Dynamics Examples

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1 Newton's Laws

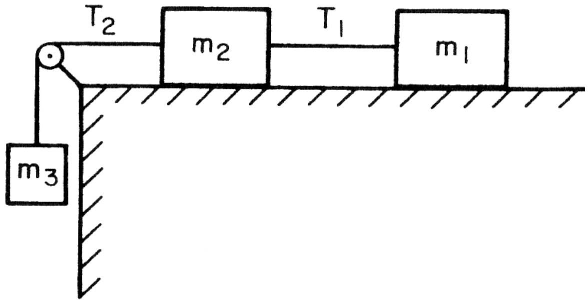

Figure 1: 3 connected blocks

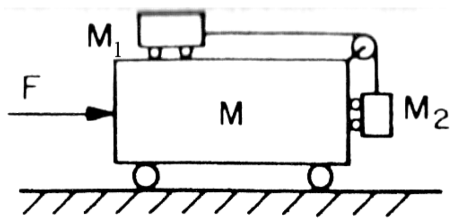

Figure 2: Masses on a trolley

1. Two blocks of mass $m_1 = 1\text{kg}$ and $m_2 = 2\text{kg}$ on a frictionless horizontal surface are connected by a massless string. See Figure 1. m_2 is connected by another string to $m_3 = 2\text{kg}$ hanging over a frictionless pulley. (a) Sketch the free-body diagrams for all masses, showing the forces acting. (b) Find the acceleration of the masses and the tension in the strings.

2. What horizontal force F must be constantly applied to M, in Figure 2, so that M_1 and M_2 do not move relative to M? Neglect friction.

3. In the system shown in Figure 3, $M_1 = 400\text{g}$ slides without friction on the inclined plane of angle θ, and $M_2 = 400\text{g}$. Find the acceleration of M_2 and the tension in the cords. Assume the pulley is light.

4. A mass of M_1 hangs from the ceiling of a lift of mass M_2 that is being accelerated upwards by a constant force $f > (M_1 + M_2)g$. See Figure 4. The mass M_1 is initially a distance s above the floor of the lift.
(a) find the acceleration of the lift.
(b) What is the tension in the string connecting M_1 to the ceiling?
(c) If the string breaks, to what does the acceleration of the lift change? What is the acceleration of M_1?
(d) How long is it before M_1 hits the floor of the lift?

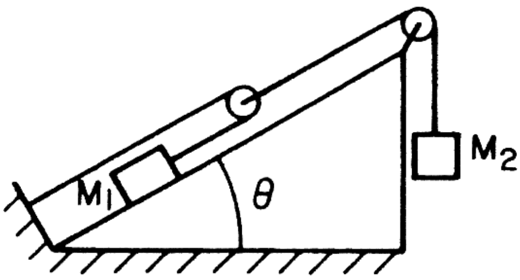

Figure 3: An inclined pulley system

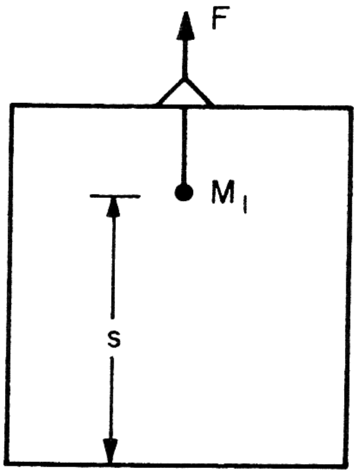

Figure 4: Mass in a lift

5. A painter of mass 100kg is in a cradle of mass 20kg and wishes to quickly ascend. See Figure 5 He pulls on the fall rope with sufficient force that he only presses against the cradle with a force of 50kg $\times$ g, where g is the acceleration due to gravity.
(a) Find the acceleration of the painter and cradle
(b) What is the total force supported by the pulley?

