{1+(z-2)\nu}{2}}$$

Condensate growth

$$|\psi|^2(t) \sim \tilde{\varepsilon}(t) e^{a_2 \bar{t}^{1+z\nu}}$$

$$\tilde{\varepsilon}(t) \equiv \zeta t_{\rm freeze} \ell_{\rm co}^{-d}(t)$$

Adiabatic evolution
$t = t_{eq} \gg t_{freeze}$

$$|\psi|^2(t = t_{\rm eq}) \sim |\psi|^2_{\rm eq}(\epsilon(t_{\rm eq}))$$

 # Defects

$$\rho(t_{\rm eq}) \sim 1/\ell_{\rm co}^{d-D}(t_{\rm eq}) \sim \left[\log(\zeta^{-1}\tau_Q^{\Lambda})\right]^{-\frac{(d-D)(1+(z-2)\nu)}{2(1+z\nu)}} \rho_{\rm KZ}$$

Fast quenches

$$t_f \ll t_{eq}$$

$$q_{max}(T_f) = \epsilon(t_f)^{d\nu}$$

Breaking of τ_Q scaling

KZ $\quad t_f < t_{freeze}$

US $\quad t_{freeze} \ll t_f \ll t_{eq}$

Exponential growth

$$|\psi|^2(t) \sim \epsilon_f^{(d-z)\nu} \zeta \exp\left[2b(t - t_{\text{freeze}})\epsilon_f^{\nu z}\right]$$

Number of defects

Independent of τ_Q

$$\rho \sim \begin{cases} \epsilon_f^{(d-D)\nu} & R_f \lesssim O(1) \\ \epsilon_f^{(d-D)\nu} \log^{-\frac{d-D}{2}} R_f & R_f \gg 1 \end{cases}$$

$$R_f \equiv \frac{\epsilon_f^{2\beta}}{\zeta \epsilon_f^{(d-z)\nu}} \qquad \epsilon_f \equiv \frac{T_c - T_f}{T_c}$$

Holography?

Defects survive large N limit

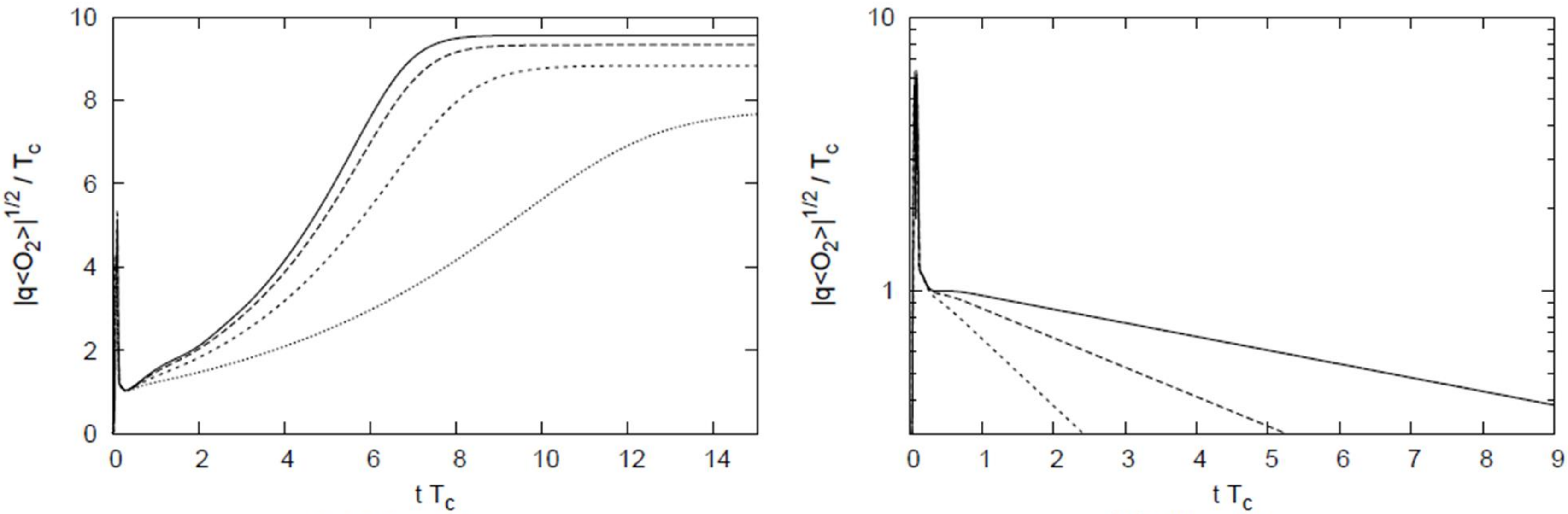
Universality

Real time

Holographic SC out of equilibrium

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{Newton}}} \int d^4x \sqrt{-G} \left[R + \Lambda + \frac{1}{e^2} \left(-\frac{1}{4} F_{MN} F^{MN} - |D\Phi|^2 - m^2 |\Phi|^2 \right) \right]$$

