Smaller is different and maybe holographic

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Pedro Ribeiro
Dresden

Santos & Way
Santa Barbara

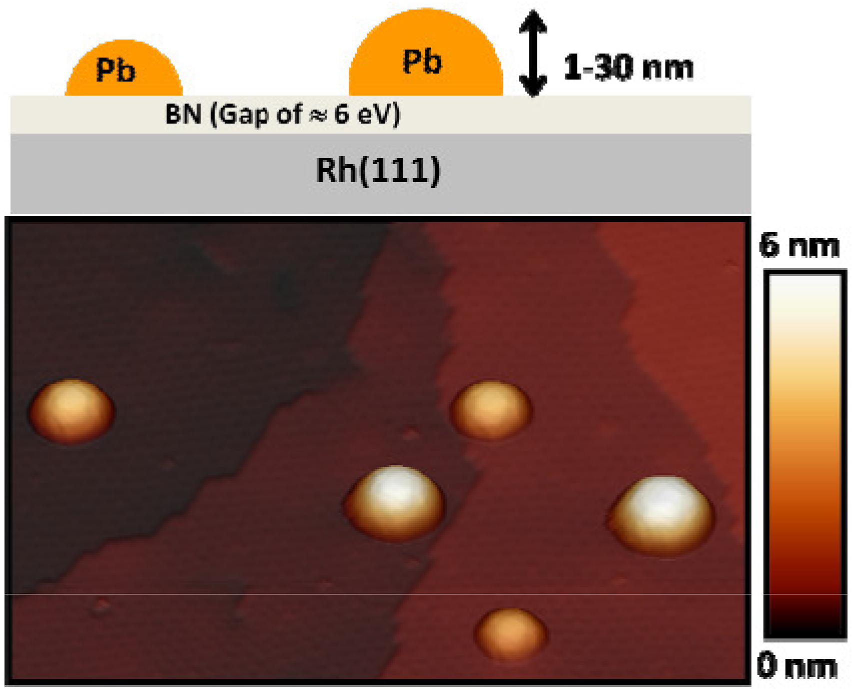
PRL 108, 097004 (2012)

arXiv:1204.4189

PRB 84,104525 (2011)
Editor‘s Suggestion

Sangita Bose
Bombay

Klaus Kern
Stuttgart



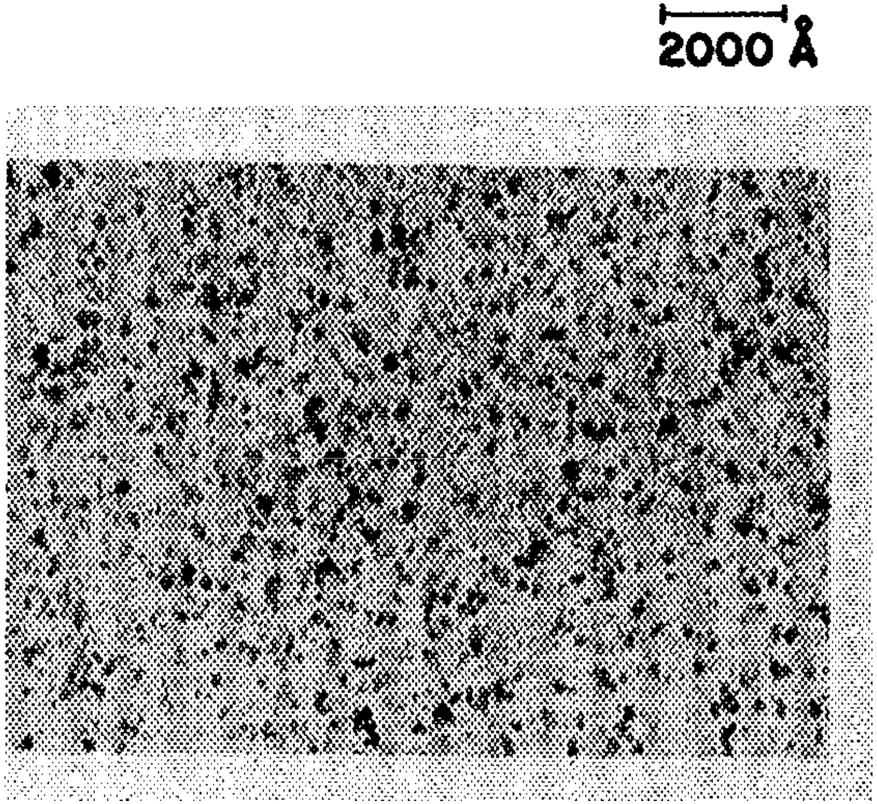
$\Delta \geq \delta$

L ~ 10nm

Thermal and quantum fluctuations in weakly coupled superconductors	Finite size effects in strongly coupled superconductors
PRL 108, 097004 (2012)	arXiv:1204.4189

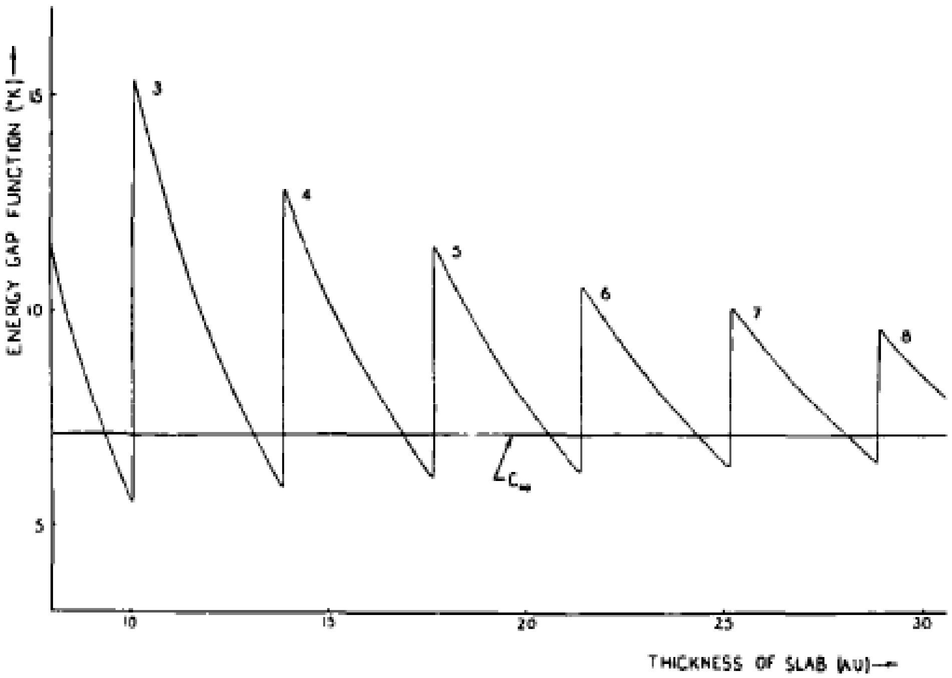
Early days

Metal	T_c (°K)	T_c/T_{c0}	d (Å)	ρ_0
Al	3.0	2.6	40	0.19
Ga	7.2	6.5	⋯	0.20
Sn	4.1	1.1	110	0.31
In	3.7	1.1	110	0.36
Pb	7.2	1.0	⋯	0.53



Abeles, Cohen, Cullen, Phys. Rev. Lett., 17, 632 (1966)
A.M. Goldman, Giaver.....

Thin Films $\Delta < \delta$ Single grains



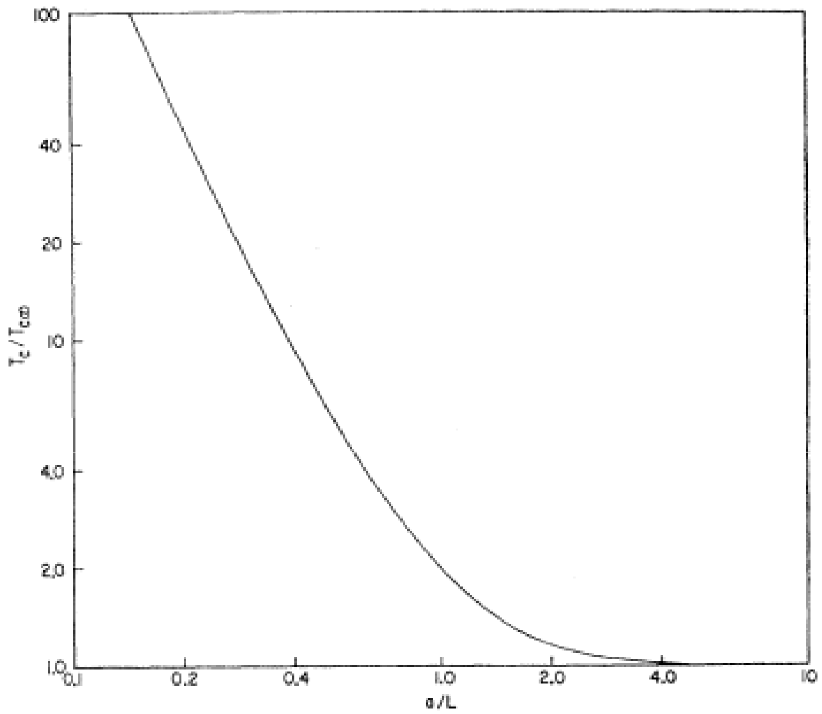


FIG. 1. $(T_c/T_{c\infty})$ versus (a/L) (see Ref. 17).

