
Practical calculations using first-principles QM
Convergence, convergence, convergence

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Results of First-Principles Simulations

Synopsis

Convergence

Structural Calculations

Summary

First-principles methods may be used for subtle, elegant and accurate computer experiments and insight into the structure and behaviour of matter.



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First principles results may be worthless nonsense

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2. How to avoid some of the common pitfalls and to avoid computing nonsense.

Further reading: *Designing meaningful density functional theory calculation in materials science - a primer* Anne E Mattson *et al.* *Model. Sim. Mater. Sci Eng.* **13** R1-R31 (2005).

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Plane-wave cutoff
Pseudopotentials and
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Force and Stress
Iterative Tolerances
K-point convergence

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- Fortunately well converged *properties* may frequently be computed using an incomplete basis.

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- Reaction energy computed as

$$\Delta E = E_{\text{product}} - \sum E_{\text{reactants}} = E_{\text{Mg}(\text{OH})_2} - (E_{\text{MgO}} + E_{\text{H}_2\text{O}})$$

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- **Always use same cutoff for all reactants and products.**

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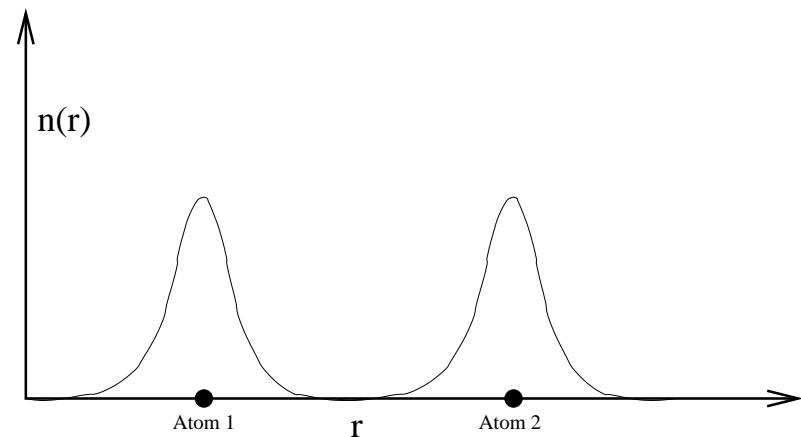
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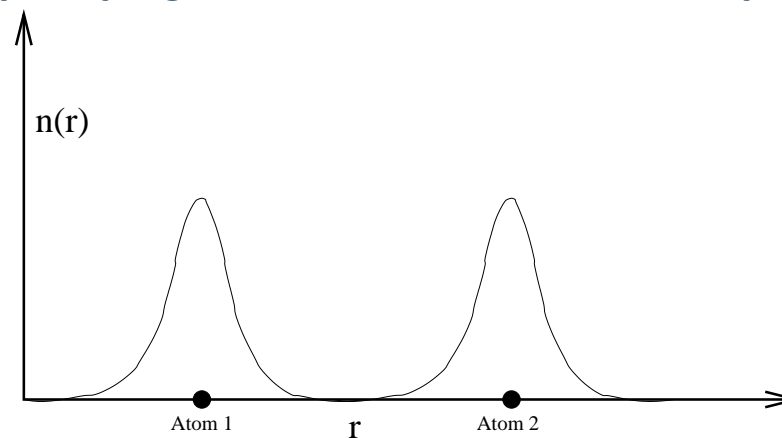
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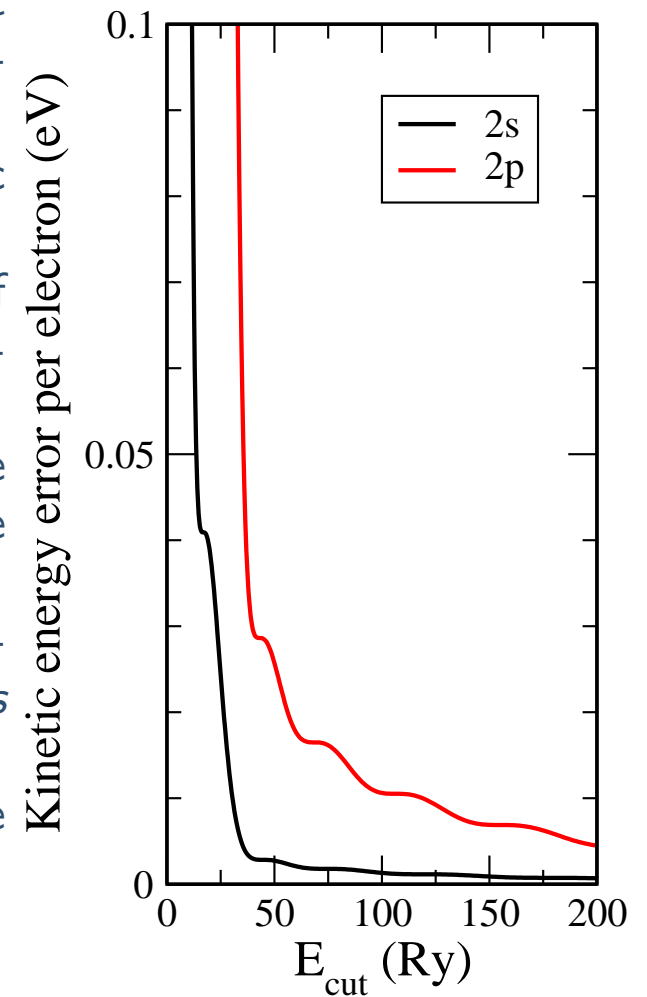