Gubser,Herzog, Horowitz, Hartnoll



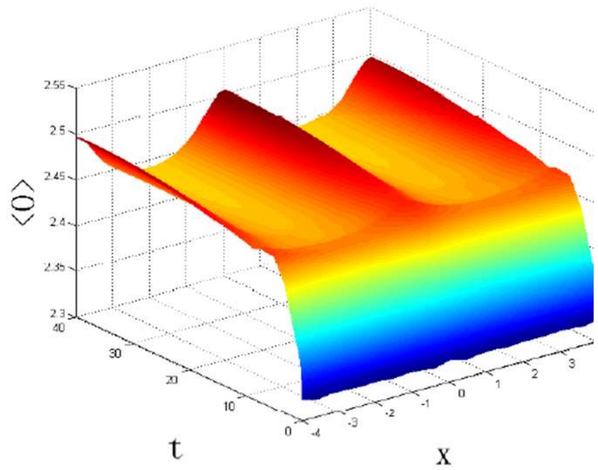
(a) $T < T_c$ (b) $T > T_c$

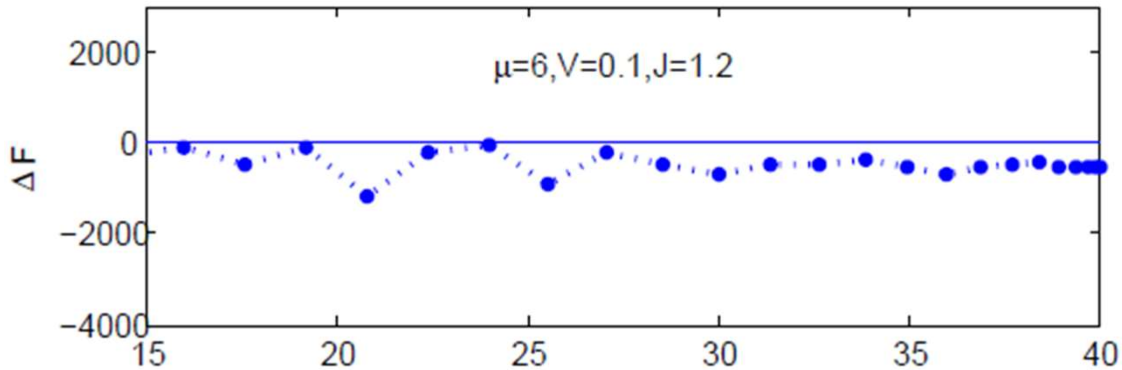
$$|\langle \mathcal{O}_2(t) \rangle| = C_1 \exp\left(-t/t_{\text{relax}}\right) + C_2 \qquad |\langle \mathcal{O}_2(t) \rangle| = C \exp\left(-t/t_{\text{relax}}\right)$$

Exponential growth/decay

Murata, et al.,
arXiv:1005.0633

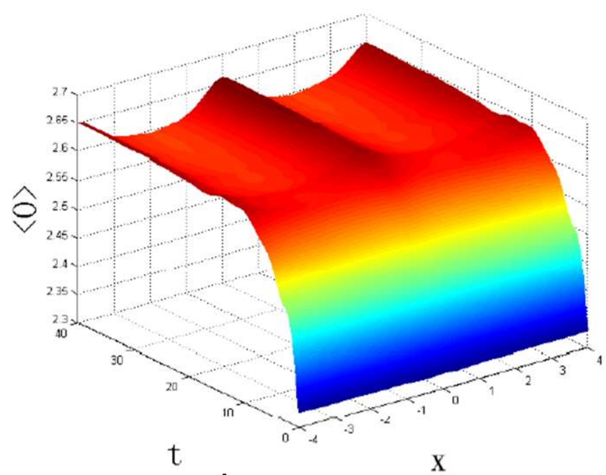
Oscillations in space





$$\Psi \approx z\psi_1 + \psi(x,t)z^2$$

$$\psi_1(t) = \mathrm{J}(1 - \tanh vt)$$



Basu et al., arXiv:1308.4061

AGG, Zhang, Bi,arXiv:1308.5398

$$\langle O \rangle \sim \psi(x,t)$$

Probe limit

Conservation laws!

Oscillations in space: BdG

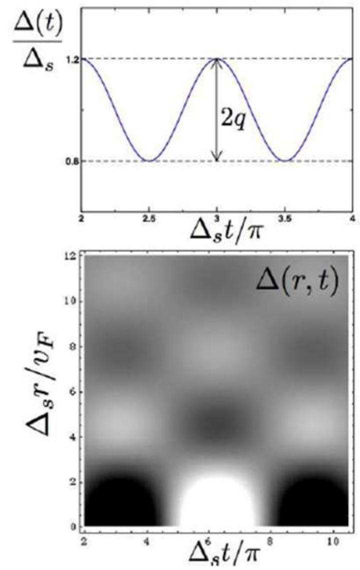
$$\hat{\xi} = -\vec{\nabla}^2/2m - \mu$$

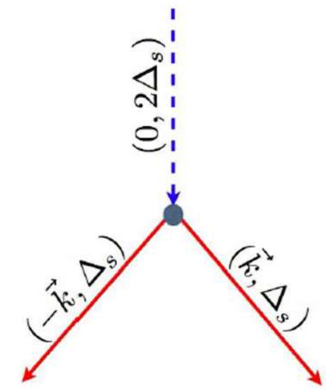
$$i\dot{u}_{\mathbf{p}}(\mathbf{r},t) = \hat{\xi}u_{\mathbf{p}}(\mathbf{r},t) + \Delta(\mathbf{r},t)v_{\mathbf{p}}(\mathbf{r},t),$$
$$i\dot{v}_{\mathbf{p}}(\mathbf{r},t) = -\hat{\xi}v_{\mathbf{p}}(\mathbf{r},t) + \bar{\Delta}(\mathbf{r},t)u_{\mathbf{p}}(\mathbf{r},t)$$

$$\Delta(\mathbf{r},t) = \Delta(t) + \delta\Delta(\mathbf{r},t)$$

$$\delta\Delta(\vec{r},t) \approx \frac{Ce^{\nu_m t}\cos[\Delta_s(t-\tau)]}{\sqrt{\Delta_s t}}\,\frac{\sin(k_m R)e^{-R^2/l^2(t)}}{k_m R}$$

$$l(t) \approx \xi\sqrt{\Delta_s t} \qquad \nu_m \approx 2q\Delta_s$$





Conservation laws

Instability to spatial inhomogeneity

Dzero et al., EPL 85 (2009) 20004

KZ scaling in holographic superconductors

Quantum Quench Across a Zero Temperature Holographic Superfluid Transition,

P. Basu, D. Das, S. R. Das, T. Nishioka

arXiv:1211.7076

Kibble-Zurek Scaling in Holographic Quantum Quench : Backreaction

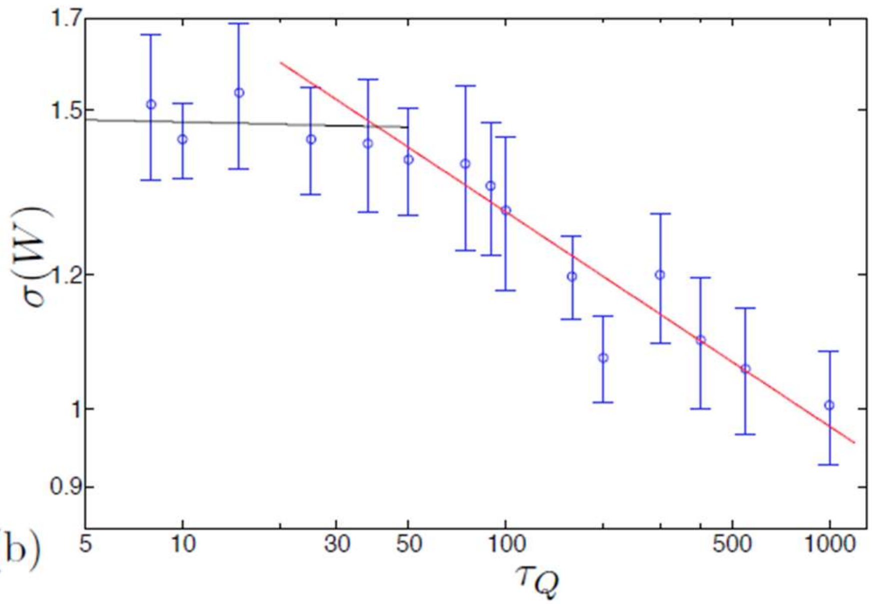
S. R. Das, T. Morita

arXiv:1409.7361

Universal far-from equilibrium dynamics of a holographic superconductor

Sonner, Campo, and Zurek

arXiv:1406.2329



(b)

