

Control and enhancement of superconductivity by finite size effects

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Ribeiro, Dresden



Sacramento, Araujo,
Lisbon

**Phys. Rev. B 84,
172502 (2011)**

**Phys. Rev.
B 84,104525 (2011)
Editor Suggestion**

**Phys. Rev. Lett. 108,
097004 (2012)**

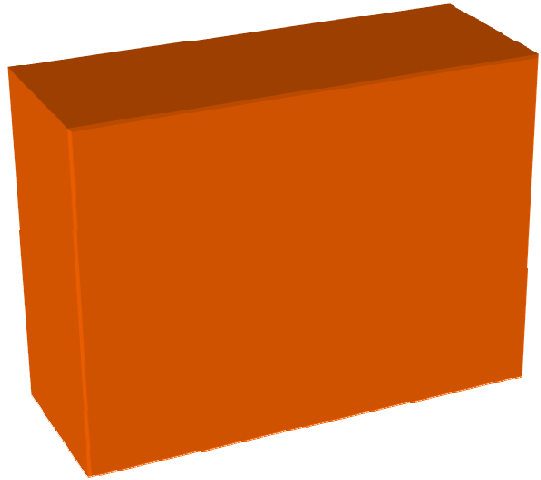
**Nature Materials 9,
554, May 2010**



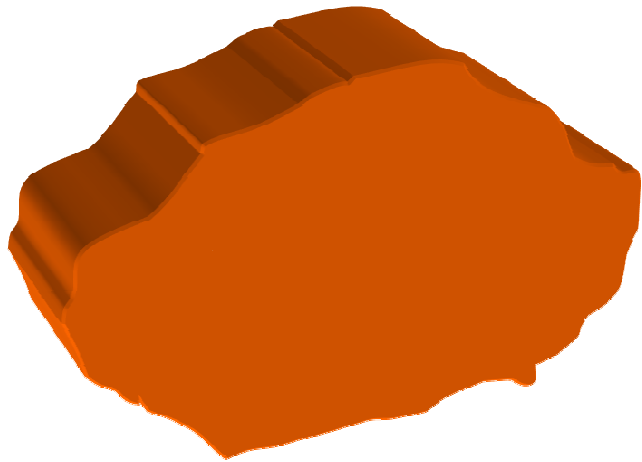
Sangita Bose
Bombay



Kern, Stuttgart Ugeda
Brihuega, Michaelis



**1. Analytical description
of a clean, finite-size
superconductor?**



**2. Are these results
applicable to realistic
grains?**



$L \sim 10\text{nm}$

**3. Is it possible to
increase T_c ?**

Experiments: A little history

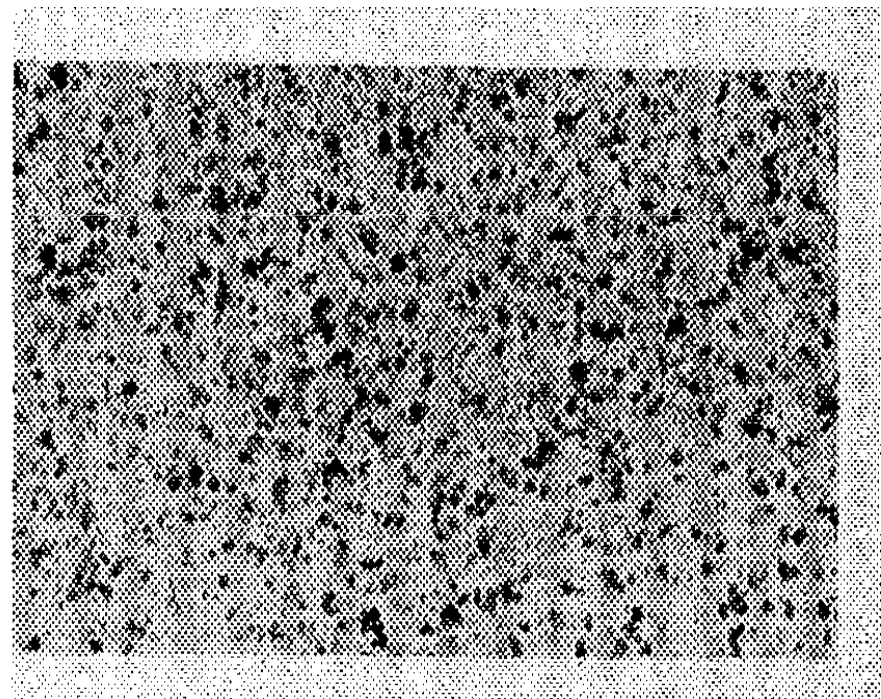
Kammerer, Strongin, Phys. Lett. 17, 224 (1965)

Abeles, Cohen, Cullen, Phys. Rev. Lett., 17, 632 (1966)

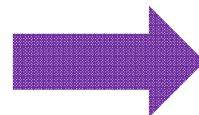
Crow, Parks, Douglass, Jensen, Giaever, Zeller....

Metal	T_c (°K)	T_c/T_{c0}	d (Å)	ρ_0
Al	3.0	2.6	40	0.19
Ga	7.2	6.5	...	0.20
Sn	4.1	1.1	110	0.31
In	3.7	1.1	110	0.36
Pb	7.2	1.0	...	0.53

2000 Å



No exp control

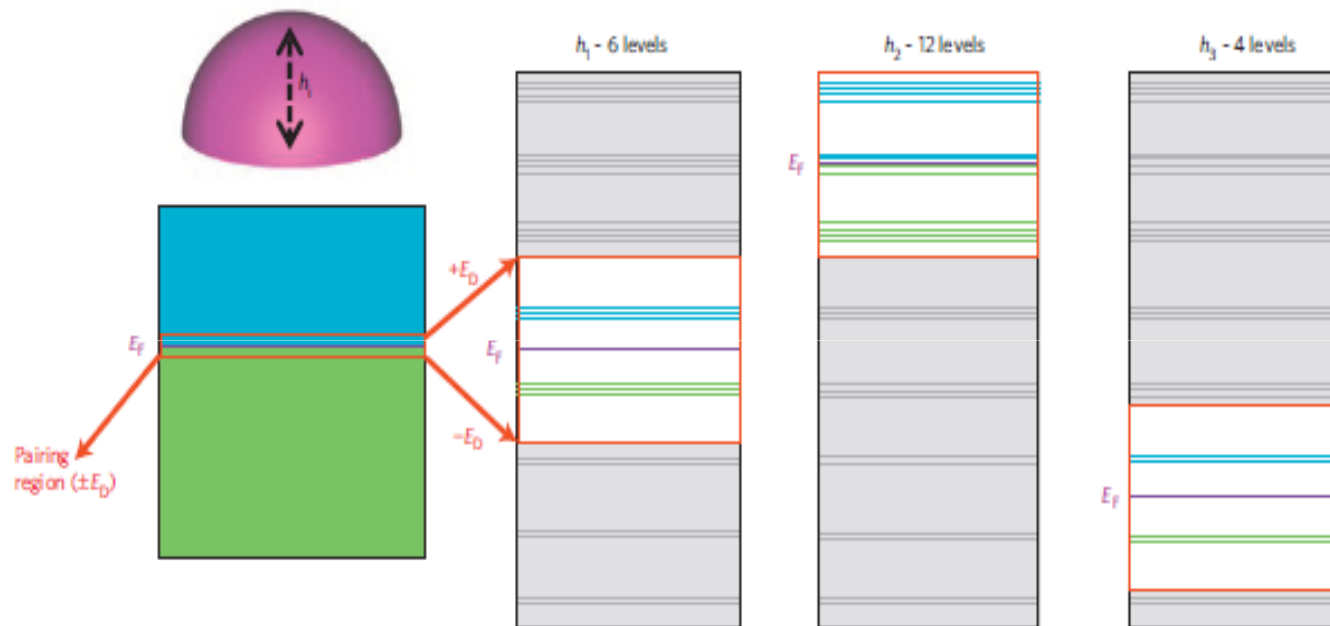


Theoretical drift

Theory: A little history

Breaking of superconductivity for $\delta/\Delta_0 > 1$? Anderson (1959)