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- E_{cut} (and G_{max}) depend only on *types* of atoms, not numbers.
- Simulation cutoff is maximum over pseudopotentials used.
- Required cutoff is *system-independent* but not *property-independent*.

- When performing cutoff test you can not always assume completely smooth approach to convergence.
- Can get plateaus or other non-asymptotic behaviour
- Sometimes cause is over-optimization of the pseudopotential too low a desired cut-off.
- In cases with this behaviour can choose criterion for convergence energy to be plateau values.
- Absolute energy convergence is rarely desirable. *Force and stress convergence is much more useful criterion.*
- (Example is *not* a pseudopotential from the CASTEP database)

KE convergence for O



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- FFT grid should be large enough to accommodate all G-vectors of density, $n(r)$, within cutoff: $G \leq 2G_{\text{MAX}}$.
- Guaranteed to avoid "aliasing" errors in case of LDA and pseudopotentials without additional core-charge density.

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- GGA XC functionals give density with higher fourier components, and need $1.75G_{MAX} - 2G_{MAX}$
- Finer grid may be needed to represent USP augmentation charges or core-charge densities.
- CASTEP incorporates a second, finer grid for charge density to accommodate core/augmentation charges while using G_{MAX} for orbitals.

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Frequently need to find crystal structure at *mechanical equilibrium*.