Dual gravity theory

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{Newton}}} \int d^4x \sqrt{-G} \left[R + \Lambda + \frac{1}{e^2} \left(-\frac{1}{4} F_{MN} F^{MN} - |D\Phi|^2 - m^2 |\Phi|^2 \right) \right]$$

$$\Lambda = -3 \qquad m^2 = -2$$

Herzog, Horowitz, Hartnoll, Gubser

AdS_4

$$ds^2 = r^2 g_{\mu\nu}(t, \boldsymbol{x}, r) dx^\mu dx^\nu + 2 dr dt$$

Eddington-Finkelstein coordinates

Probe limit

$$0 = \nabla_M F^{NM} - J^M,$$

$$0 = (-D^2 + m^2)\Phi.$$

EOM's:

PDE's in x,y,r,t

Boundary conditions:

$r \rightarrow \infty$

$$= 0$$

$$\Psi = \frac{\psi^{(1)}(x,y,t)}{r} + \frac{\psi^{(2)}(x,y,t)}{r^2} + ..$$

$$A_t = \mu - \rho/r$$

hep-th/9905104v2
1309.1439
Science 2013

Drive:

$$\epsilon(t) = t/\tau_Q \qquad t_i = (1 - T_i/T_c)\tau_Q$$

$$t \in (t_i, t_f) \qquad t_f = (1 - T_f/T_c)\tau_Q$$

No solution of Einstein equations but do not worry, Hubeny 2008

Dictionary:

$$\langle O_2 \rangle \sim \psi_2$$

Stochastic driving

$$\psi^{(1)} = \varphi(t, x)$$

$$\langle \varphi^*(t, x)\varphi(t', x') \rangle = \xi \delta(t - t')\delta(x - x')$$

Field theory:

$$\xi(T, \nu)$$

Quantum/thermal fluctuations

Gravity:

$$\xi \propto 1/N^2$$

Hawking radiation

Predictions

Mean field critical exponents

Slow quenches:

$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \quad |\psi|^2(t) \sim \tilde{\varepsilon} t_{\rm freeze} \bar{t} e^{a_2 \bar{t}^2}, \quad \ell_{\rm co}(t) \sim \xi_{\rm freeze}\sqrt{\bar{t}}$$

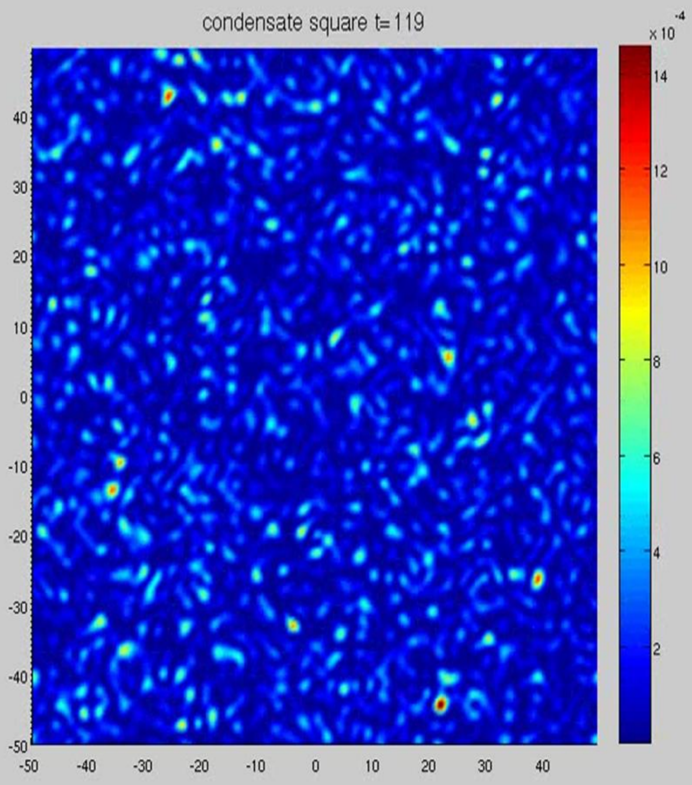
$$\frac{t_{\rm eq}}{t_{\rm freeze}} \sim \sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}} \qquad \rho \sim \frac{1}{\sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}}} \rho_{\rm KZ}$$

Fast quenches:

$$C(t,r) = |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \qquad |\psi|^2(t) \sim \zeta \exp\left[2b(t - t_{\rm freeze})\epsilon_f\right]$$

$$\ell_{\rm co}^2(t) = 4a(t - t_{\rm freeze}) \qquad \rho \sim \frac{\epsilon_f}{\log \frac{N^2}{\epsilon_f}}$$

Movies!!


condensate square t=119
x 10-4
14
12
10
8
6
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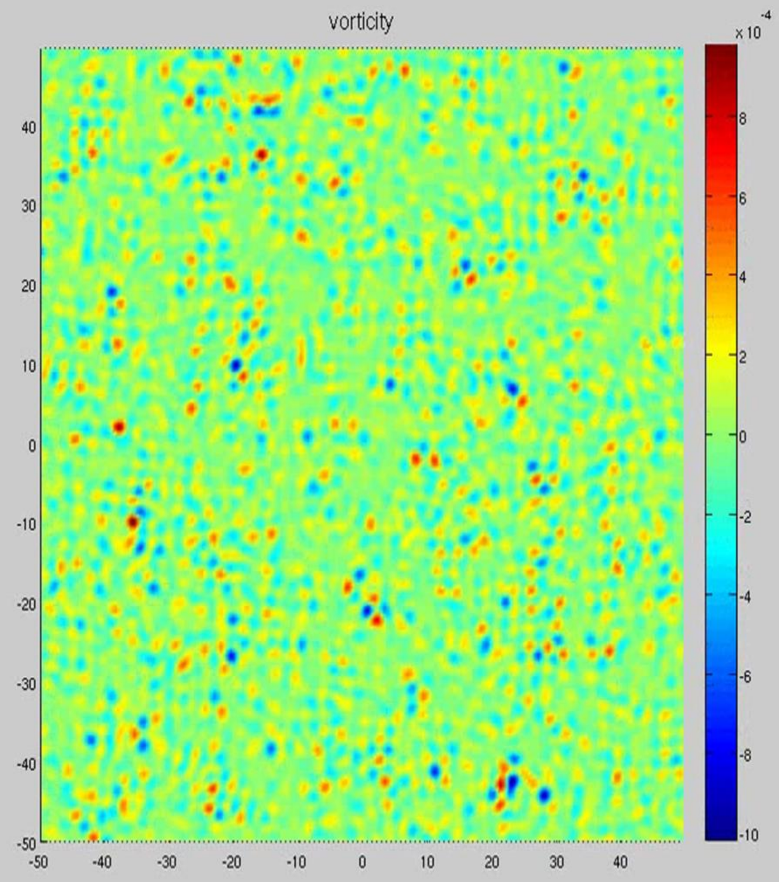
-50 -40 -30 -20 -10 0 10 20 30 40