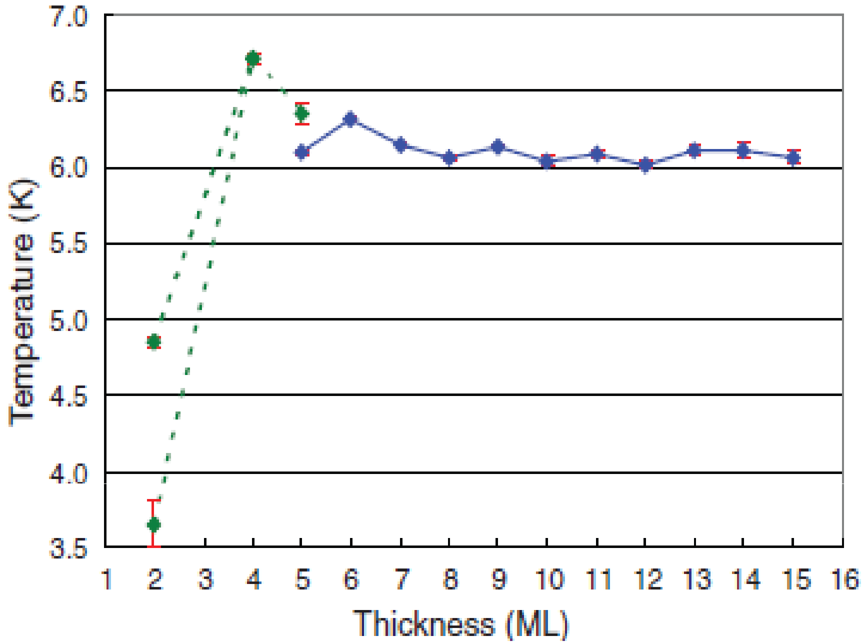
Shape Resonances

Blatt, Thompson
Phys. Lett. 5, 6 (1963)

Shell Effects

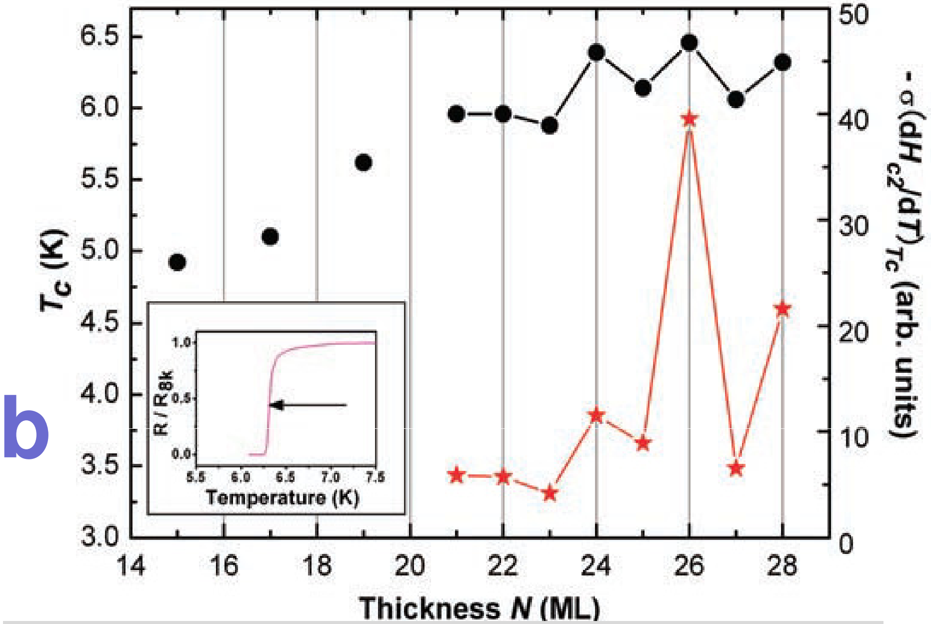
Parmenter, Phys. Rev. 166, 392 (1967)

Recent Atomic scale **control**

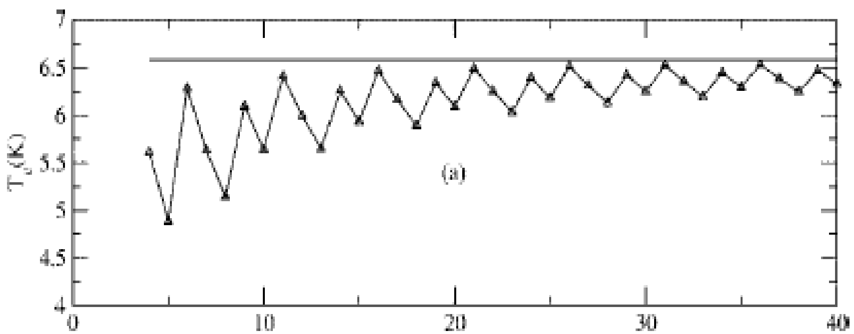


Shih et al., Science 324, 1314 (2009)

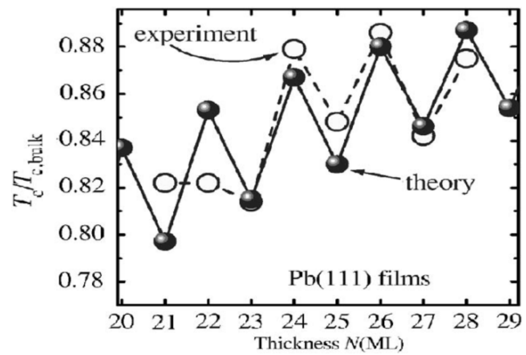
Pb



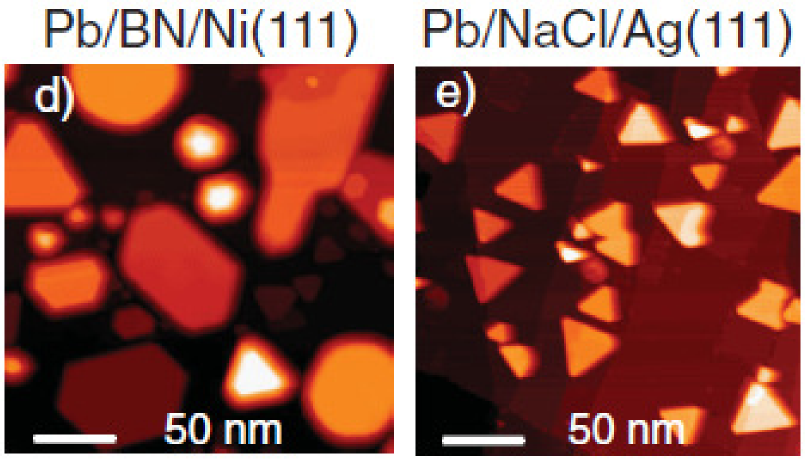
Xue et al., Science 306, 1915 (2004)