- Given guessed or exptl. initial structure, seek local minimum of Born-Oppenheimer energy surface generated by K-S functional.
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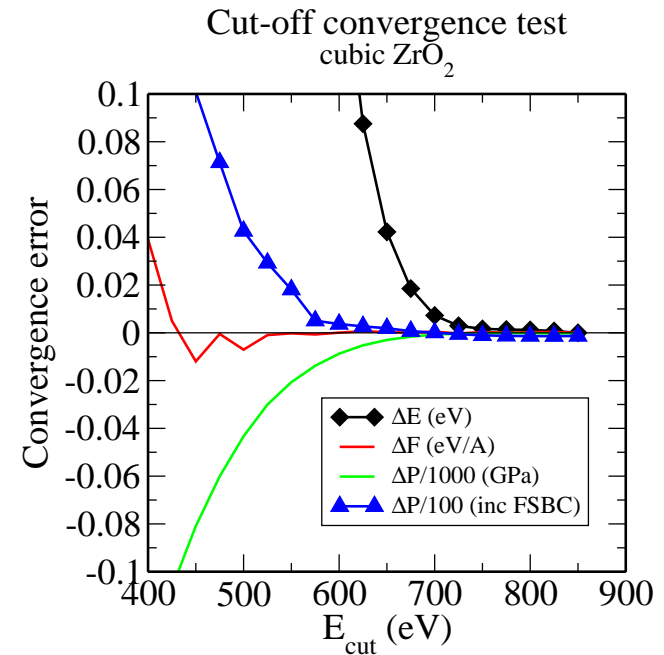
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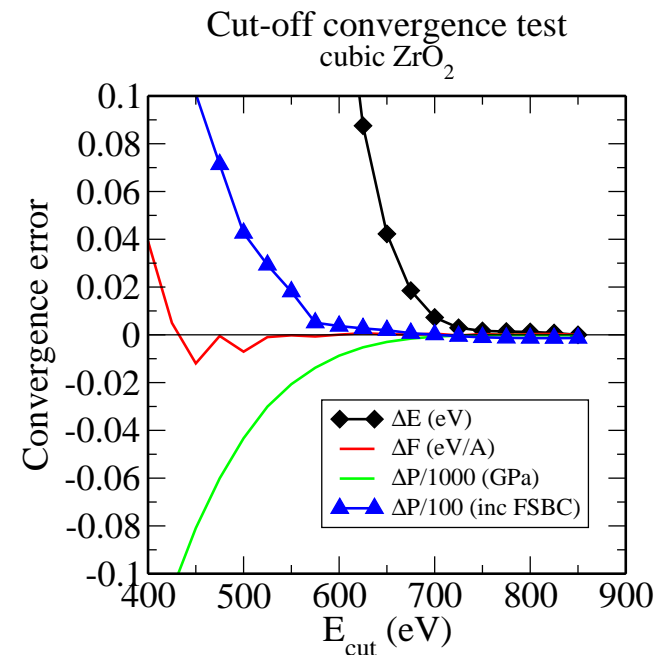


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- different physical quantities converge at different rates.
- Always test convergence specifically of **quantities of importance to your planned calculation**