Parmenter, Blatt, Thompson (60's) : BCS in a rectangular grain



Heiselberg (2002): BCS in harmonic potentials, cold atom appl

Shanenko, Croitoru, Peeters (2005-): BCS in a wire, cylinder

Devreese (2006): Richardson equations in a box

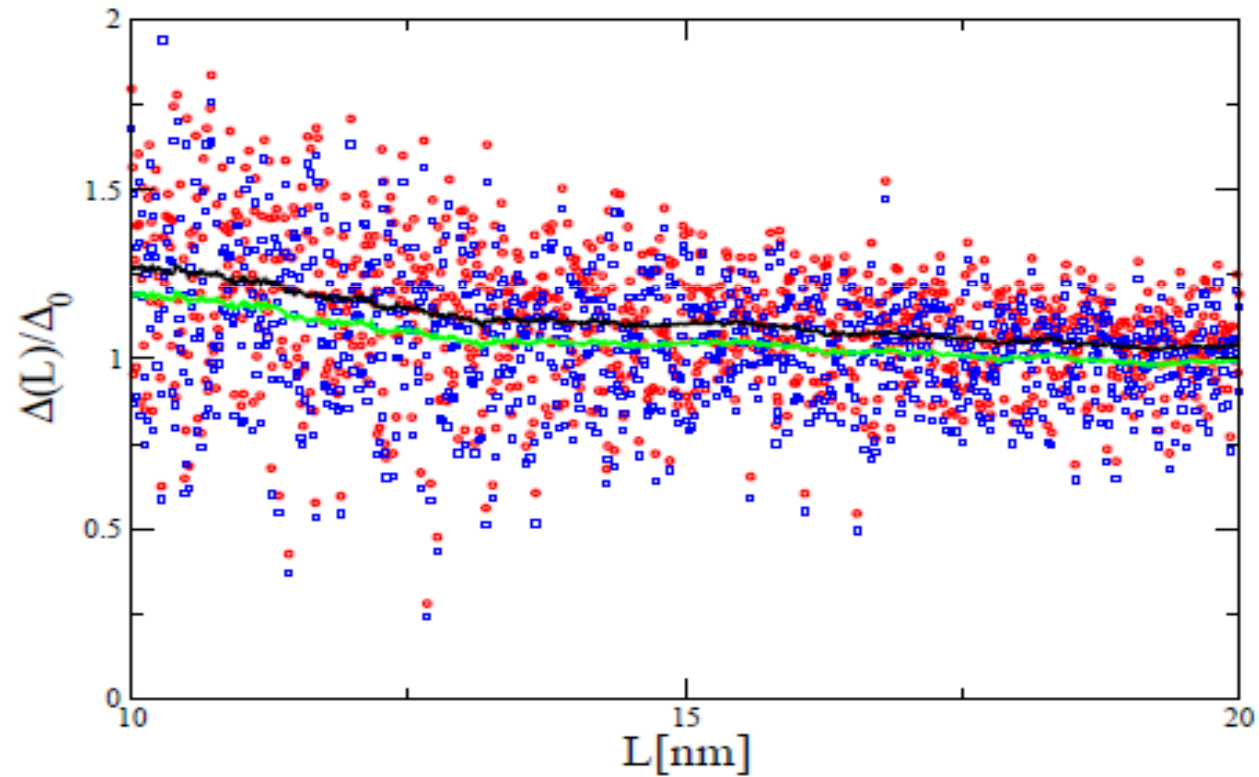
Kresin, Boyaci, Ovchinnikov (2007) : Spherical grain, high T_c

Olofsson (2008): Estimation of fluctuations within BCS

Fluctuations
 $\xi > L$

Symmetries

No fluctuations
 $\xi < L$



$$\Delta_0 \sim \delta$$



No long range order

2008

Chaotic grains?

$$\frac{2}{g} = \sum_{|\varepsilon_i| < E_D} \frac{c_i}{\sqrt{\Delta^2 + \varepsilon_i^2}}$$

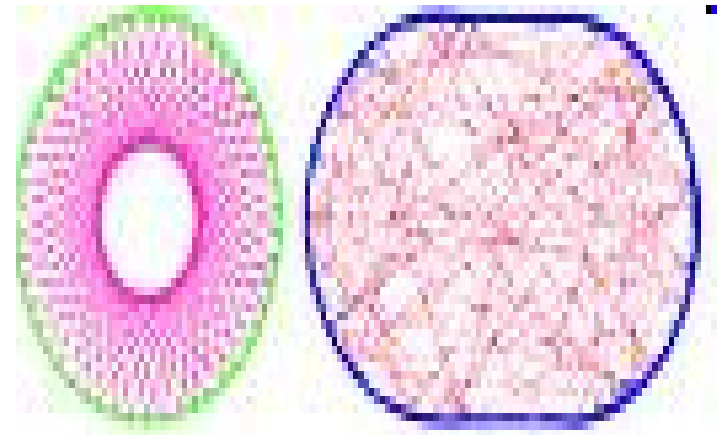
ε_i = eigenvalues 1-body problem

Analytical? $1/k_F L \ll 1$

Semiclassical techniques

Quantum observables in terms
of classical quantities

Berry, Gutzwiller, Balian, Bloch



$$\nu(\varepsilon) \Leftrightarrow L_p$$

3d chaotic

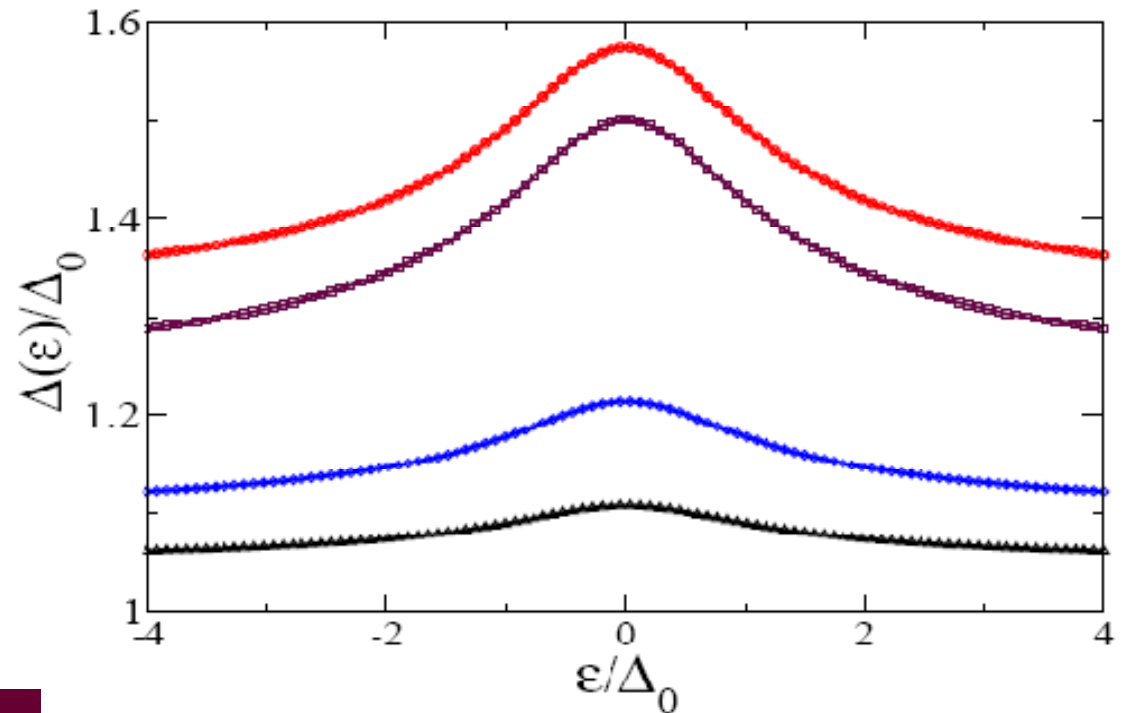
Al grain

$$k_F = 17.5 \text{ nm}^{-1}$$

$$\Delta_0 = 0.24 \text{ mV}$$

For $L < 9 \text{ nm}$ leading correction comes from matrix elements

Phys. Rev. Lett. 100, 187001
(2008)