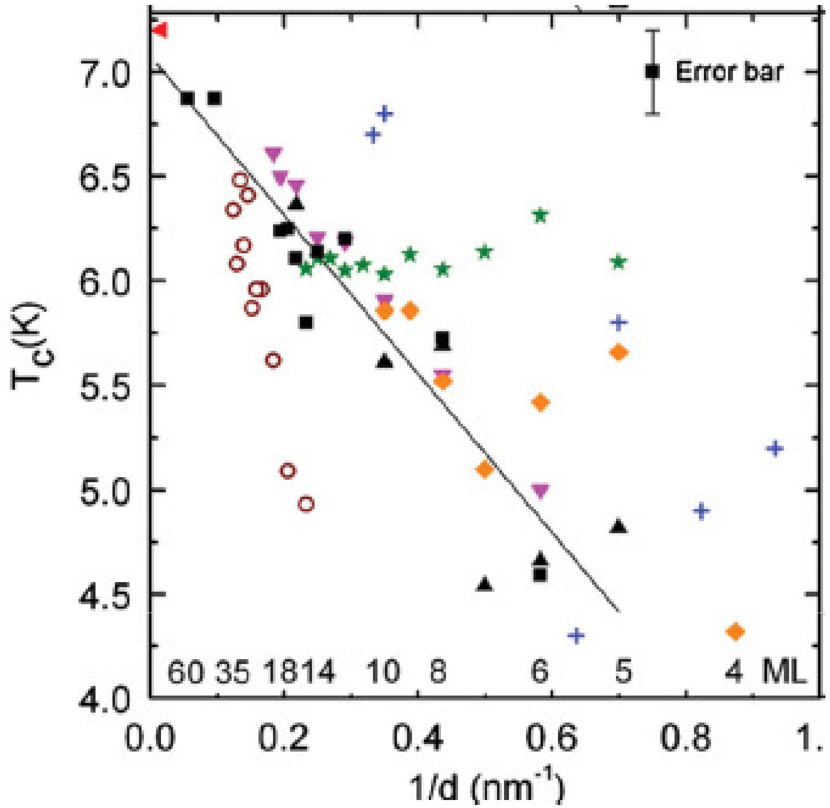


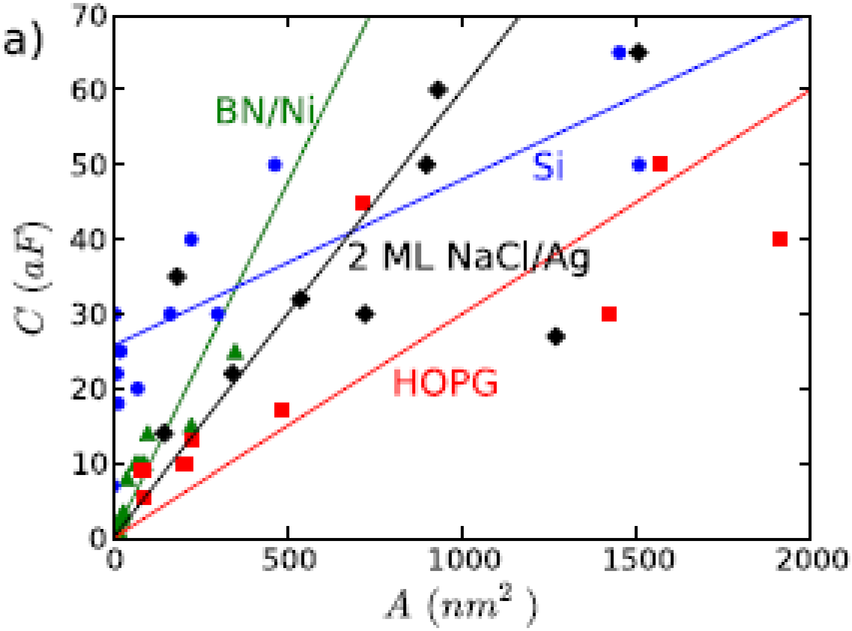
PRB 74 132504 (2006)



Peeters, et al. PRB 75 014519 (2007)







Schneider, et al.,
PRL 102, 207002 (2009)

PRL 108, 126802 (2012)

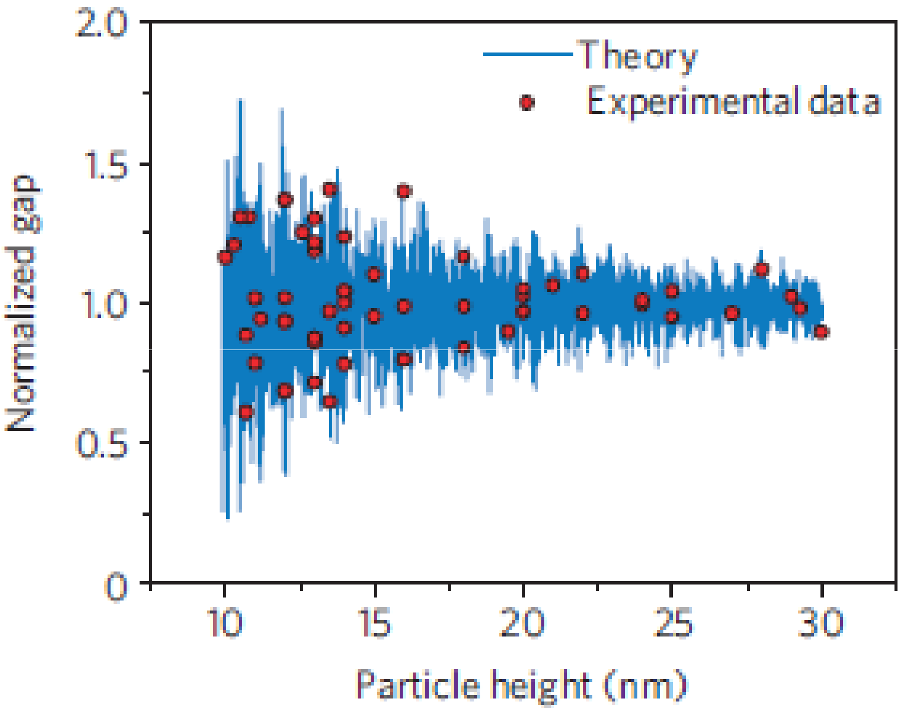
Theory:

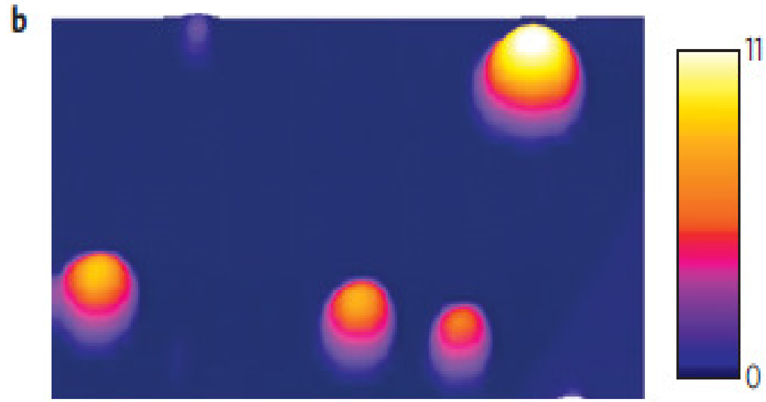
Ribeiro, AGG. PRL 108, 097004 (2012)

Grains
Wires

$\Delta >> \delta$

Sn

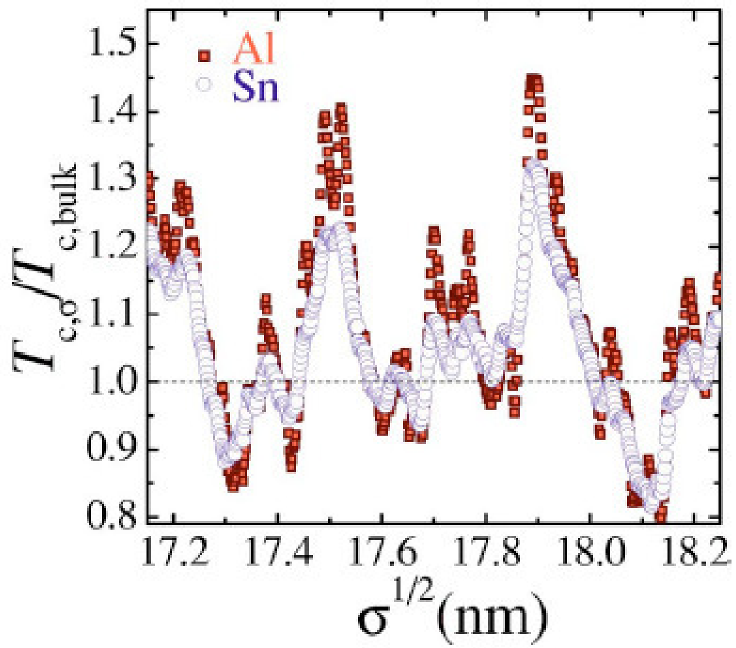




S. Bose, AGG, Nature Mat. 9, 550 (2010), PRB 84 104525 (2011)

PRL 100, 187001 (2008)

Peeters, Shanenko, Croitoru, Zgirski..

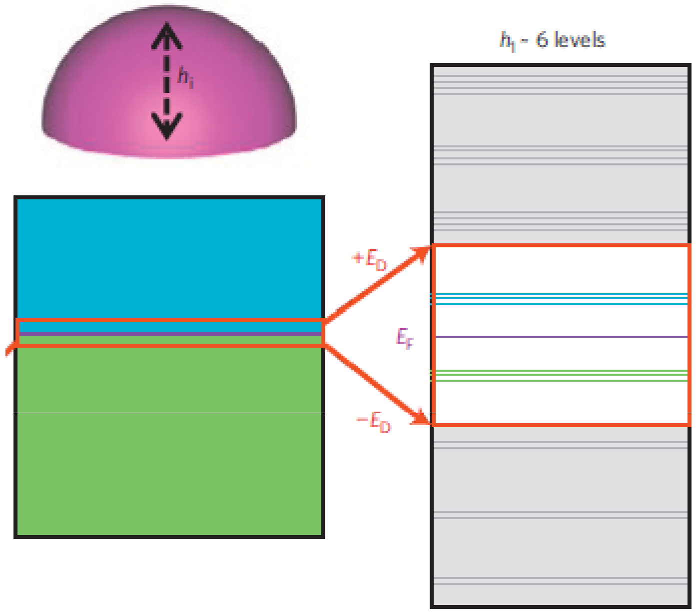


PRB 74, 052502 (2006)

$\Delta >> \delta$ L ~ 10nm

Bogouliobov de Gennes or....