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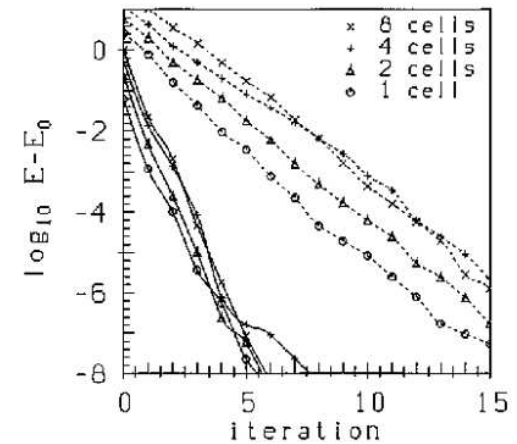
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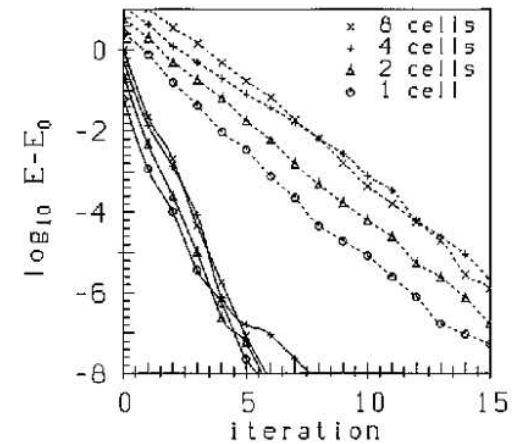
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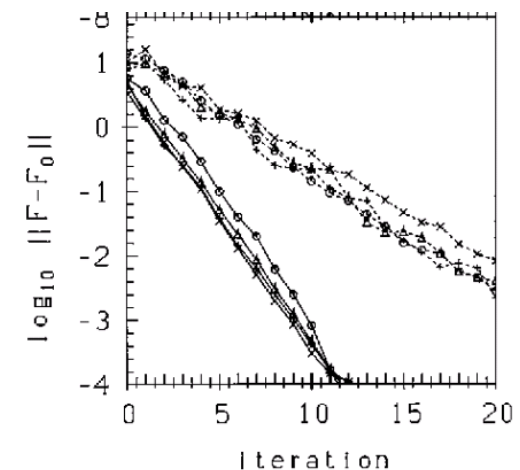
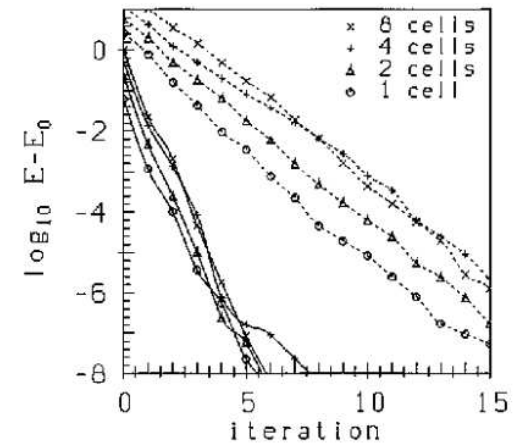
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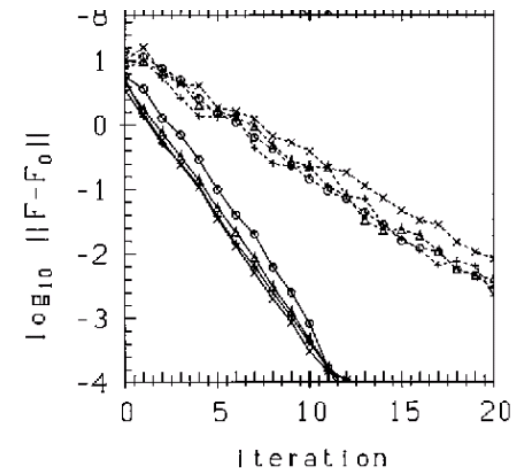
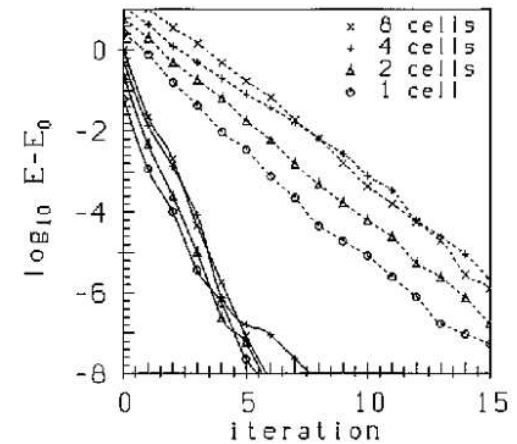
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- Inaccurate forces are common cause of geometry optimization failure.



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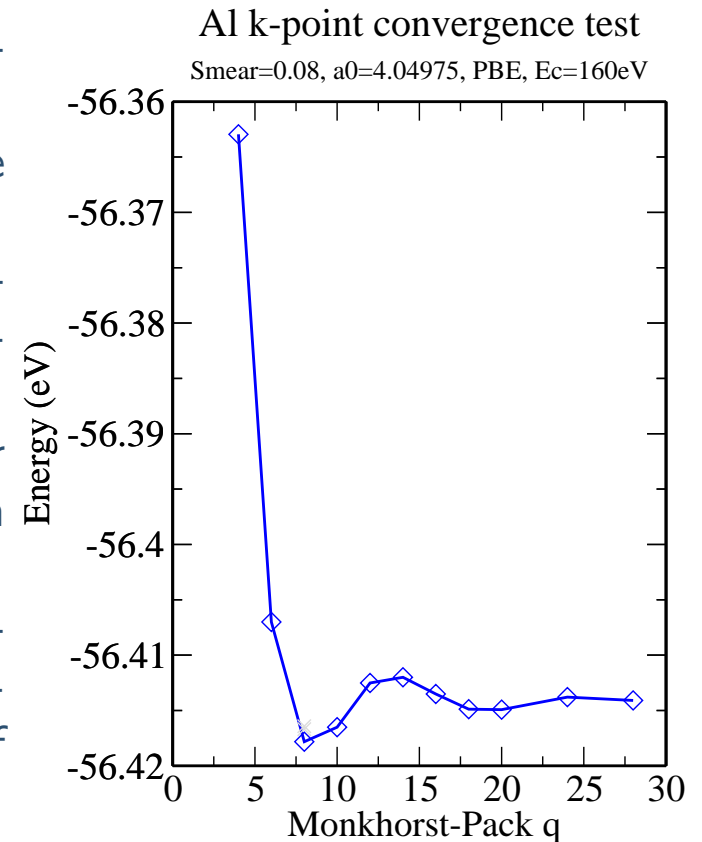
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Consequently comparative phase stability energetics and surface energetics frequently demand high degree of k-point convergence.