$L = 6 \text{ nm}$, Dirichlet, $\delta/\Delta_0 = 0.67$

$L = 6 \text{ nm}$, Neumann, $\delta/\Delta_0 = 0.67$

$L = 8 \text{ nm}$, Dirichlet, $\delta/\Delta_0 = 0.32$

$L = 10 \text{ nm}$, Dirichlet, $\delta/\Delta_0 = 0.08$

2010

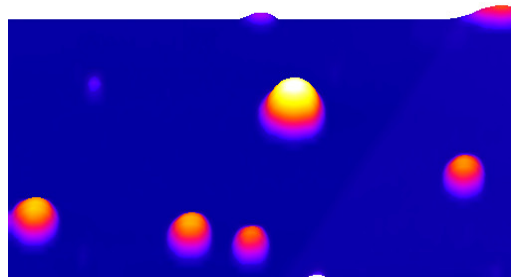


Single, Isolated

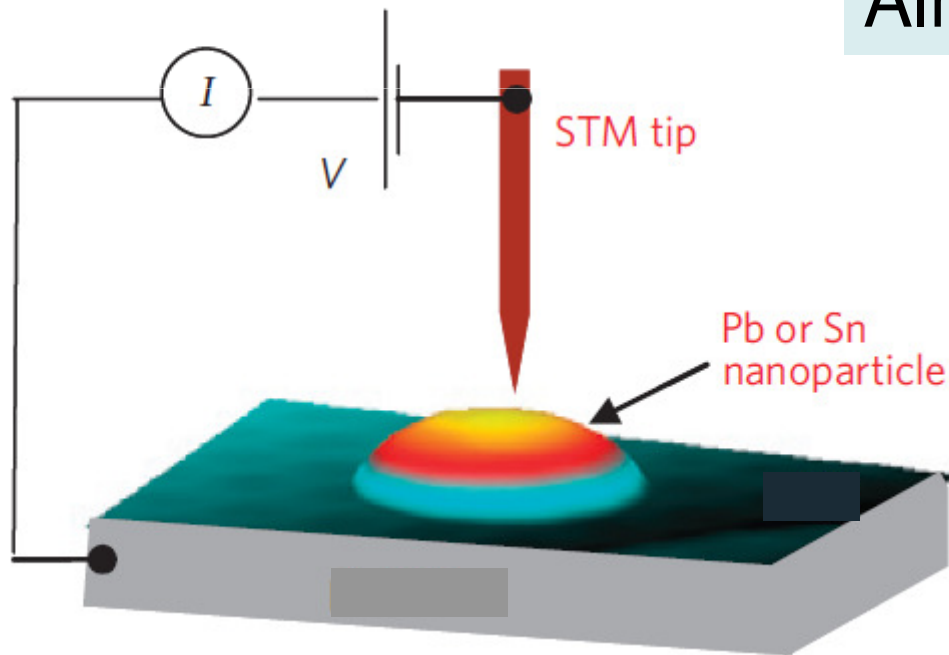
$R \sim 4\text{-}30\text{nm}$

B closes gap

Almost hemispherical

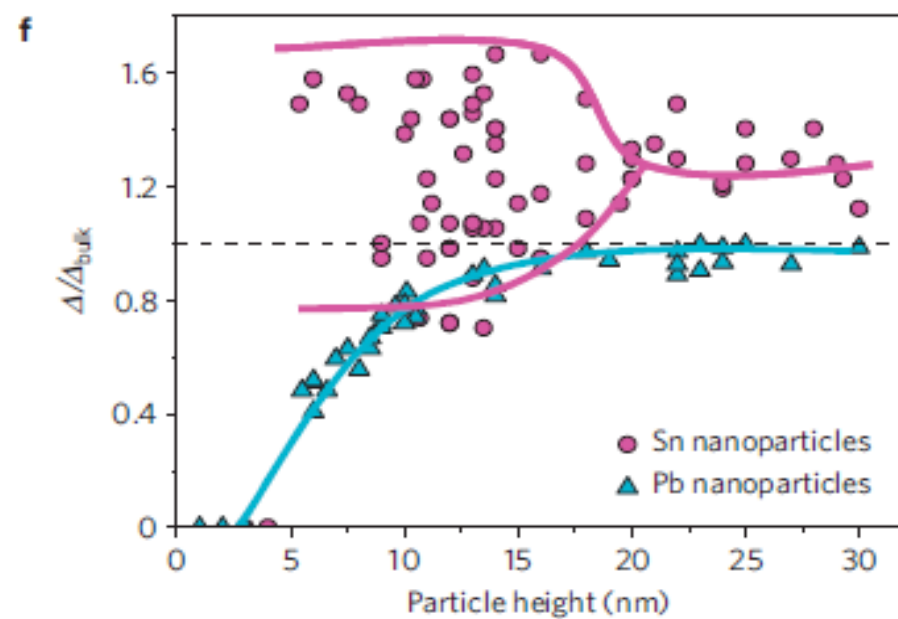
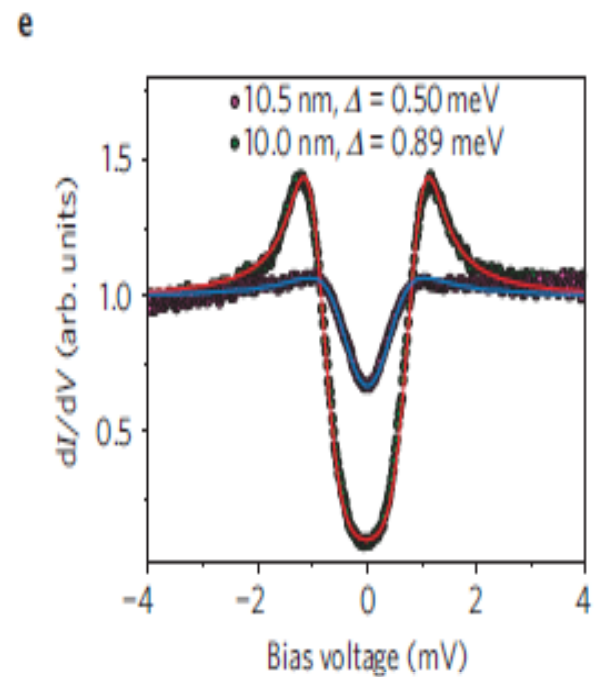
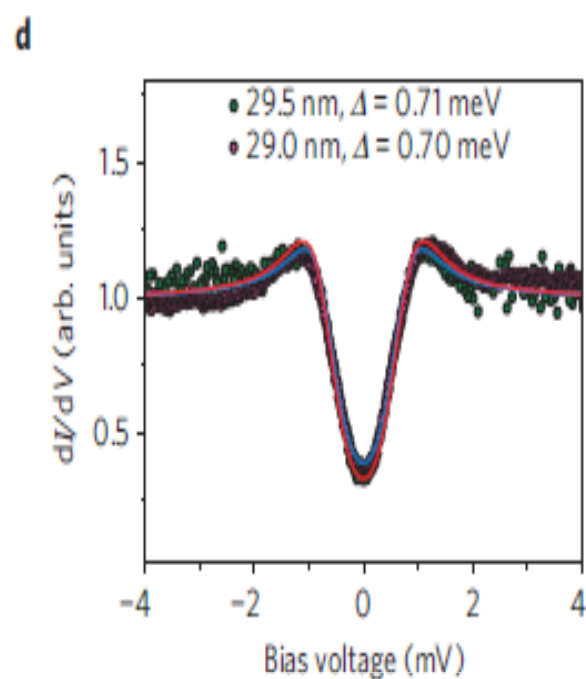
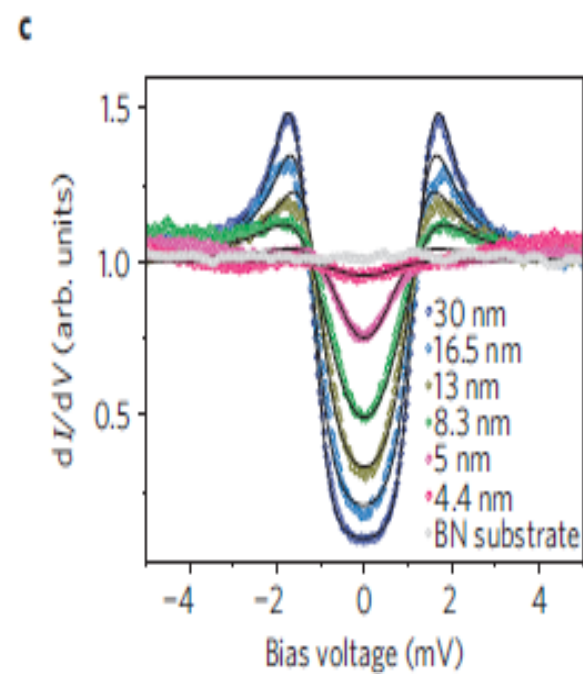