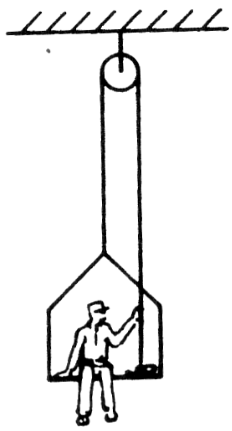
Figure 5: Painter on a cradle

6. A spring balance on Earth accurately finds that a 1kg mass (A) has a weight of 9.8N. On the Moon, at a place where the acceleration of gravity is only known approximately to be about 1/6th that on Earth, a rock (B) gives a reading of 9.8N on the spring balance. When A and B are hung over a pulley, B is observed to fall with an acceleration 1.2m/s^2 on the Moon. What is the mass of the rock B?

7. A wall of height h, thickness t and density ρ_w rests on rough ground. Air of density ρ_a and speed V blows against it.
(i) Assuming the wind is stopped when it reaches the wall, show that the wall topples over if $V > (\rho_w g/\rho_a h)^{1/2}\, t$.
(ii) If $\rho_w = 3 \times 10^3$kg/m^3, $\rho_a = 1.25$kg/m^3, $h = 2$m and $t = 0.1$m, find the critical V. Determine the minimum coefficient of friction between the ground and wall for the wall to topple instead of slide.

8*. One end of a long, thin, pliable carpet is bent back by 180 degrees and pulled from the lift edge with a constant velocity v. See Figure 6. The carpet still on the floor is at rest. Find the speed of the centre of mass of the moving part. If the carpet has width w and mass per unit area ρ. What is the minimum force needed to pull the moving part?

Figure 6: Folding carpet

2 Momentum and Energy

1. A neutron of kinetic energy E collides head on with a stationary ^{12}C
nucleus and rebounds elastically in the direction from whence it came. What is its final kinetic energy?

2. A machine gun mounted at one end of a 10,000kg platform 5m long, free to move on a frictionless track, fires bullets into a thick target mounted on the other end of the platform. 10 bullets each of mass 100g are fired every second with a muzzle speed of 500ms^{-1}.
(a) How does the platform move?
(b) In which direction?
(c) How fast?

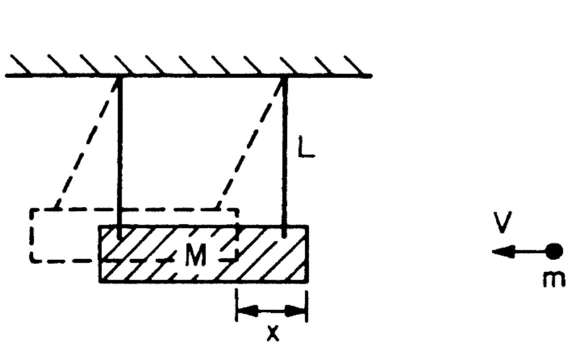

Figure 7: Ballistic pendulum

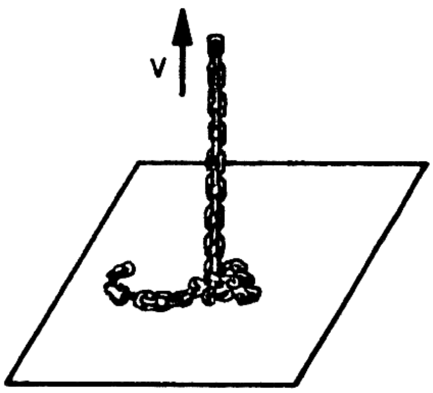

Figure 8: Lifting chain

3. A "ballistic pendulum", shown in Figure 7, can be used to measure the speed of a bullet. A bullet of known mass m and unknown speed v, embeds itself in a stationary wooden block of mass M, suspended as a pendulum on strings of length L. Derive an expression for the speed of the bullet in terms of the amplitude of the resultant swing, x, and m, M, and L.

4. The end of a chain, shown in Figure 8, of mass per unit length μ, at rest on a table top at $t = 0$, is lifted vertically at constant speed v. Evaluate the upward lifting force as a function of time.