BCS but..



$$H = \sum_{n\sigma} \epsilon_n c^\dagger_{n\sigma} c_{n\sigma} - \sum_{n.n'} I_{n,n'} c^\dagger_{n\uparrow} c^\dagger_{n\downarrow} c_{n'\downarrow} c_{n'\uparrow}$$

$$I(\epsilon_n, \epsilon_{n'}) = \lambda V \delta \int \psi^2_n(\vec{r}) \psi^2_{n'}(\vec{r}) d\vec{r}$$

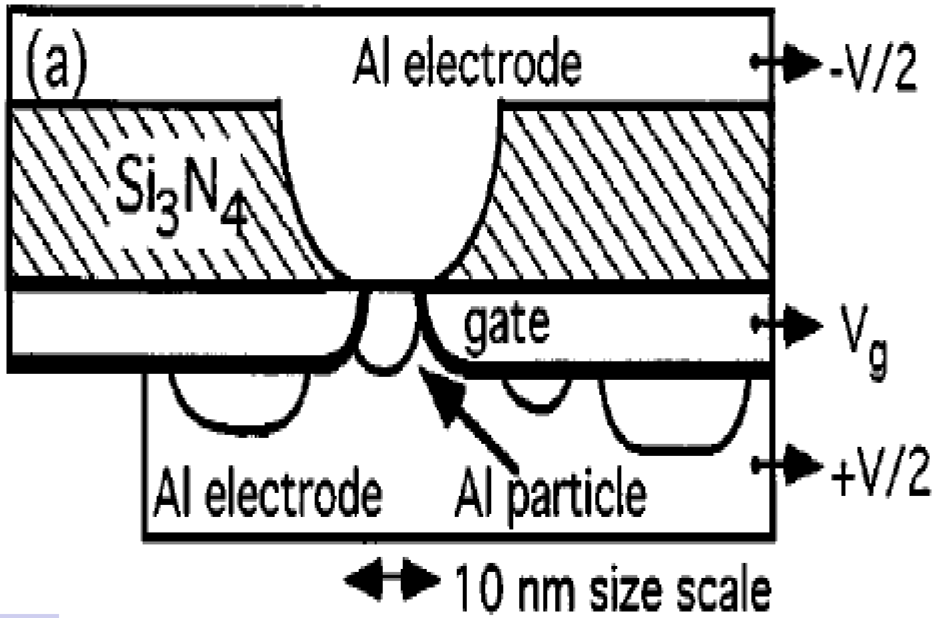
$$\Delta(\epsilon) = \frac{1}{2} \int_{-\epsilon_D}^{\epsilon_D} \frac{\Delta(\epsilon') I(\epsilon, \epsilon')}{\sqrt{\epsilon'^2 + \Delta^2(\epsilon')}} \nu(\epsilon') d\epsilon'$$

Expansion in $1/k_F L$

$\Delta \leq \delta$

Supercon ductivity?

1959



Yes, superconductivity

Ralph, Black,Tinkham,
Superconductivity in Single Metal Particles
PRL, 74, 3241-3244 (1995).

Odd-even effects

Isolated grain?

Beyond mean field

Fluctuations

J. von Delft et al., Phys. Rep., 345, 61 (2001)

Beyond mean field

Quantum Fluctuations	Thermal fluctuations
Random Phase Approx	Path Integral Static Path Approx
Richardson Eqs	Muhlschlegel, Scalapino (1972)

Disorder, Coulomb....

Larkin, Gorkov ....

Fluctuations → $T < T_c$ finite resistivity
Stronger e-e interaction

Thermal fluctuations → Path integral

$$Z = \int \mathcal{D}c^\dagger \mathcal{D}c \; e^{-\int_0^\beta d\tau \left[\sum_{k,\sigma} c^\dagger_{k,\sigma}(\partial_\tau + \varepsilon_k)c_{k,\sigma} - \lambda\delta\left(\sum_k c^\dagger_{k,1}c^\dagger_{-k,-1}\right)\left(\sum_{k'} c_{-k',-1}c_{k',1}\right)\right]}$$

$$= \int \mathcal{D}\Delta^\dagger \mathcal{D}\Delta \mathcal{D}c^\dagger \mathcal{D}c \; e^{-\int_0^\beta d\tau \left\{\sum_{k,\sigma} c^\dagger_{k,\sigma}[\partial_\tau + \varepsilon_k]c_{k,\sigma} + \sum_k \left(c^\dagger_{k,1}c^\dagger_{-k,-1}\Delta(\tau) + \Delta^\dagger(\tau)c_{-k,-1}c_{k,1}\right) + (\lambda\delta)^{-1}\Delta^\dagger(\tau)\Delta(\tau)\right\}}$$

Hubbard-Stratonovich transformation

0 d grains

Δ homogenous

↔

Static path approach (SPA)

Scalapino et al. 70's

$$\frac{Z}{Z_0} = \int d|\Delta| \, |\Delta| \, e^{-\beta\mathcal{A}(|\Delta|)}$$

$$\xi_k = \sqrt{\varepsilon_k^2 + \Delta^\dagger\Delta}$$

$$\mathcal{A}(|\Delta|) = \left\{(\lambda\delta)^{-1}|\Delta|^2 + \sum_{k'}\left[(|\varepsilon_{k'}| - \xi_{k'}) - \frac{2}{\beta}\log\left(\frac{e^{-\beta\xi_{k'}} + 1}{e^{-\beta|\varepsilon_{k'}|} + 1}\right)\right]\right.$$

T=0 deviations from mean field

Richardson's equations

Explicit?

Von Delft, Braun, Dukelsky, Marsiglio, Sierra

$$-\frac{1}{\lambda d}+\sum_{j=1}^{m}{}'\frac{1}{E_i-E_j}=\frac{1}{2}\sum_{k=1}^{n}\frac{1}{E_i-\epsilon_k}\qquad i=1,\ldots,m$$

Ground state

$$E=2\sum_{i=1}^{m}E_i+\sum_{B}\epsilon_B$$

Pair breaking

$$e^b=\epsilon_a+\epsilon_b+E_{g.s.}(\epsilon_a,\epsilon_b)-E_{g.s.}$$

OK expansion in δ/Δ_0 !

Richardson ~ 1968,
Yuzbashyan, Altshuler ~ 2005

Pair breaking energy

Energy gap Δ

Path integral?

RPA + Mean field ?
Matveev, Larkin

$D \equiv E_D$

$d \equiv \delta$

$$\Delta^b = 2\Delta_0 - d\sqrt{1+\frac{\Delta_0^2}{D^2}} + \frac{d\Delta_0}{D}\left[1+\phi(\lambda)\right]$$