a

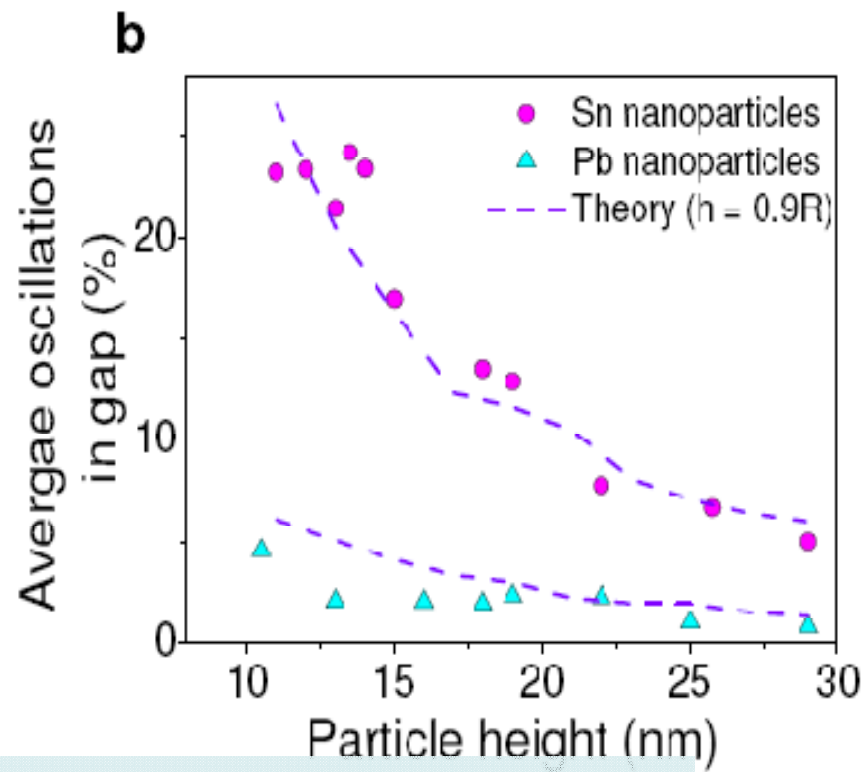
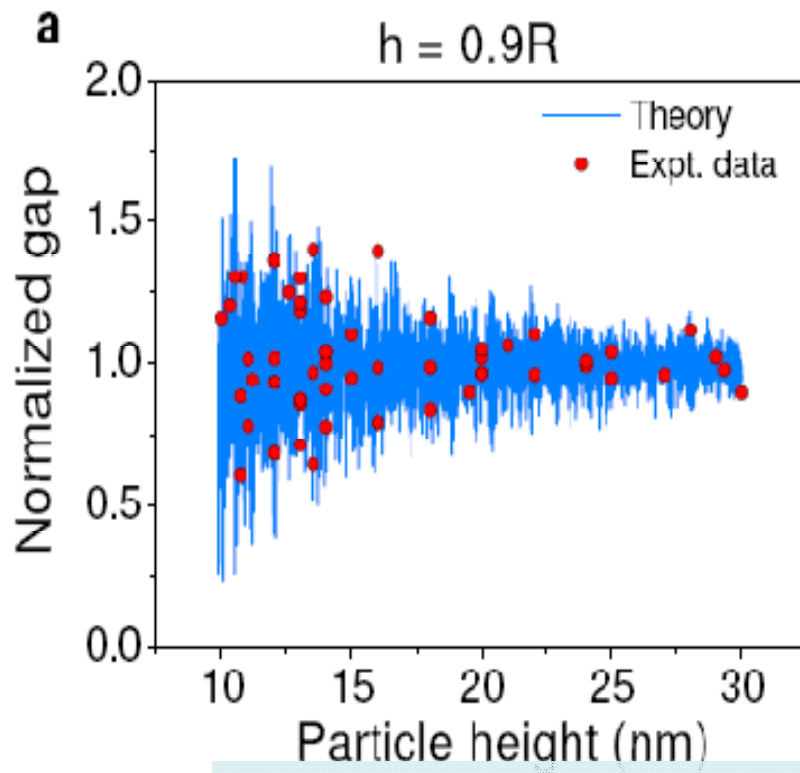


**Experimental
output**

Tunneling
conductance



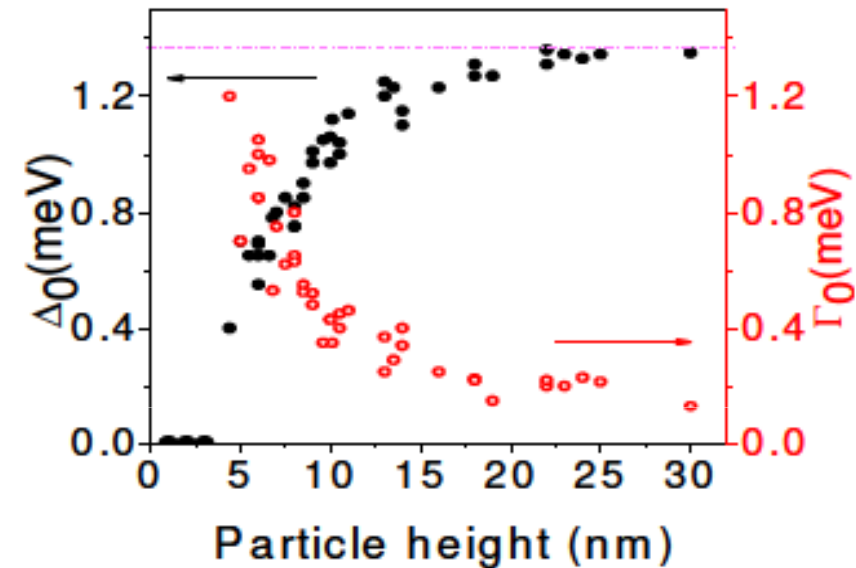
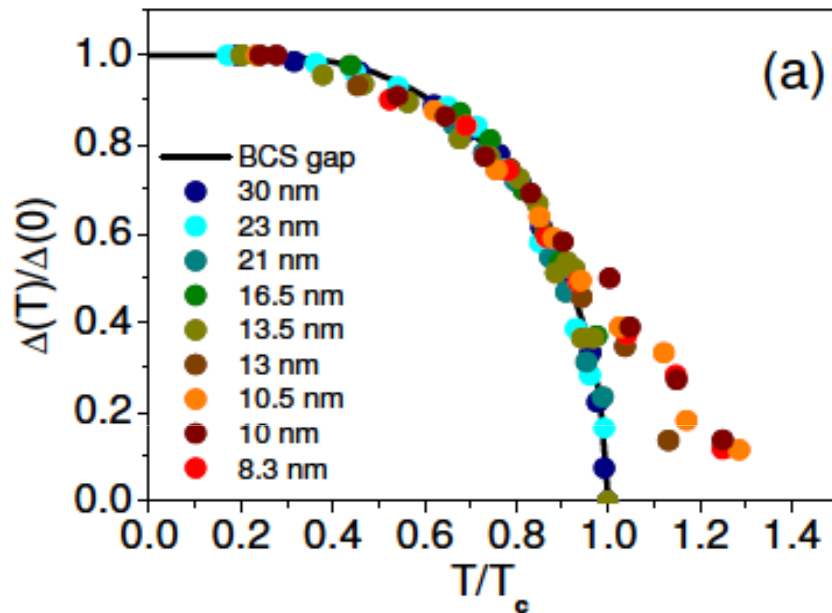
g



Nature Materials 9, 554, (2010)



Superconductivity in single isolated Pb nanograins

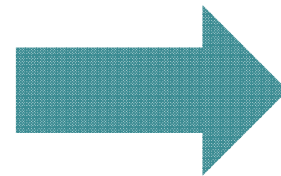


Phys. Rev. B 84,104525 (2011)

Editor Suggestion

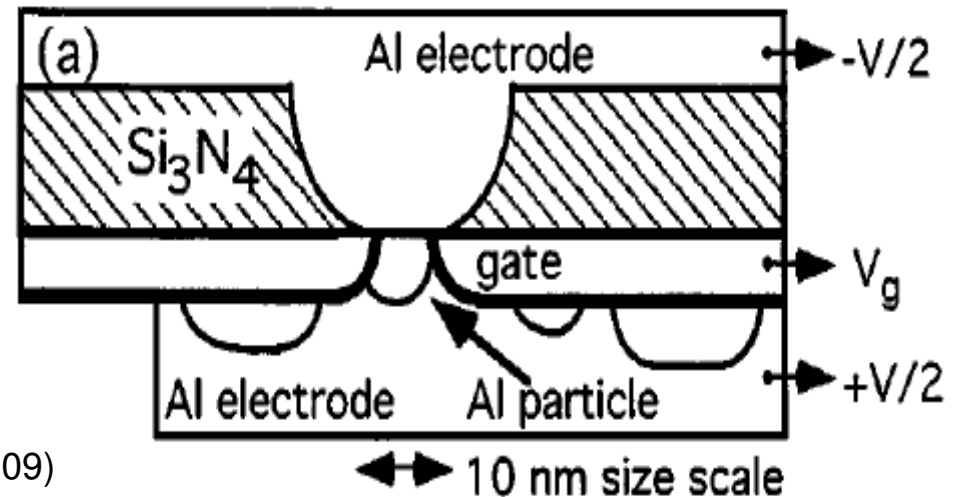
$$\Delta(T) > 0 \text{ for } T > T_c$$

$$\Delta(0) \downarrow\downarrow \text{ for } L < 10\text{nm}$$



Physics
beyond
mean-field

Ralph, Black, Tinkham,
Superconductivity in
Single Metal Particles
PRL 74, 3241-3244 (1995).