vorticity
x 10-4
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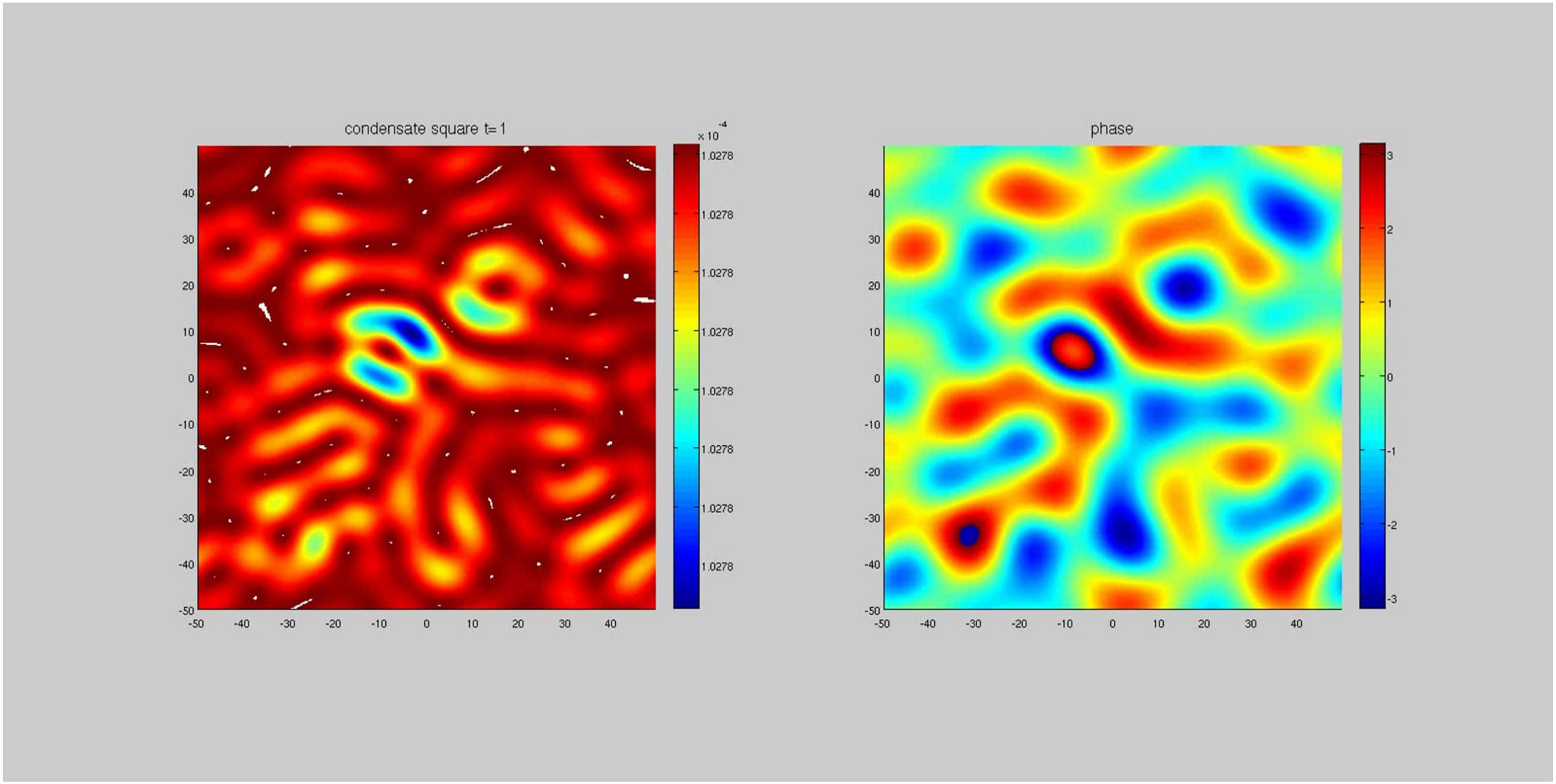
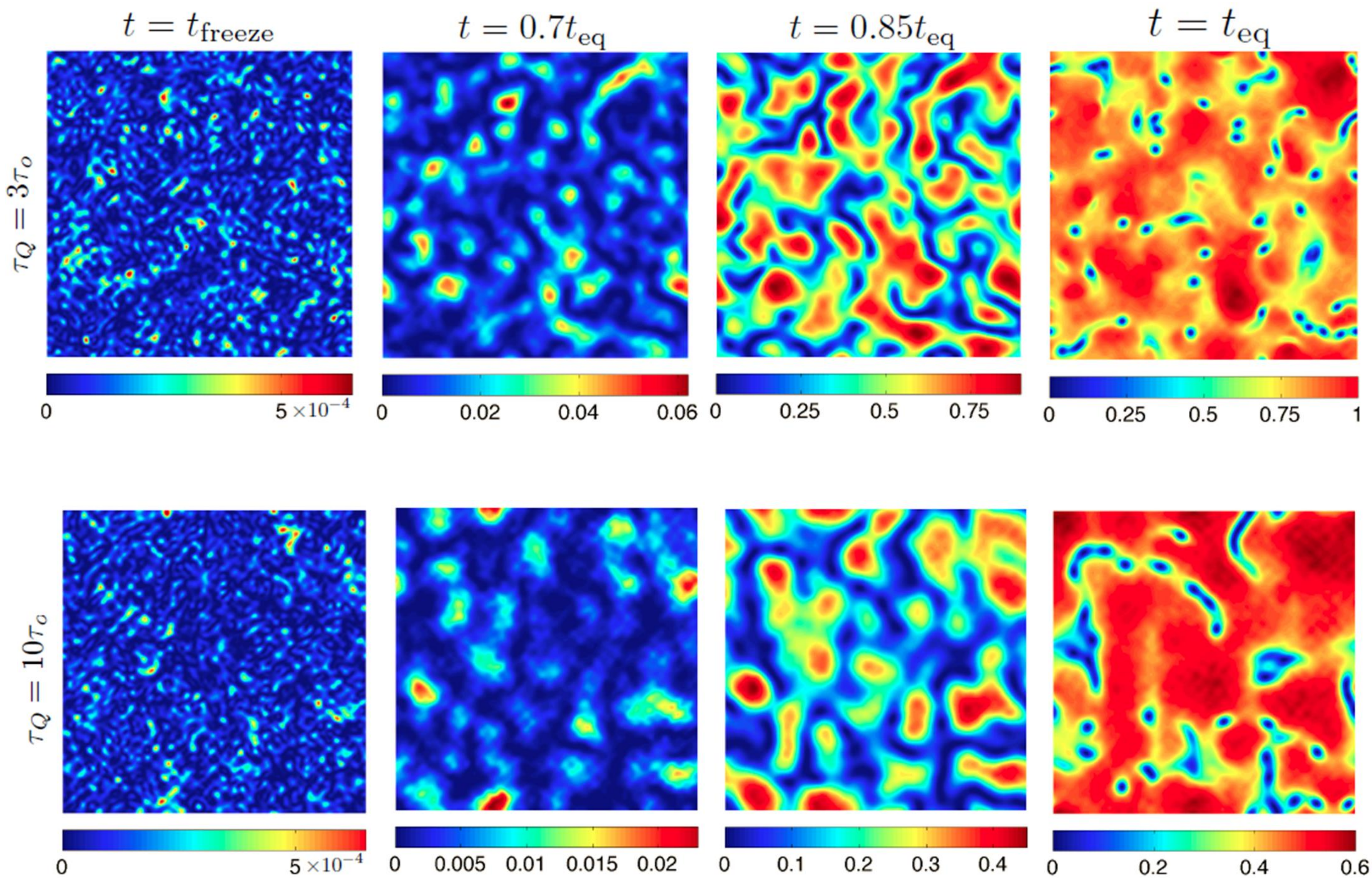