Blocking effect

Remove levels closest to E_F

>>

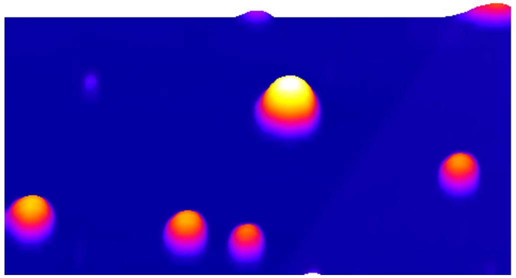
Quantum fluctuations

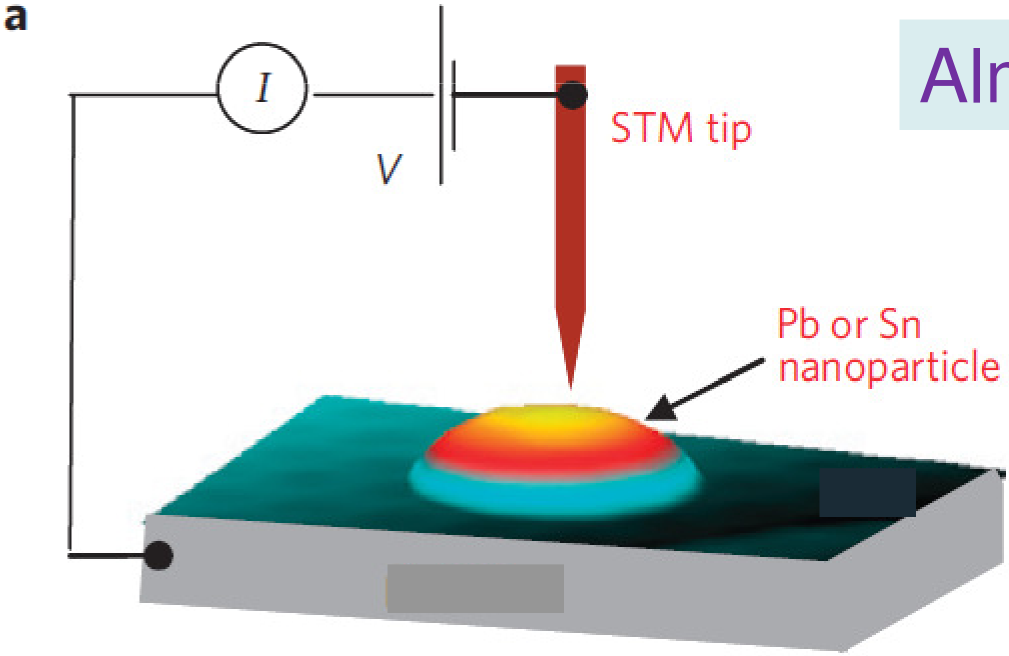
Important
$\delta \sim \Delta$
L<5nm

Single, Isolated Pb grains


Kern


Bose





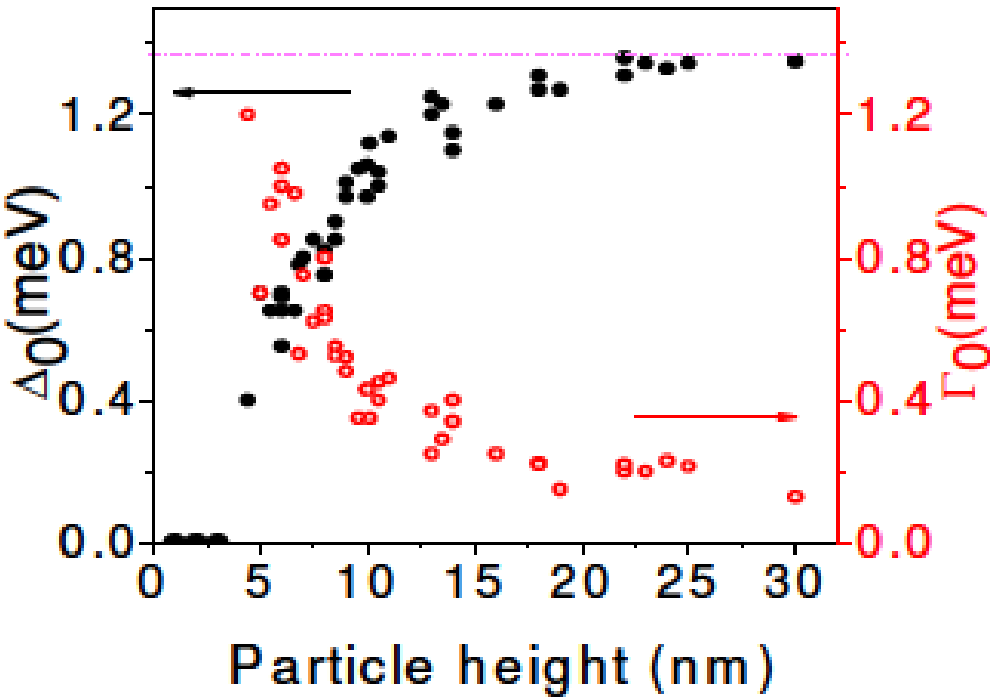
R ~ 4-30nm

B closes gap

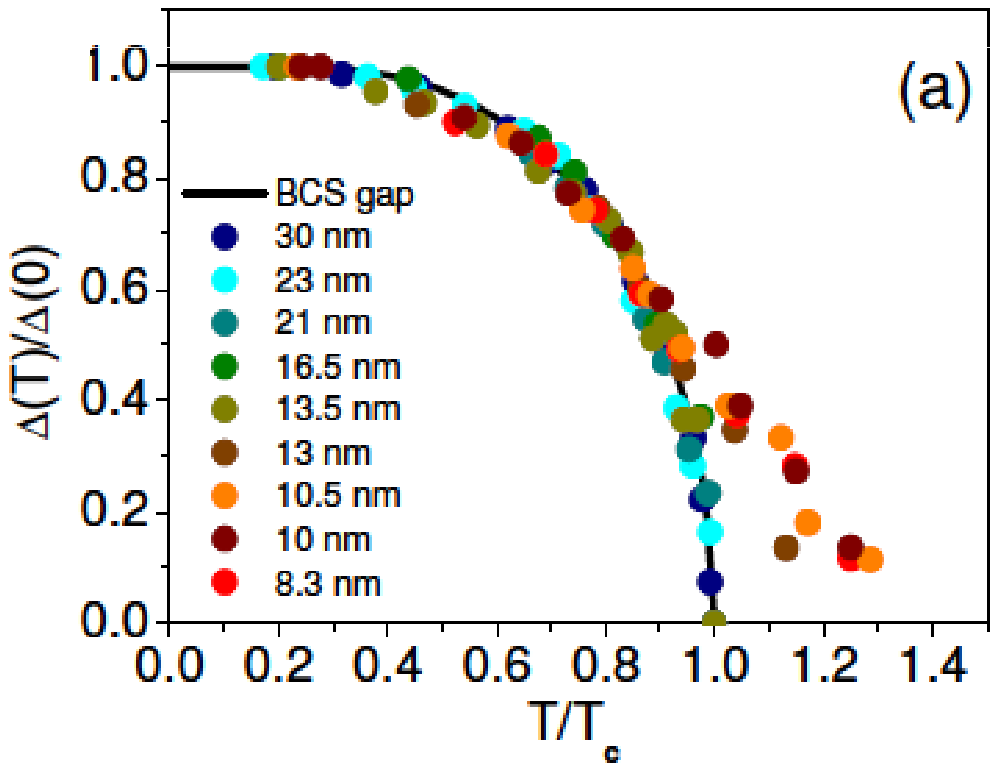
Almost hemispherical

Experimental output

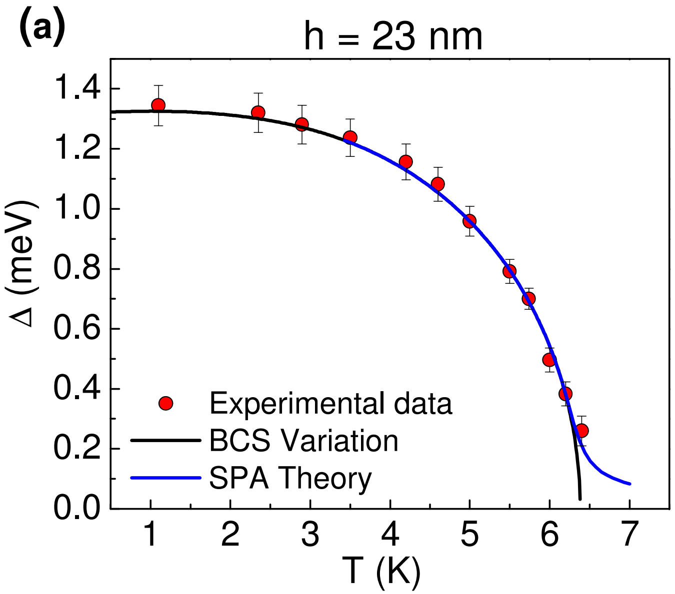
Tunneling conductance

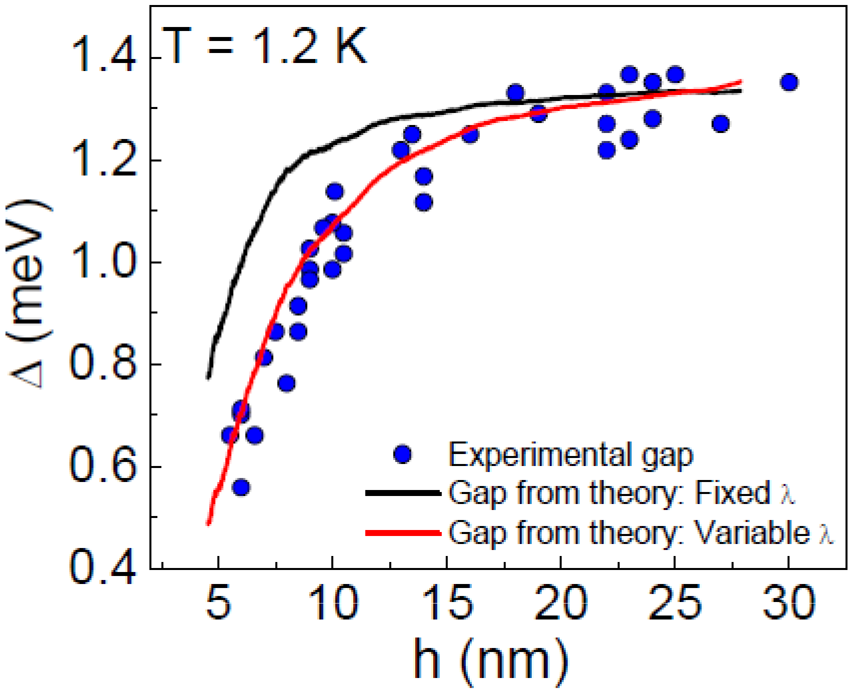


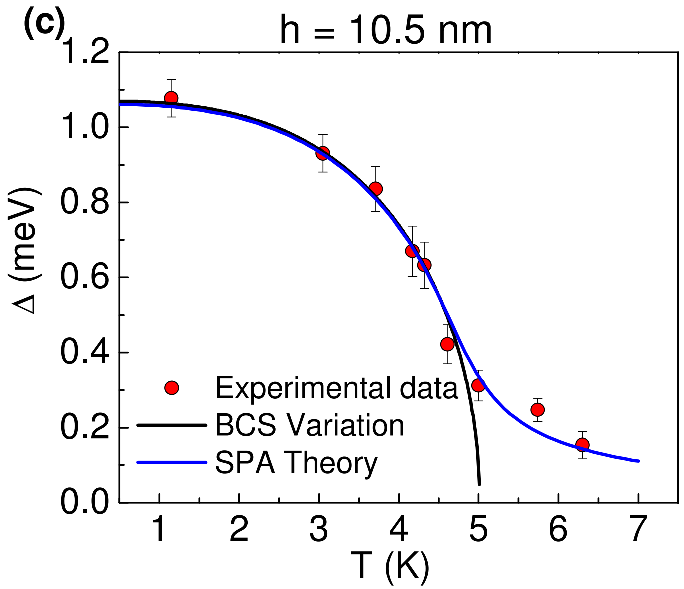
Δ0(meV)
Γ0(meV)
0.0
0.4
0.8
1.2
0
5
10
15
20
25
30
Particle height (nm)



(a)
BCS gap
30 nm
23 nm
21 nm
16.5 nm
13.5 nm
13 nm
10.5 nm
10 nm
8.3 nm
Δ(T)/Δ(0)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
T/Tc







Phys. Rev. B 84,104525 (2011)
Editor's Suggestion

Quantum + Thermal?

Any δ/Δ_0
T=0

Richardson solution

Coulomb?

Dynamical phonons?

Exact low energy

BCS OK $\delta/\Delta_0 \sim 1/2$

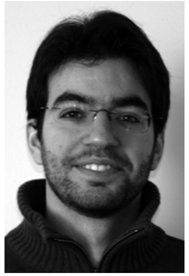
$\delta/\Delta_0 << 1$
Any T

SPA+RPA?