Nature 404, 971974 (2000), Science 324, 1314 (2009)

Even for $\delta / \Delta_0 \leq 1$ there is
superconductivity
Odd-even effects

No isolated, size/shape unknown

**Theory
beyond
mean field**

**$\delta / \Delta_0 \ll 1$
Any T**

**$T = 0$
RPA**

**$T \sim T_c$
Static Path
Approach**

RPA+SPA
Ribeiro and AGG,
Phys. Rev. Lett. 108,
097004 (2012)

**Any δ / Δ_0
 $T=0$**

Richardson
exact solution
Von Delft, Braun, Dukelsky, Sierra
Exact low energy excitations
BCS fine until $\delta / \Delta_0 \sim 1/2$
Coulomb, phonons?



**Any δ / Δ_0
Any T**

**Not
Yet**

Pb
 $L < 10-15\text{nm}$

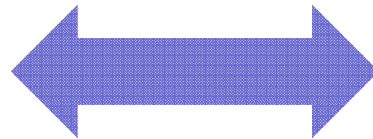
Strong
coupling



Eliashberg
theory

Scattering, recombination,
phonon spec., Coulomb

Thermal
fluctuations δ/T_c



Path integrals

SPA, Scalapino

Quantum
fluctuations
 δ/Δ , E_D



Richardson
equations, RPA

Von Delft, Sierra,
Braun, Dukelsky

BCS Finite-size
corrections



Semiclassical

AGG, et al., Phys. Rev. Lett. 100,
187001 (2008)

Finite $T \sim T_c$

Thermal fluctuations

Static Path Approach

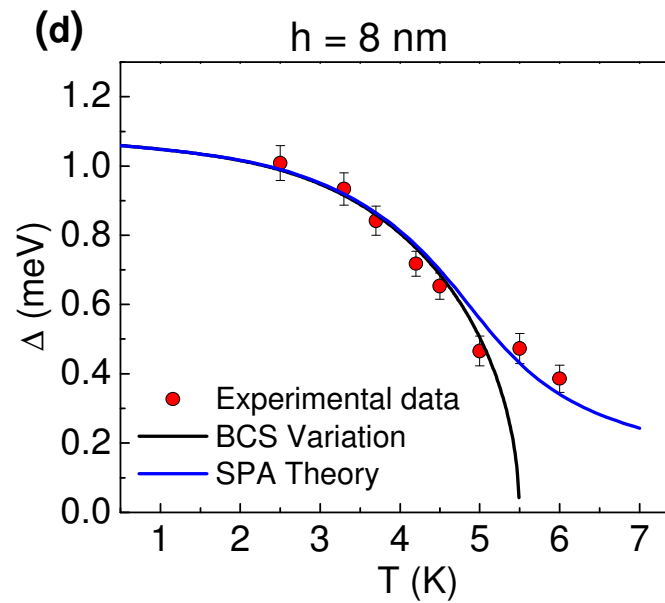
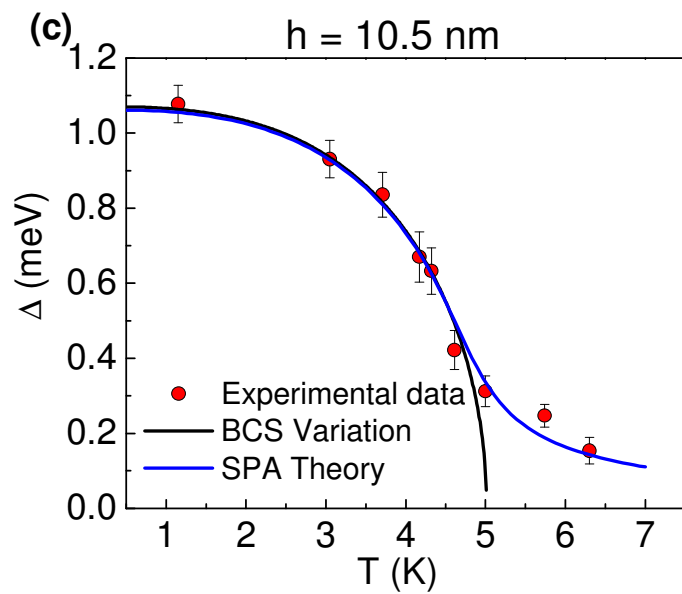
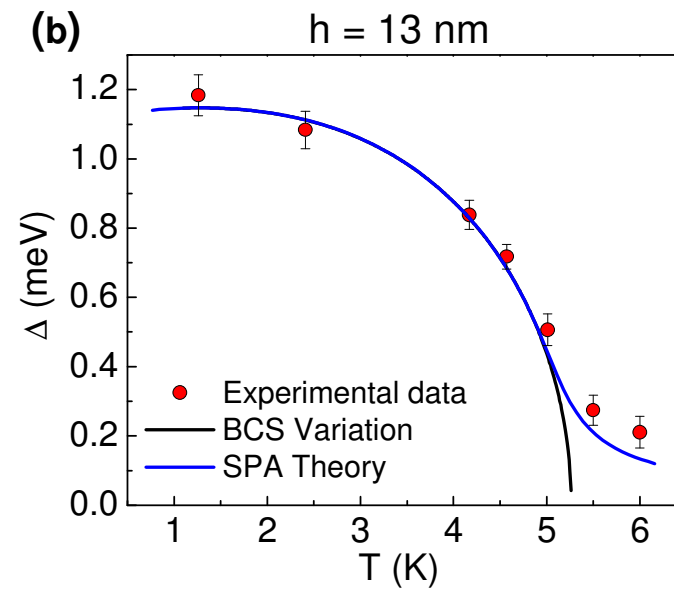
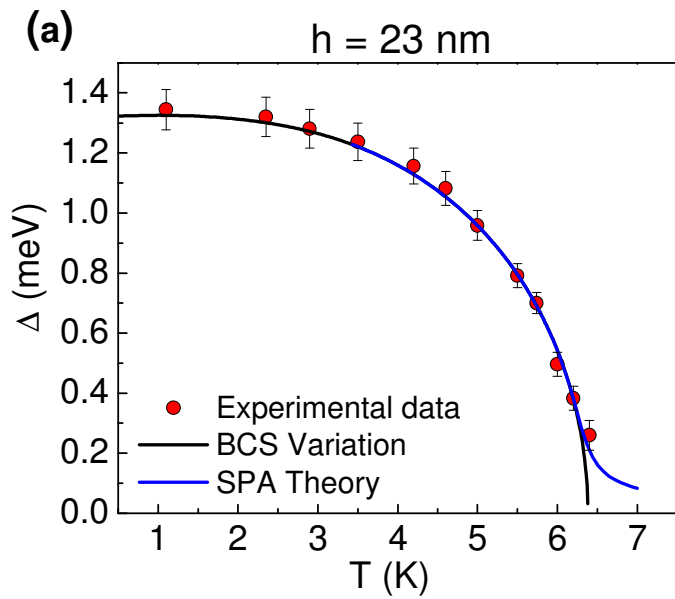
BCS finite size effects

Parmentier, Blatt, 60's
Leboeuf, Peeters, Shanenko, 00's
AGG, et al., PRL 100, 187001 (2008)