5. An Earth satellite of mass 10kg and average cross-sectional area 0.5m^2 moves in a circular orbit at 200km altitude where the molecular mean free paths are several metres and the air density is about $1.6 \times 10^{-10}\text{kgm}^{-3}$. Assume that on impact molecules stick, then detach from the satellite with low relative speed. Calculate the retarding force that the satellite would experience due to air friction. How should the frictional force vary with velocity? Would the satellite's speed decrease as a result of the net force on it? Check the speed of a circular satellite orbit versus height.

6. Two elastic balls of masses m_1 and m_2 are placed with m_1 on top, with a small vertical gap between them, and then dropped to the ground. What is the ratio m_1/m_2 for which the

upper ball receives the largest fraction of the total energy? What ratio of masses is necessary if the upper ball is to bounce as high as possible?

7*. Two steel balls of masses M and m are suspended by vertical strings so that they are just in contact with their centres at the same height. The ball of mass M is pulled to one side in the original plane of the balls and released from a height h above the original position, with string still taut.
(a) Show that, whatever the value of m, the second ball cannot rise to a height greater than $4h$.
(b) A similarly suspended third steel ball of mass μ is now added, just touching that of mass m when at rest. The ball M is again drawn aside and released. Show that the kinetic energy transferred to the third ball is a maximum if m is chosen as $m = (M\mu)^{1/2}$. Assume all collisions are elastic.

3 Circular Motion and Orbits

1. From the fact that the Moon orbits the Earth in appoximately 28 days, estimate the distance of the Moon from the Earth's centre.

2. An Earth satellite of mass 20kg is in a circular orbit at a height small compared to the radius of the Earth.
If air resistance causes its total energy to decrease by 10kJ per revolution, what is the fractional change in its speed per revolution? Does the speed increase or decrease?

3. Sketch the four orbits resulting when a satellite in a circular orbit is given a small impulse in each of the four directions shown in Figure 9. In each case, state the changes in energy, angular momentum, and period of the satellite. *How do you transfer a satelite from near Earth orbit (circular) to a higher geostationary orbit? What set of impulses are required and when should they be applied?

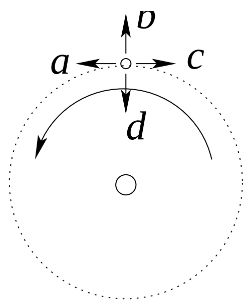

Figure 9: Impulses in orbit

4. Two similar spacecraft are launched from a space station orbiting the Earth by rockets which burn for only a few minutes. Spacecraft A is launched so that it just escapes from the solar system. Spacecraft B is launched in such a way that it falls into the centre of the Sun. Show that spacecraft B requires a more powerful rocket to launch it than does spacecraft A. Assume that the Earth moves in a circular orbit round the Sun, and ignore both the velocity of the space station relative to the Earth and the Earth's gravitational field.

5*. According to Einstein's theory of General Relativity a small correction $6GMmv^2/r^2c^2$ should be added to the Newtonian gravitational force between a sun of mass M and a planet of mass m moving with a speed v in a circular orbit of radius r around the sun.
(a) Show that this correction decreases the period of motion of the planet by a factor of

approximately $1 - 3GM/c^2r$.
(b) Estimate the advance of the planetary motion of Mercury in a century due to this correction.
The mass of the Sun is 2.0×10^{30}kg and the radius of Mercury's orbit, which is assumed to be circular, is 5.8×10^{10}m.

6**. The energy E of a mass in a central force is given by

$$
\begin{aligned}
E &= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r) \\
&= \frac{1}{2}m\dot{r}^2 + \underbrace{\frac{L^2}{2mr^2} + V(r)}_{V_{\text{eff}}(r)}.
\end{aligned}
$$

As discussed in lectures, this of the same form as a particle in an effective potential V_{eff}. Sketch the forms of effective radial potential for a power-law central potential $V(r) = -Ar^{-(1+\alpha)}$ with α being (a) zero and (b) $\alpha \ll 1$. Find the condition for a circular orbit and its frequency, and the frequency of the radial oscillations resulting from a radial perturbation. Hence describe the motion in each case.