Divergences at intermediate T

Rossignoli and Canosa
Ann. of Phys. 275, 1, (1999)

RPA+SPA ,Ribeiro and AGG, **Phys. Rev. Lett. 108, 097004 (2012)**



$$\Delta(\tau) = \Delta_0 + \delta\Delta(\tau)$$

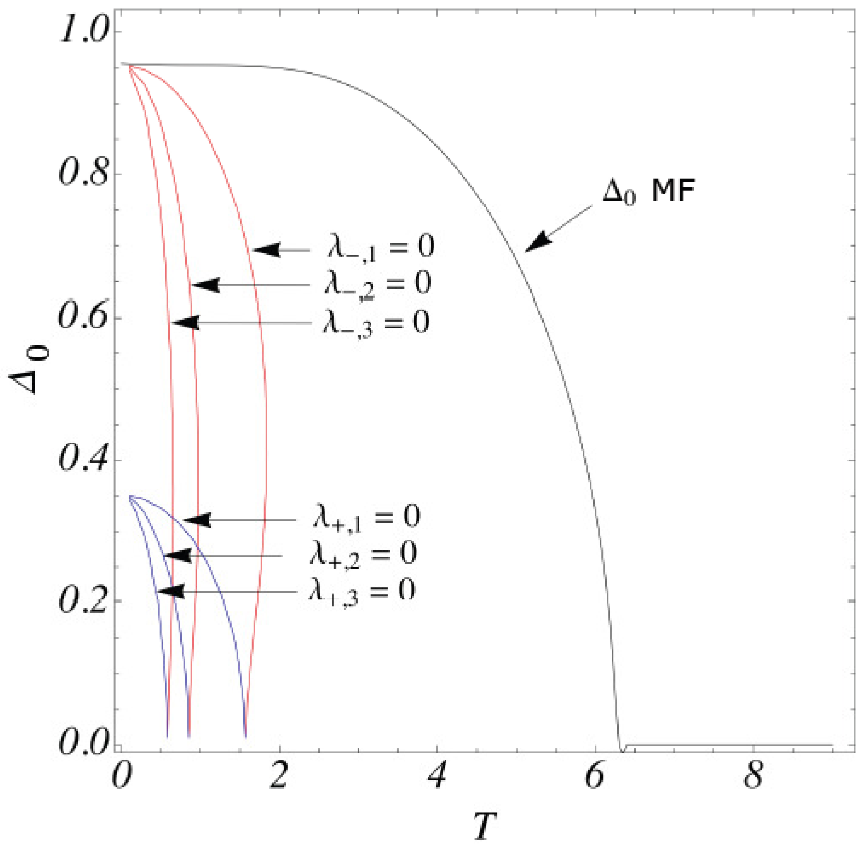
SPA+RPA	Δ_0 in SPA	$\delta\Delta(\tau)$ Small
Rossignol et al.	Cartesian!	

$$Z = \int d|\Delta_0|^2 \int \mathcal{D}' \delta\Delta \, e^{-\beta\mathcal{A}[\Delta]}$$

$$\beta\mathcal{A}[\Delta] = \beta\mathcal{A}(\Delta_0) + \sum_{n\neq 0} \begin{pmatrix} \Delta_n \\ \bar{\Delta}_n \end{pmatrix}^\dagger \Xi_m \begin{pmatrix} \Delta_n \\ \bar{\Delta}_n \end{pmatrix}$$

$$\Delta_n = \frac{1}{\beta} \int d\tau e^{i\Omega_n \tau} \Delta(\tau)$$

$$\Omega_n = 2\pi\beta^{-1} n$$



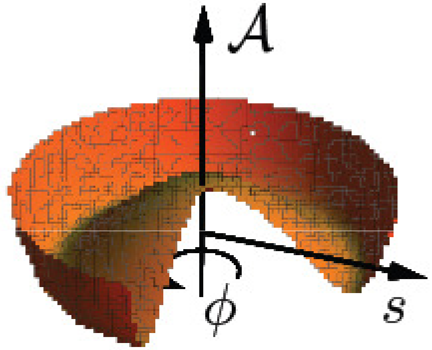
Where's the problem?

Of course the coordinates!!!

$$\Delta(\tau) = s(\tau) e^{i\phi(\tau)}$$

$$s^2(\tau) = s_0^2 + \delta s^2(\tau)$$

$$\phi(\tau) = \phi_0 + 2\pi M \tau/\beta + \delta\phi(\tau)$$



$$\mathcal{A}[s,\phi,M] = \mathcal{A}_0(s_0) + i\pi \sum_k \left(1 - \frac{\varepsilon_k}{\xi_{0k}}\right) \frac{1}{\beta} M + \left(\sum_k \frac{s_0^2}{2\xi_{0k}^3}\right) \frac{1}{\beta^2} (\pi M)^2 + \frac{1}{2} \sum_{m \neq 0} \begin{pmatrix} \tilde{s}^2_{-m} \\ \phi_{-m} \end{pmatrix} . \Xi(s_0)_m \cdot \begin{pmatrix} \tilde{s}^2_m \\ \phi_m \end{pmatrix}$$

$$s^2_m = \frac{1}{\beta} \int d\tau \, e^{i\Omega_m \tau} \delta s^2(\tau)$$

$$\phi_m = \frac{1}{\beta} \int d\tau \, e^{i\Omega_m \tau} \delta\phi(\tau)$$

$$\tilde{s}^2_m = \left[\beta \sum \frac{1}{2\xi_{0k}} \tanh\left(\frac{\xi_{0k}}{2}\right)\right] s^2_m$$

No divergences!!

$$\Xi\left(s_0\right)_m = \begin{pmatrix} \Xi_m^{\tilde{s}^2\tilde{s}^2} & \Xi_m^{\tilde{s}^2\phi} \\ \Xi_m^{\phi\tilde{s}^2} & \Xi_m^{\phi\phi} \end{pmatrix}$$

$$r_k = \frac{1}{2\xi_{0k}} \tanh\left(\frac{\beta\xi_{0k}}{2}\right)$$

$$\Xi_{i,j} = \beta \frac{\delta^2}{\delta X_i \delta X_j} \mathcal{A}[s, \phi]$$ with $$X_i = s(\tau), \phi(\tau)$$

$$\Xi_m^{\phi\phi} = \sum_k r_k \frac{2\beta s_0^2 \Omega_m^2}{\Omega_m^2 + \left(2\xi_{0k}\right)^2} \qquad \sim \frac{\beta}{\delta}\Omega_m^2$$

$$\Xi_m^{\tilde{s}^2\tilde{s}^2} = \frac{\sum_k r_k \left[1 - \frac{4(\varepsilon_k)^2}{\Omega_m^2 + (2\xi_{0k})^2}\right]}{2\beta s_0^2 \left[\sum_k r_k\right]^2}$$

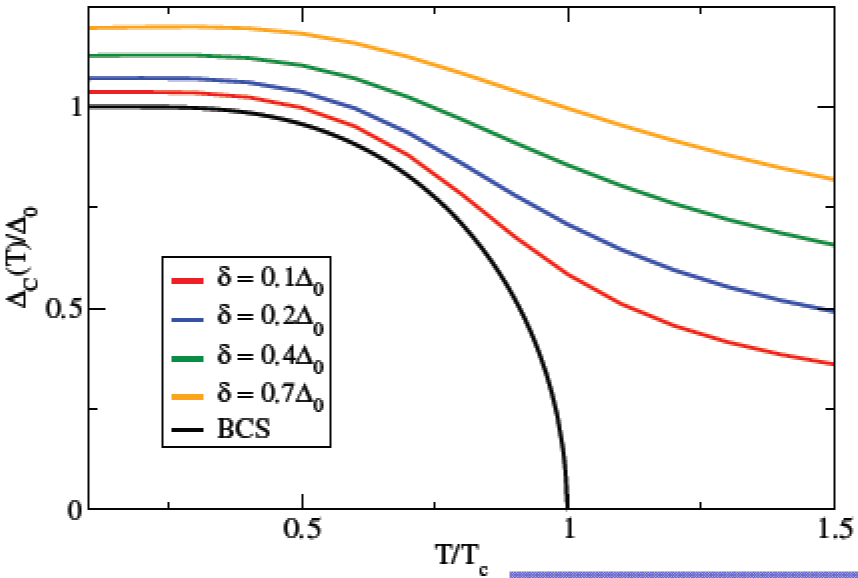
Amplitude fluctuations gapped

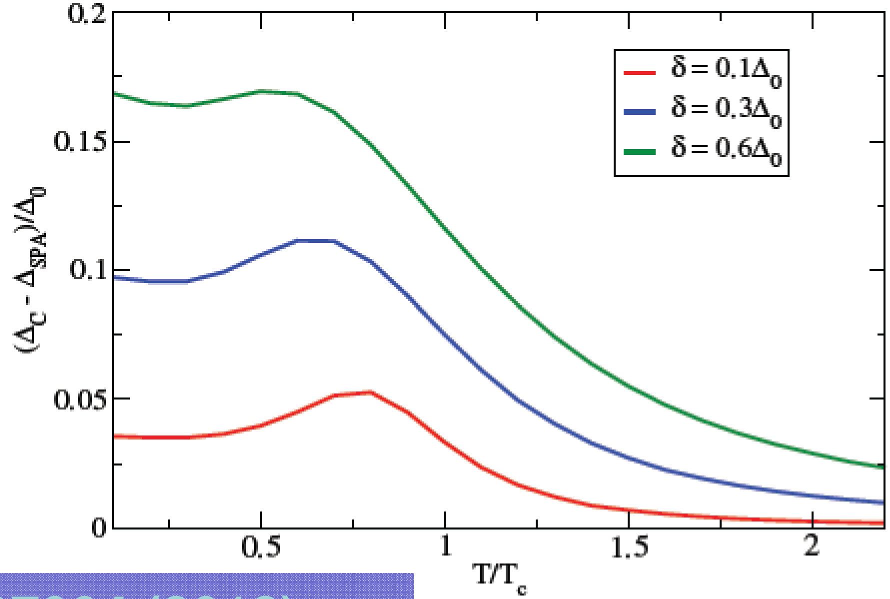
$$Z/Z_0 = \int_0^\infty ds_0^2 \; e^{-\beta[\mathcal{A}_0(s_0)+\mathcal{A}_1(s_0)]}$$