Blocking

Richardson formalism

$\lambda(T)$ **simple**
interpolation



T=0

BCS finite size effects

Shell effects are suppressed

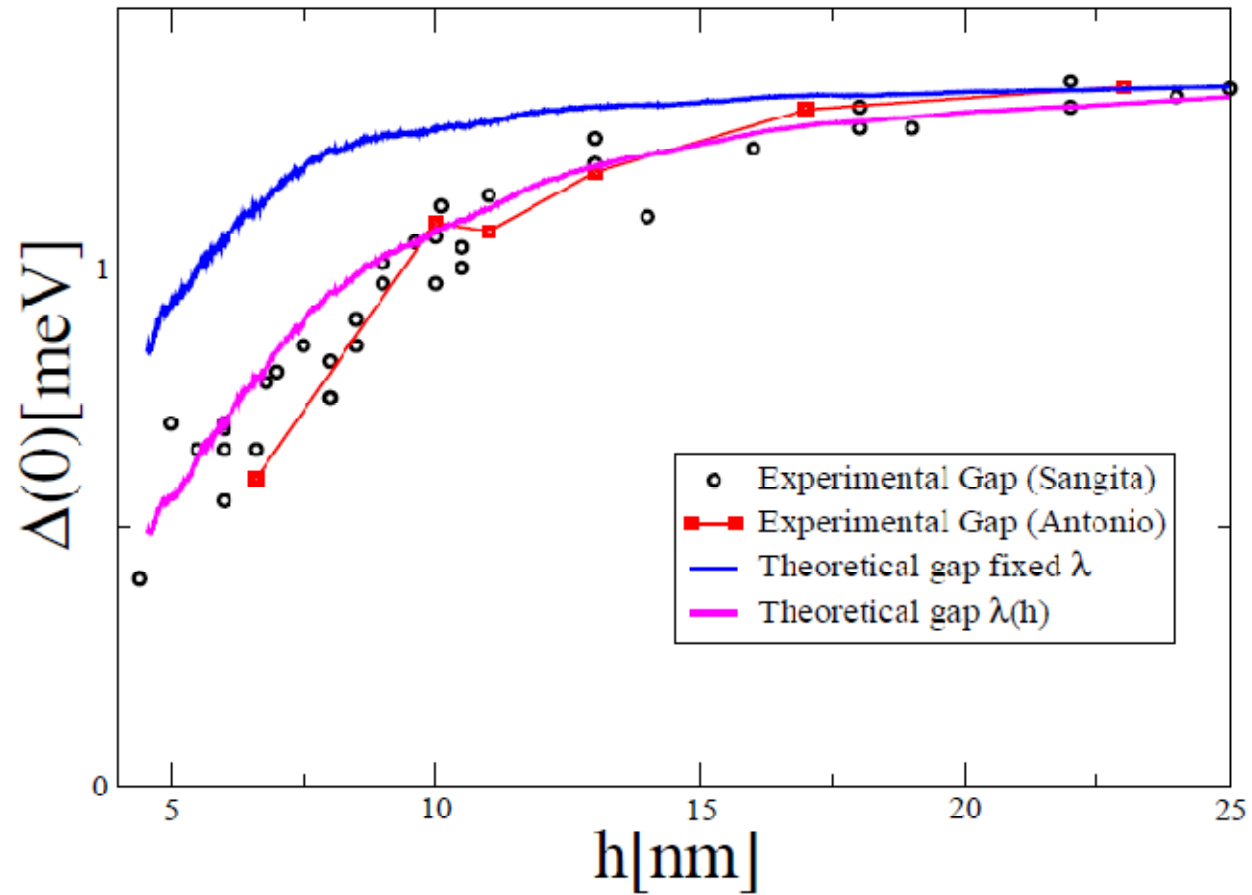
Deviations from BCS

Blocking effect only

No fluctuations!

Not important $R > 5\text{nm}$

$$\lambda = \lambda(h)$$



Phys. Rev. B **84**, 104525 (2011)
Editor Suggestions

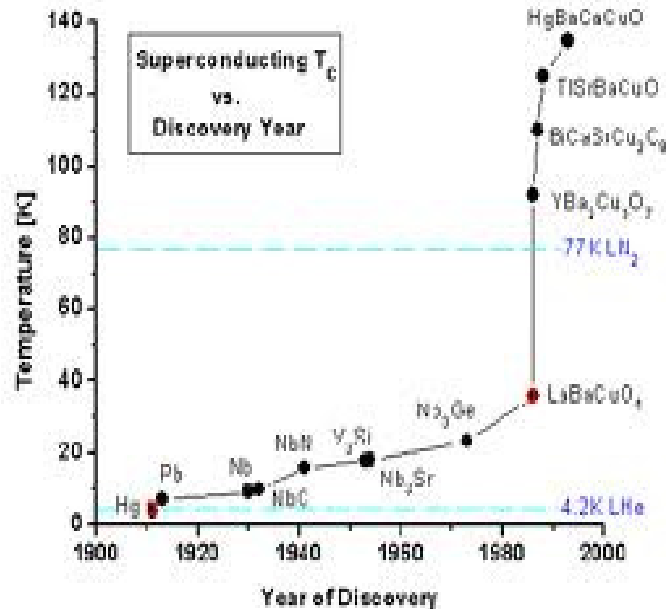
Enhancement of superconductivity in iron pnictides by finite size effects

Phys. Rev. B 84, 172502 (2011)

Pedro Sacramento, Lisbon
Miguel Araujo, Lisbon

Iron Pnictides ?

Phys. Rev. B 84, 172502 (2011)



Wishes

Fears

Robustness

No Fermi liquid

Experiments

No decent model

$\xi \sim 2$ nm

Mean field is OK
in iron pnictides

We have got
the code

Is technically
feasible?

Any
reasonable
model?



2 band model

Bang, Choi,
Raghu,
Scalapino

$$\epsilon_h(k) = t_1^h (\cos k_x + \cos k_y) + t_2^h \cos k_x \cos k_y + \epsilon^h$$

$$\epsilon_e(k) = t_1^e (\cos k_x + \cos k_y) + t_2^e \cos \frac{k_x}{2} \cos \frac{k_y}{2} + \epsilon^e$$

Gap equation

$$\Delta_h = - \sum_{k'} V_{hh} \Delta_h \frac{\tanh(\frac{E_h(k')}{2T})}{2E_h(k')} + V_{he} \Delta_e \frac{\tanh(\frac{E_e(k')}{2T})}{2E_e(k')}$$

$$\Delta_e = - \sum_{k'} V_{eh} \Delta_h \frac{\tanh(\frac{E_h(k')}{2T})}{2E_h(k')} + V_{ee} \Delta_e \frac{\tanh(\frac{E_e(k')}{2T})}{2E_e(k')}$$

Critical Temp

$$T_c(L) \approx 1.136 \omega_{\text{AFM}} \exp(-1/\lambda_{\text{eff}})$$

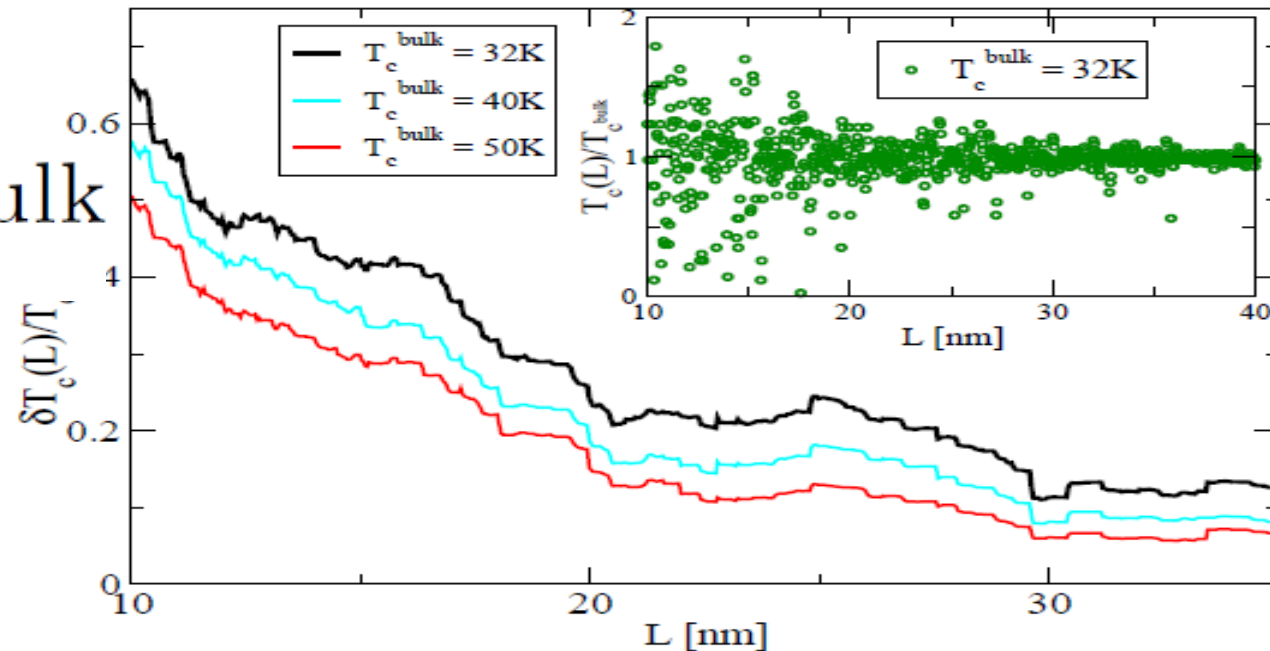
$$\lambda_{\text{eff}} = \lambda_{\text{bulk}} \sqrt{\frac{N_{\xi_e}^e(0) N_{\xi_h}^h(0)}{N^e(0) N^h(0)}}$$