Slow

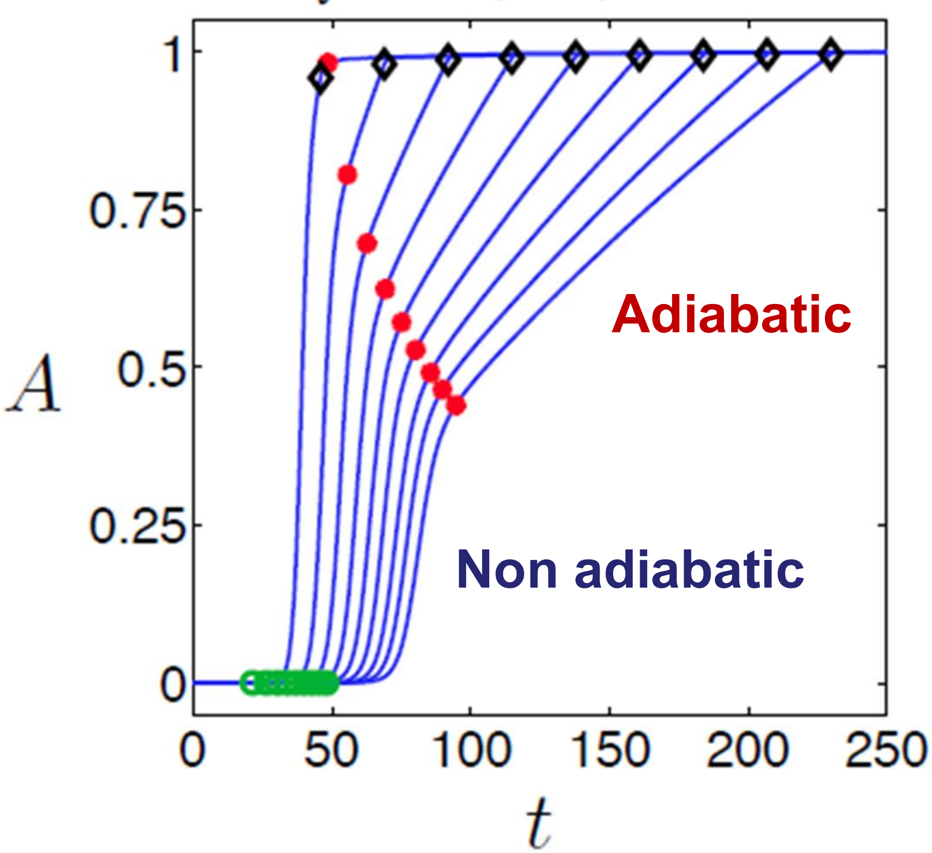


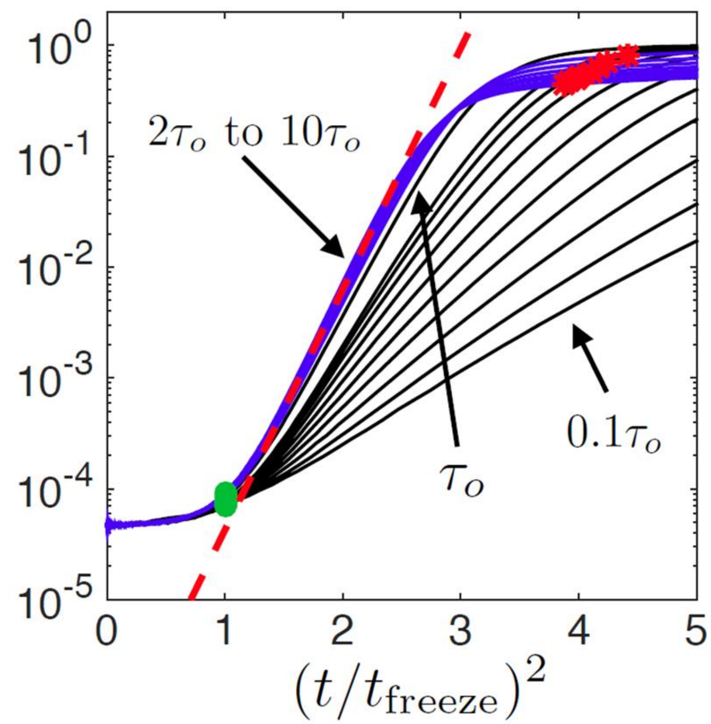
t_{eq} *is the relevant scale*

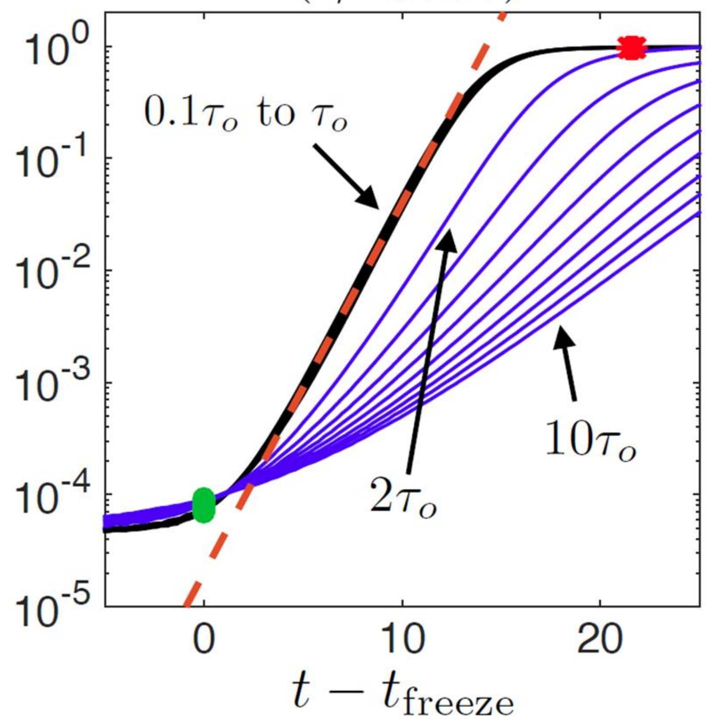