$$\mathcal{A}_1[s_0] = \frac{1}{2}\int d\nu \left[n_b(\nu) - \frac{1}{\beta\nu}\right]\frac{1}{2\pi i}\left\{\ln\left[\widetilde{C}(\nu+i0^+)\right] - \ln\left[\widetilde{C}(\nu-i0^+)\right]\right\}$$

$$\widetilde{C}(z) = \left(-z^2+4s_0^2\right)\left(-z^2\right)\left[\int_D d\varepsilon\,\varrho(\varepsilon)\,\frac{r(\xi)}{-z^2+(2\xi)^2}\right]^2 + \left(-z^2\right)\left[\int_D d\varepsilon\,\varrho(\varepsilon)\,\frac{2\varepsilon\, r(\xi)}{-z^2+(2\xi)^2}\right]^2$$

$$r(\xi) = \frac{1}{2\xi}\tanh\left(\frac{\beta\xi}{2}\right)$$





PRL 108, 097004 (2012)

Charging effects?

It seems to be the same

Perturbative

Charging effects

$$\Xi_m^{\phi\phi} = \sum_k r_k \frac{2\beta s_0^2 \Omega_m^2}{\Omega_m^2 + (2\xi_{0k})^2} \sim \frac{\beta}{\delta}\Omega_m^2 \longrightarrow \delta^{-1}\int_0^\beta d\tau\, (\partial_\tau \delta\phi)^2$$

Non perturbative

$$\phi(\tau) = \phi_0 + 2\pi M\tau/\beta + \delta\phi(\tau)$$

Odd-Even at T=0

Relevant if interactions are considered?

Finite Size

+

Strong interactions ?

Tough for even conventional superconductors

Santos

Holographic superconductivity in confined geometries?

Holographic principle

Maldacena's conjecture

AdS/CFT correspondence

t'Hooft, Susskind, Weinberg, Witten....

Strongly coupled field theory in d	⟷	Weakly coupled gravity in d+1
N=4 Super-Yang Mills CFT		Anti de Sitter space AdS

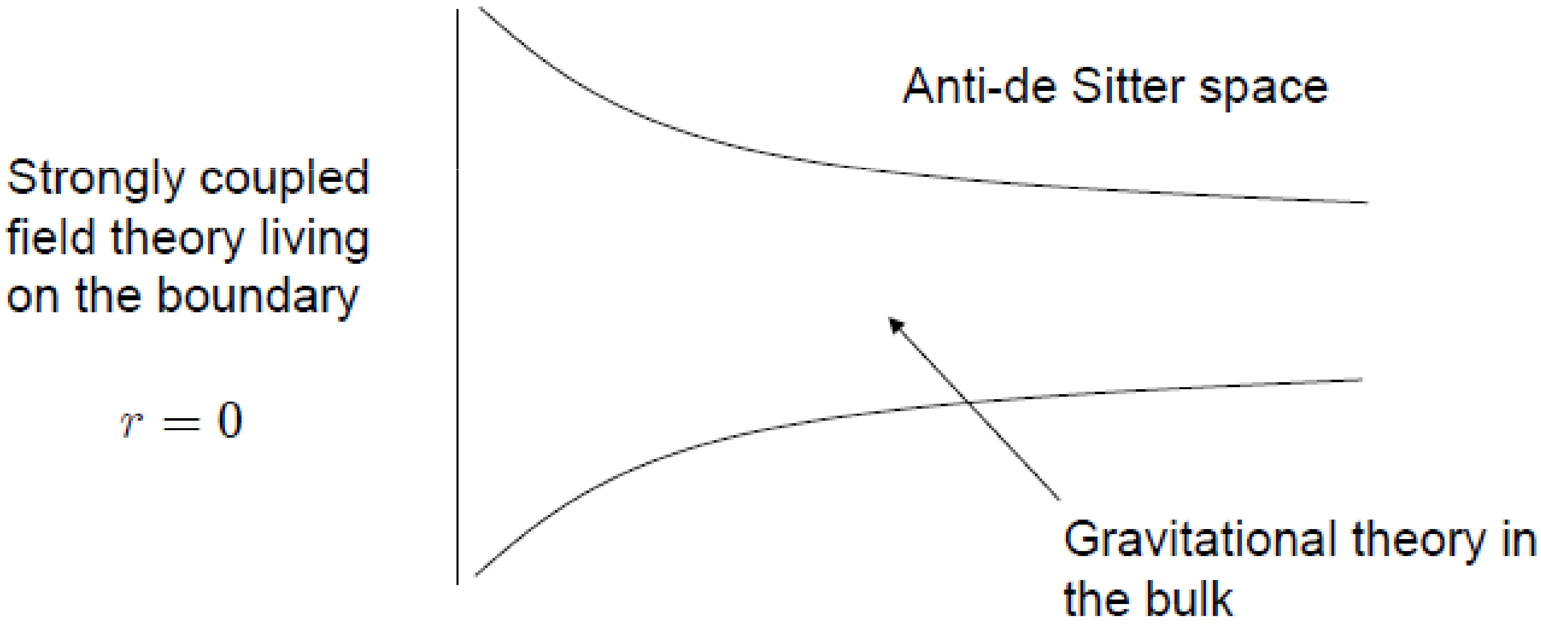
Holography beyond string theory

2003	QCD Quark gluon plasma	Gubser, Son
2008	Holographic superconductivity	Hartnoll, Herzog, Horowitz
2012	Quantum criticality, non-equilibrium..	Zaanen, Sachdev, Tesanovic, Philips

Easy to compute in the gravity dual & Detailed dictionary

Extra dimension?

Geometrization of Wilson RG



An answer looking for a question

H = ?

I do not know

Complex scalar

I know that

Spontaneous breaking U(1) at low T

Finite μ

Simplest dual gravity theory

$$S = \int d^4x\, \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\psi|^2 - m^2 |\psi|^2 \right]$$

$$D = \nabla - iqA$$

$\psi \equiv$ complex scalar

Metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

$$= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dR^2 + R^2 d\theta^2)$$

$$f(r) = \frac{r^2}{L^2} \left(1 - \frac{r_0^3}{r^3} \right) ,$$

Equations of motion:

$$\partial_r^2|\psi| + \frac{1}{r^2 f}\partial_x^2|\psi| + \left(\frac{f'}{f} + \frac{2}{r}\right)\partial_r|\psi| + \frac{1}{f}\left(\frac{A_t^2}{f} - m^2\right)|\psi| = 0$$

$$\partial_r^2 A_t + \frac{1}{r^2 f}\partial_x^2 A_t + \frac{2}{r}\partial_r A_t - \frac{2|\psi|^2}{f}A_t = 0$$

Boundary conditions:

$r \to \infty$

$$|\psi| = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2} + O\left(\frac{1}{r^3}\right)$$