Rectangle

$$\delta T_c(L)/T_c^{\text{bulk}}$$

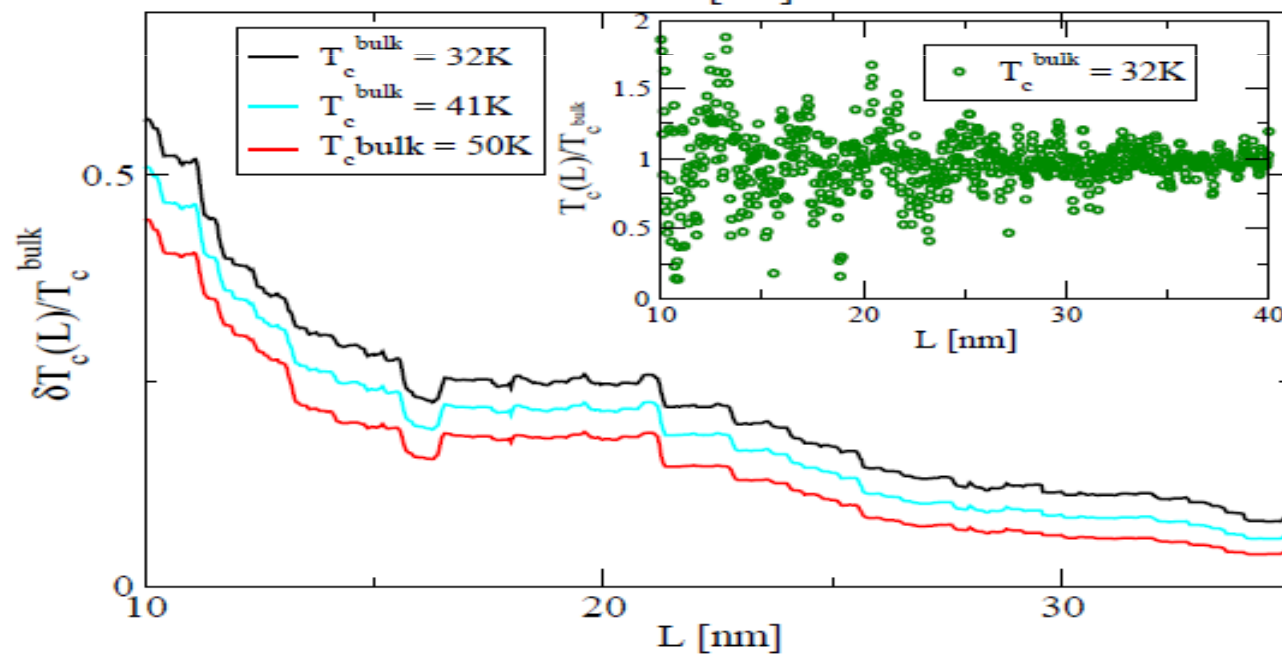
2 Band

Raghu, et al.



5 Band

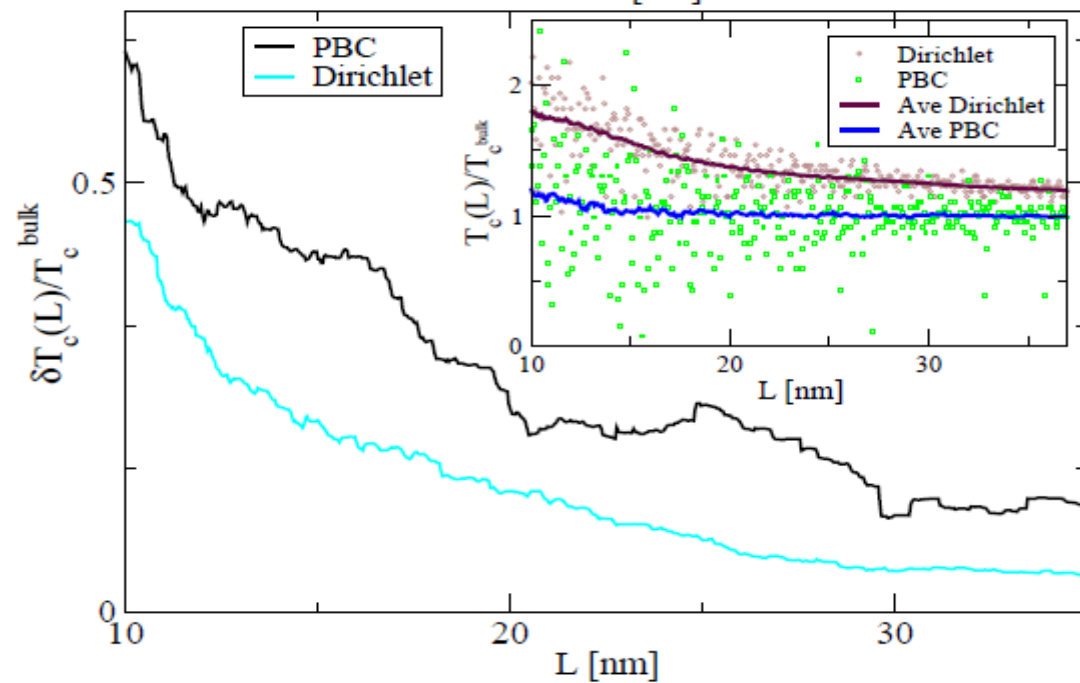
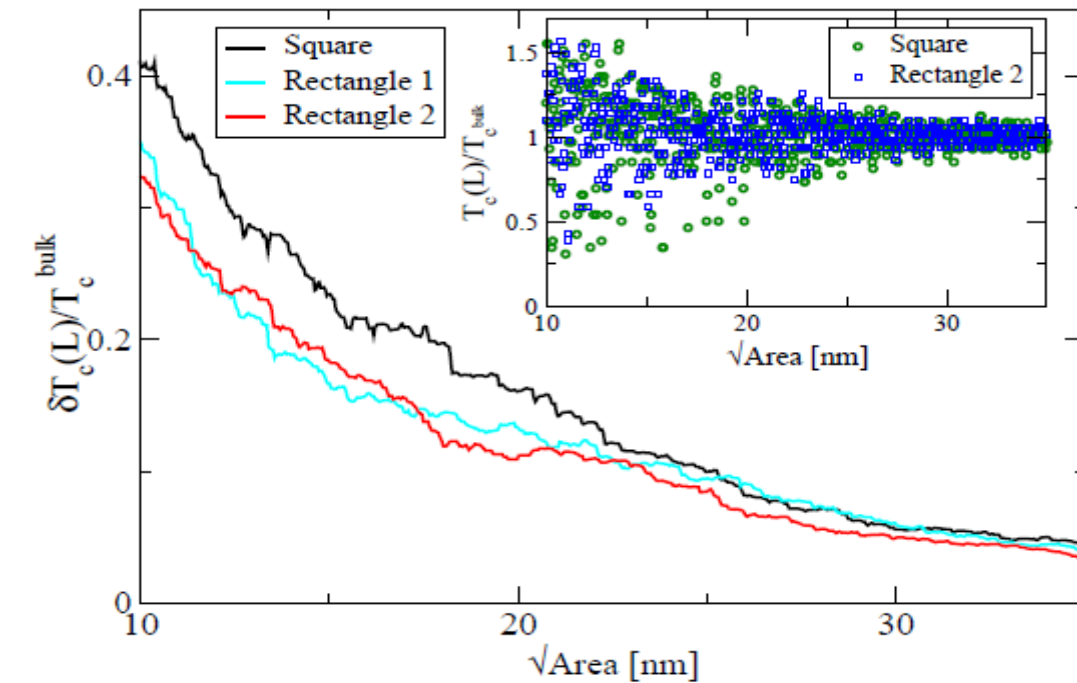
Graser,
Scalapino