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- Convergence is **not** variational and frequently oscillates.
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 - Finite-temperature *smearing* can accelerate convergence, but must extrapolate the result back to 0K.
 - In case of insulators some k-point error cancellation occurs but only between *identical* simulation cells.
- Consequently comparative phase stability energetics and surface energetics frequently demand high degree of k-point convergence.



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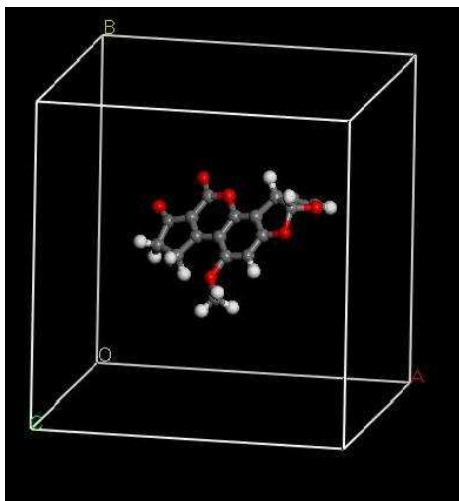
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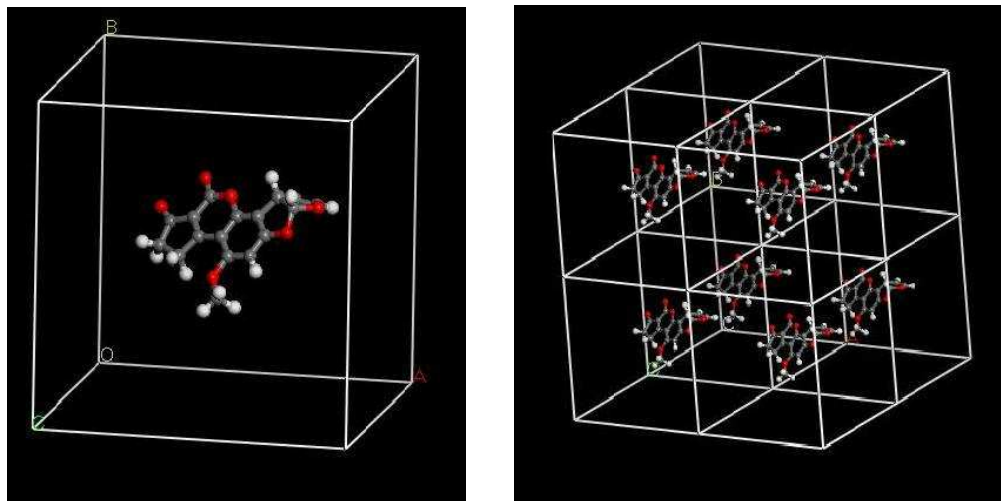
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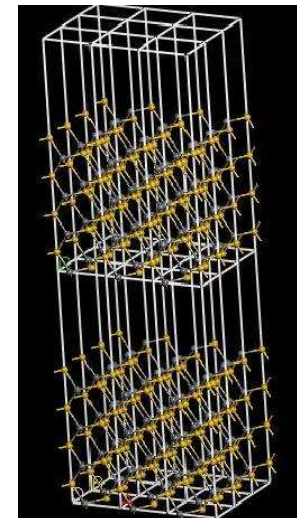
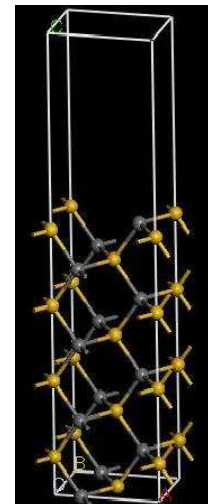
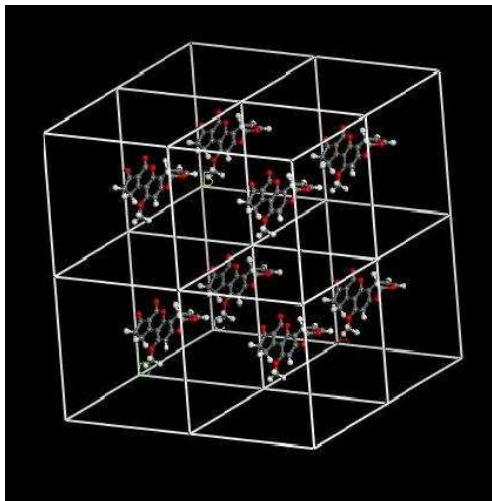
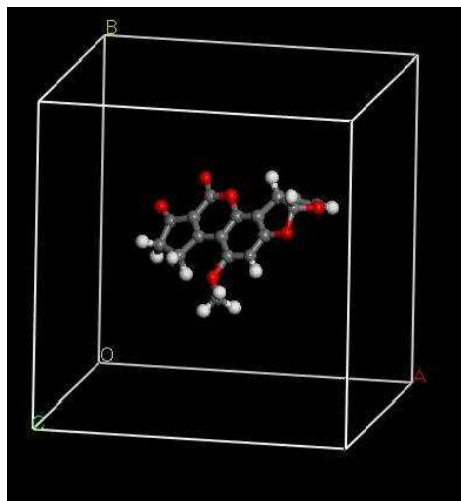
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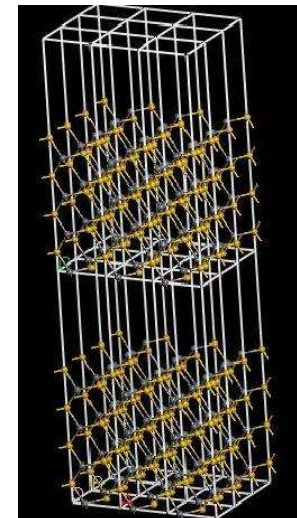