$$A(t) = \frac{1}{M}\sum_{i=1}^{M}\frac{a_i(t)}{a_i(\infty)}$$

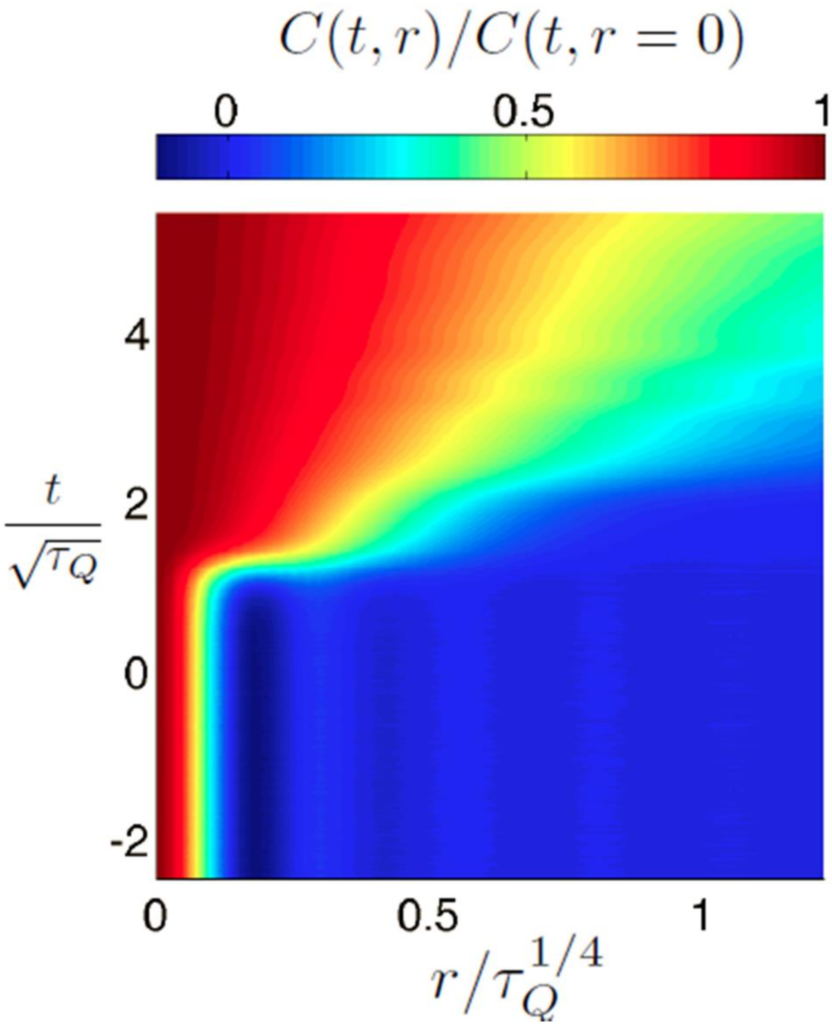
$$a_i(t) \equiv \int d^2x\, |\psi_i(t,\boldsymbol{x})|^2$$