Robustness

Geometry

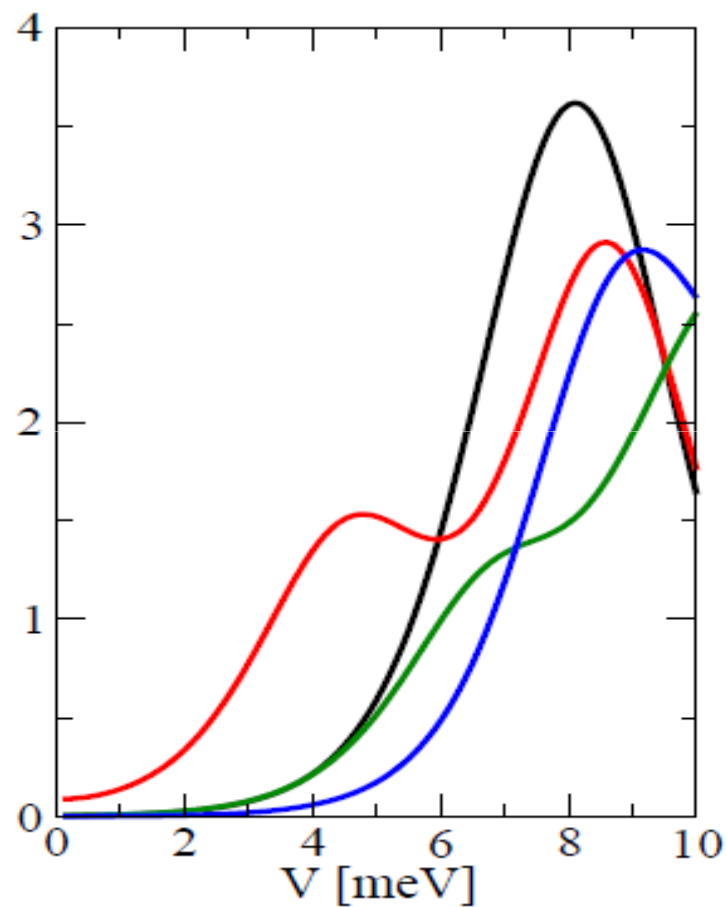
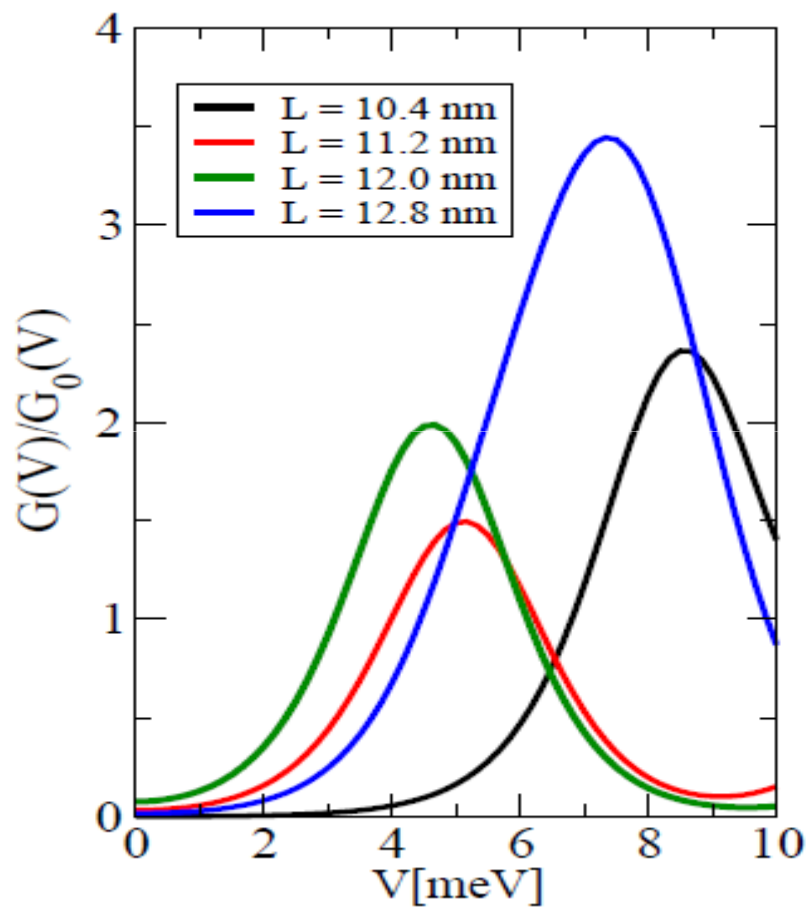
Boundary conditions



Experiments



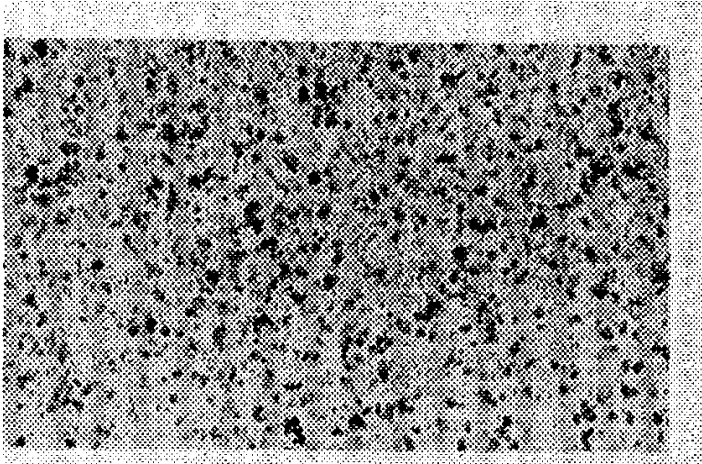
STM



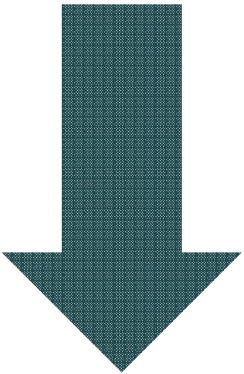
Differential
conductance

$$G(V) = \frac{1}{4Tk_B} \int_{-\infty}^{\infty} d\omega N_s(\omega) \left[\frac{1}{\cosh^2 \left(\frac{\omega+V}{2k_B T} \right)} \right]$$

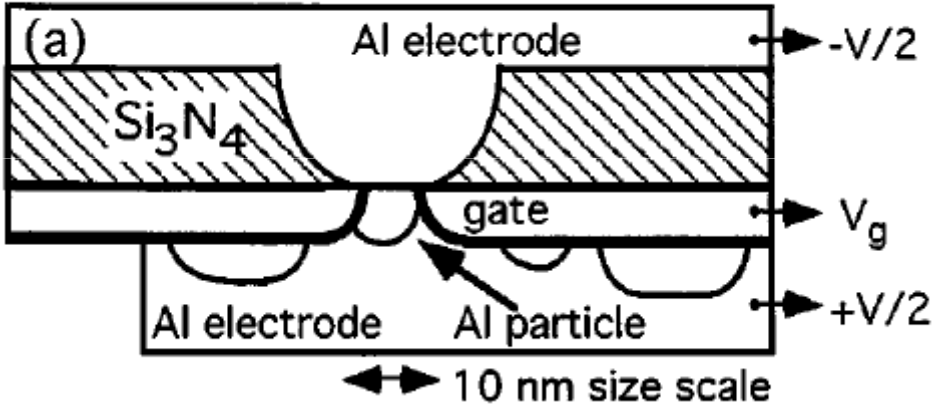
+Experimental control



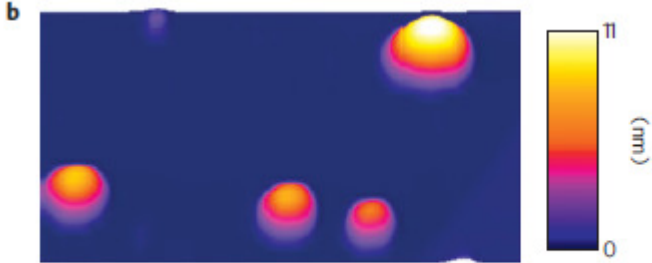
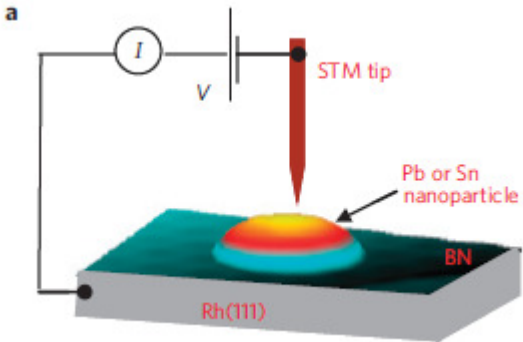
1966



+Predictive power



1995

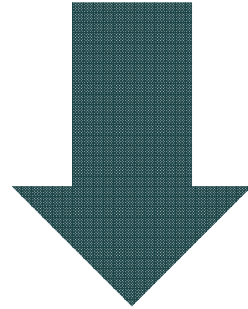


2010

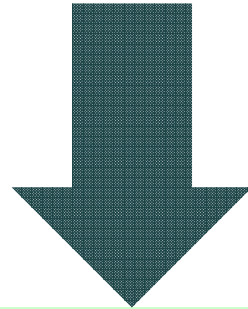
Nature Physics **6**, 104 (2010), *Science* 324, 1314 (2009).

2020

CONTROL



PREDICTIVE POWER



**Enhancement of
superconductivity**

THANKS!