$$A_t = \mu - \frac{\rho}{r} + O\left(\frac{1}{r^2}\right)$$

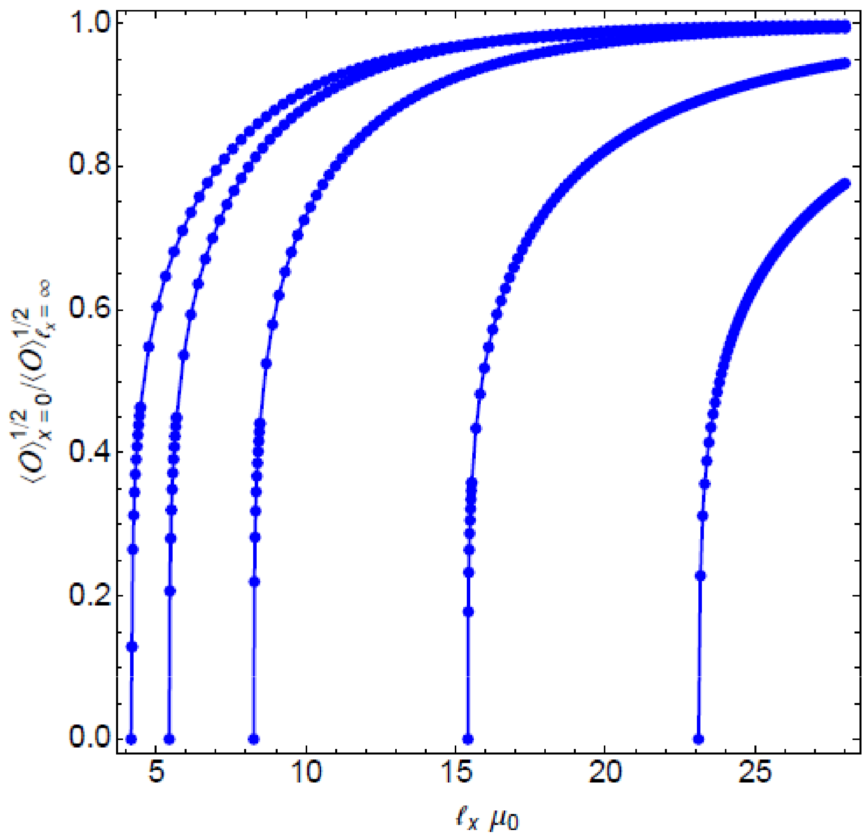
$r = r_0$ **$A_t = 0$**

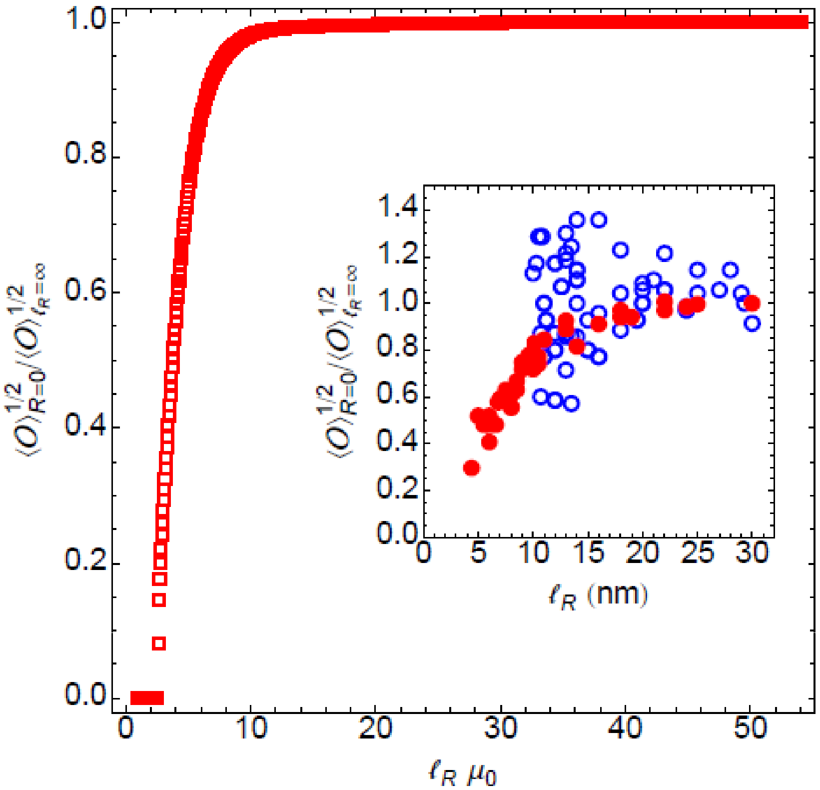
How small?

$$\mu(x) = \mu_0\left[\frac{1 - \epsilon + \epsilon\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}{\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}\right]$$

Dictionary:

$$\langle\mathcal{O}\rangle = \sqrt{2}\psi^{(2)}$$

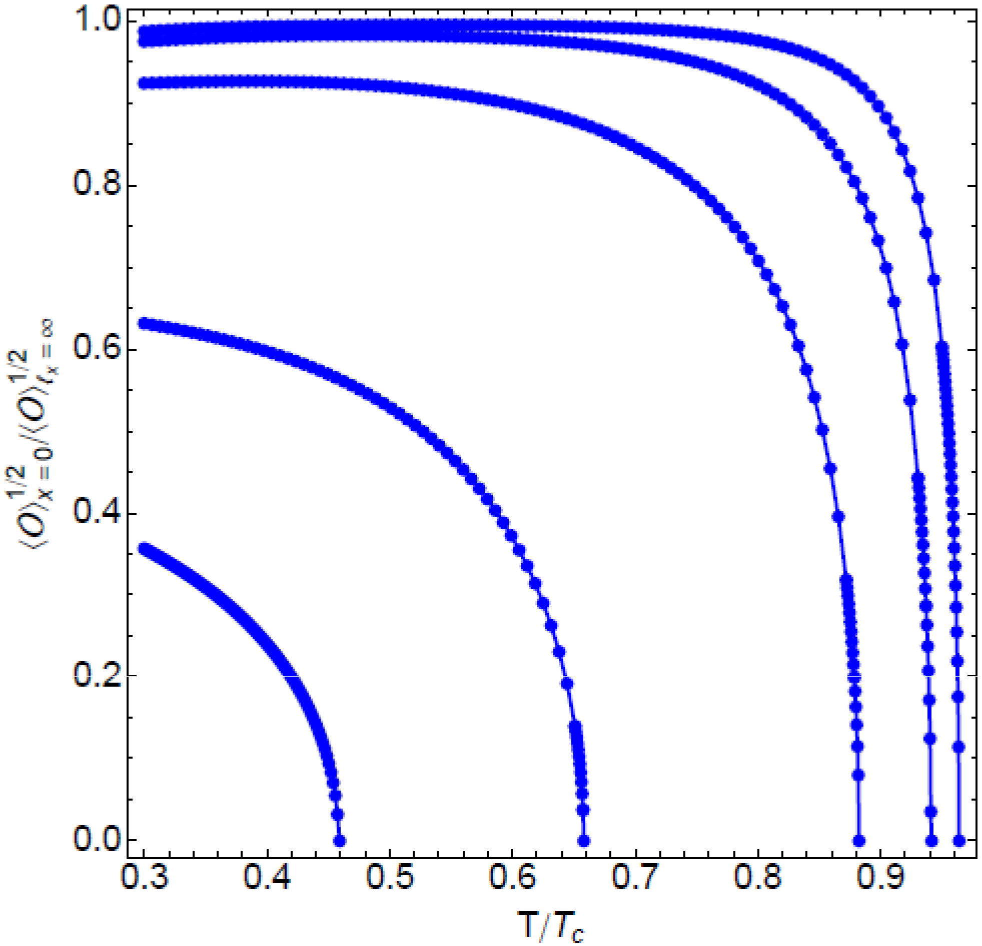




“Superconductivity” only for $l < l_c$

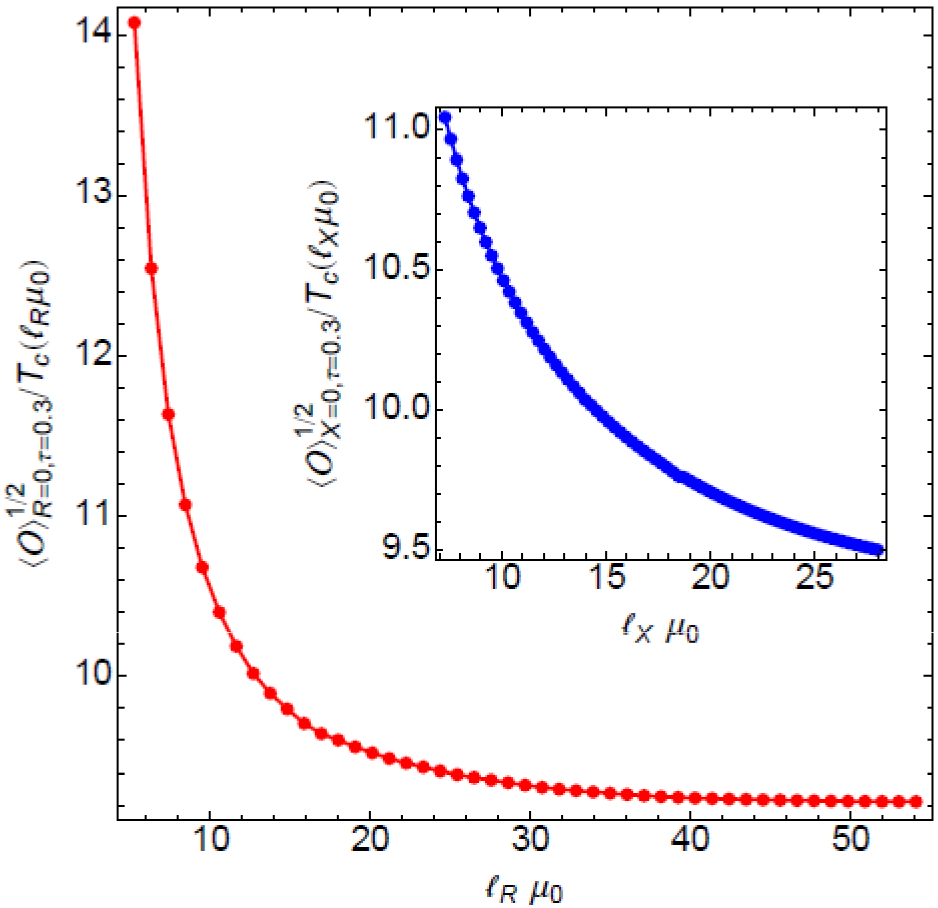
Mean field behavior

Fluctuations?



No thermal fluctuations

Large N artefact



Interactions depends on system size!

arXiv:1204.4189

THANKS!

THANKS!