$\tau_Q = 2\tau_o, \ldots, 10\tau_o$





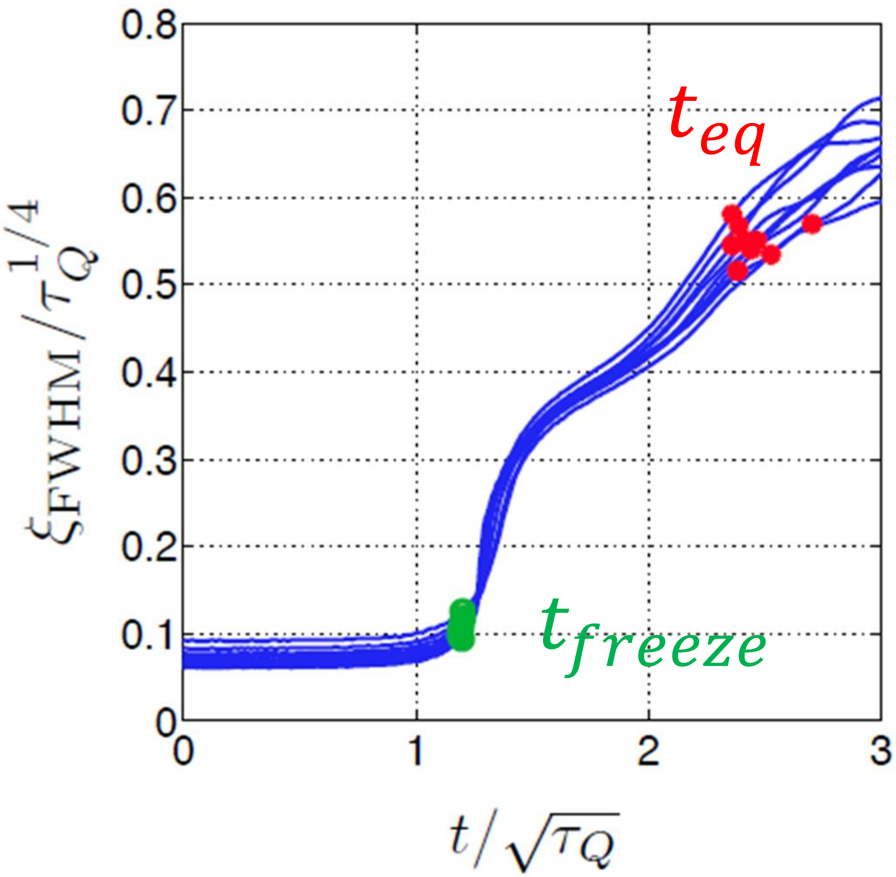




Strong coarsening

$t > t_{freeze}$

Full width half max of $C(t,r)$

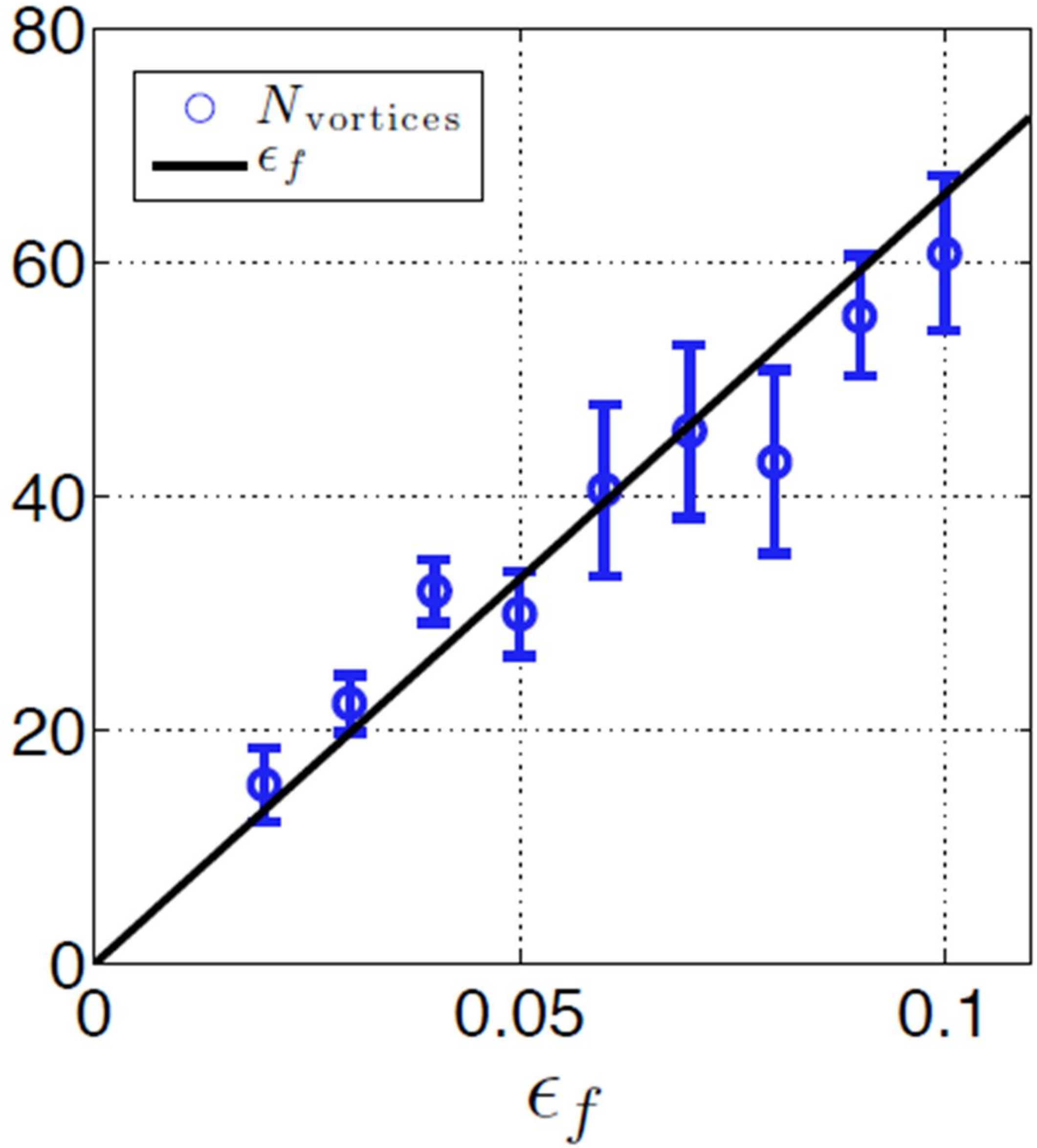


$$\ell_{co}(t) \sim \xi_{freeze}\sqrt{t/t_{freeze}}$$

Fast quenches

$$\rho \sim \frac{\epsilon_f}{\log(\frac{N^2}{\epsilon_f})}$$

$T<T_c$ dynamic irrelevant

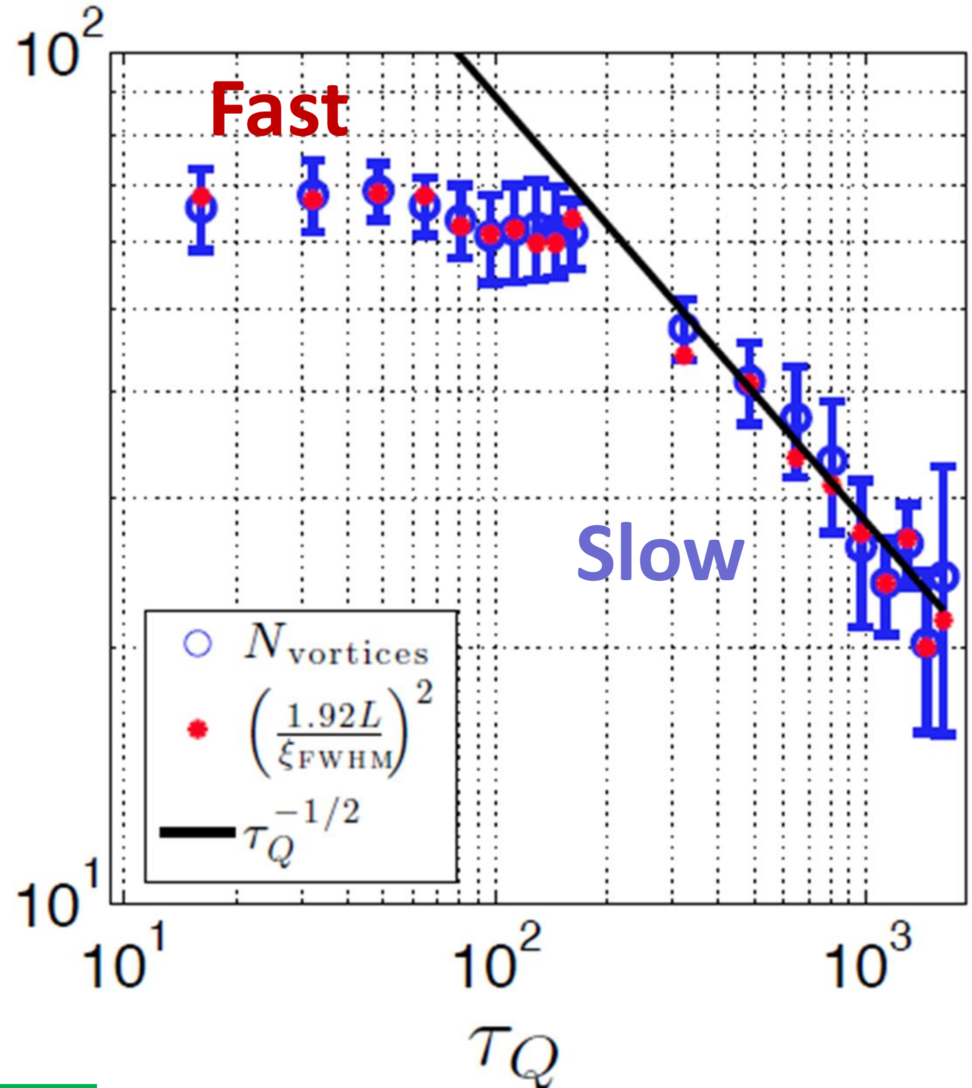


Slow

$$\rho \sim \frac{\rho_{KZ}}{\left(\log\left(N^2/\tau_Q{}^{1/2}\right)\right)^{1/2}}$$

Fast

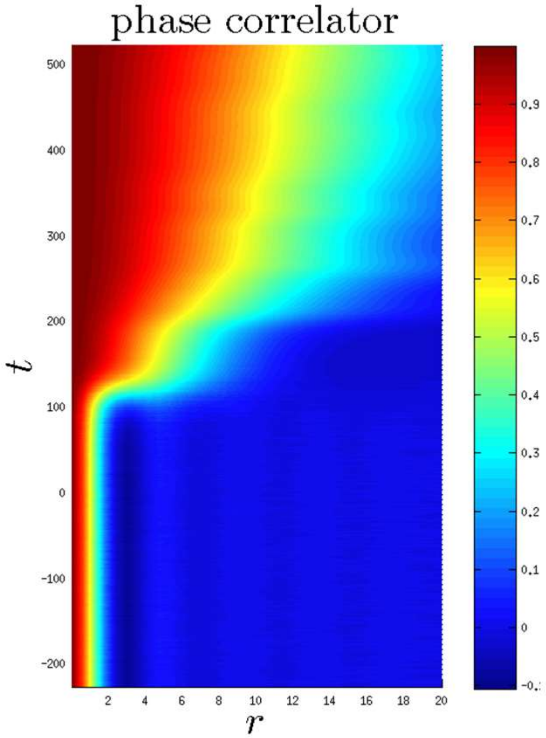
$$\rho \sim \frac{\epsilon_f}{\log\left(\frac{N^2}{\epsilon_f}\right)}$$

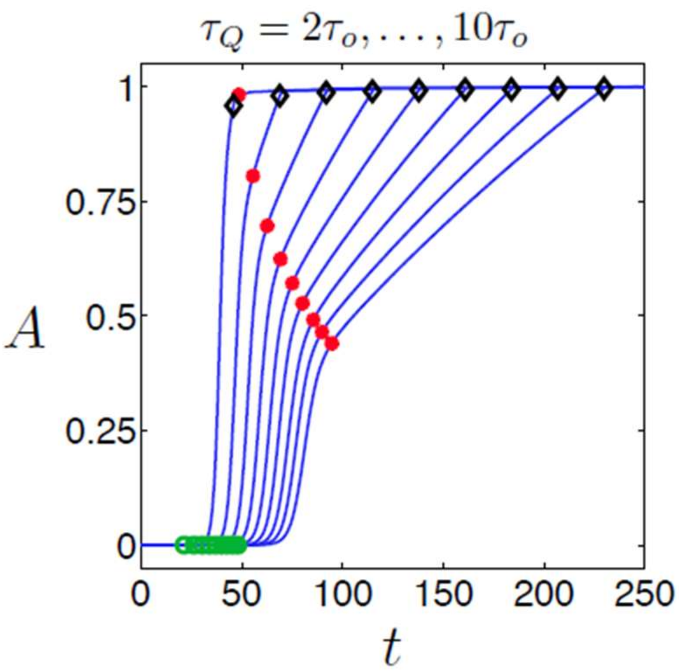


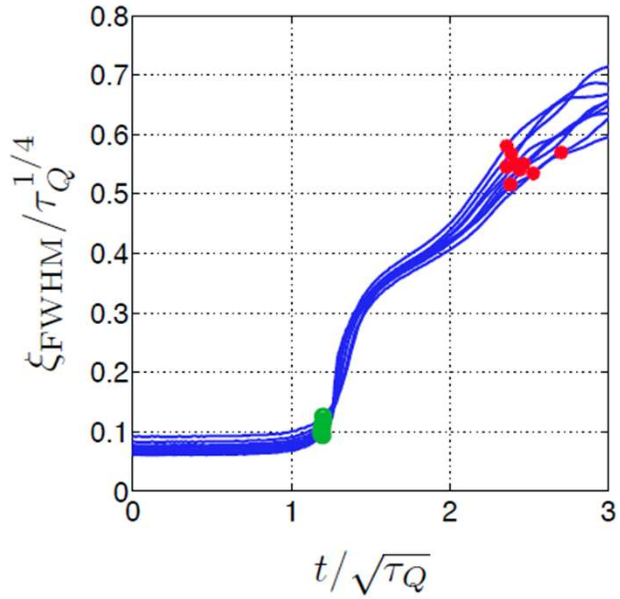
Relevant for ^{4}He ?

$$t_{\text{eq}} \sim [\log \tau_Q]^{1/(1+\nu z)} t_{\text{freeze}}$$

~25 times less defects than KZ prediction!!







time

Freezing

Condensate formation

Defect generation

Phase coherence ?

Physics beyond Kibble-Zurek

Novel dynamical region

$t_{\text{eq}} > t > t_{freeze}$

Holographic duality helpful to discover and model this region

More efficient than SGPE?

NEXT

Cracking thermalization?

^{4}He?

Cosmology

Vortex physics

BKT transition

Thanks!