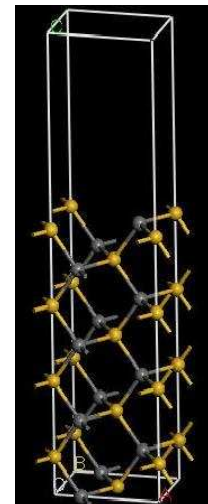
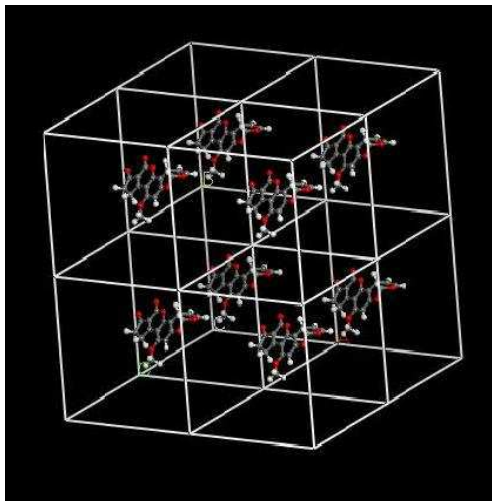
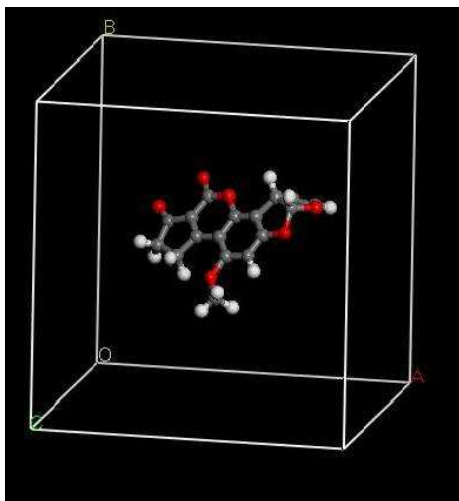
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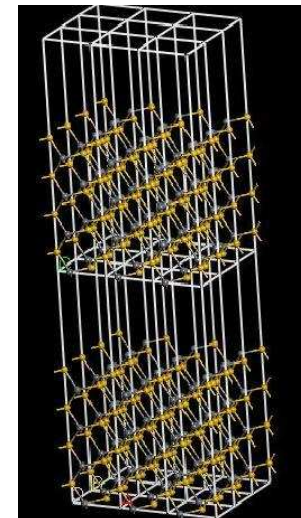
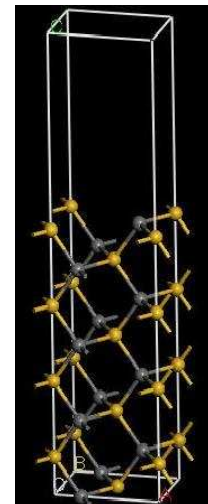
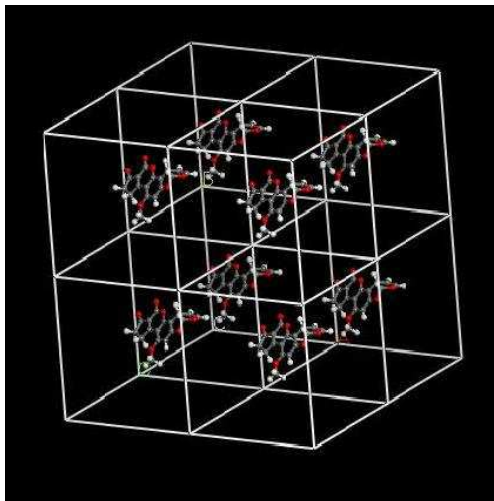
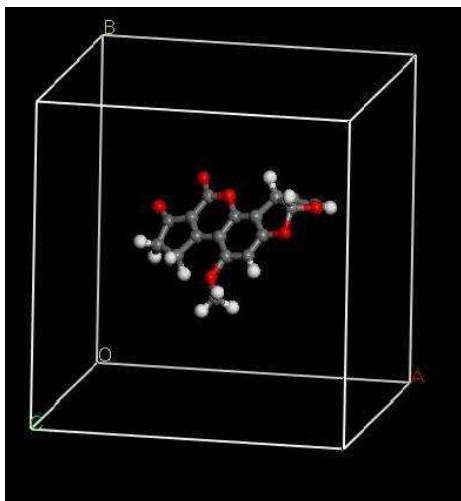
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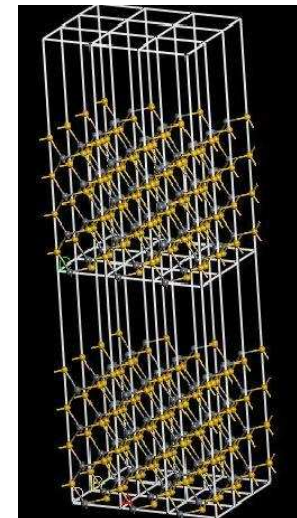
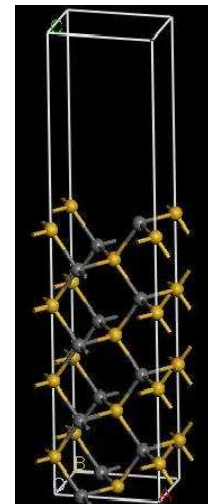
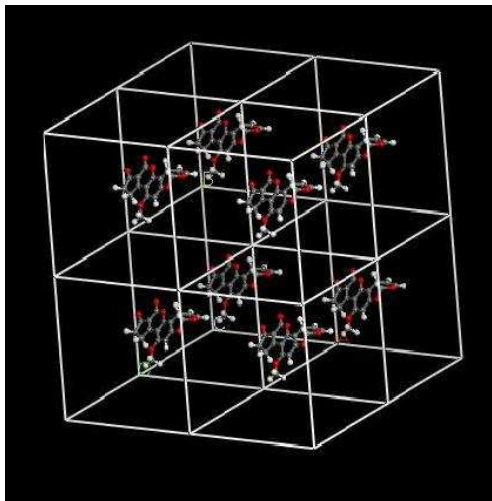
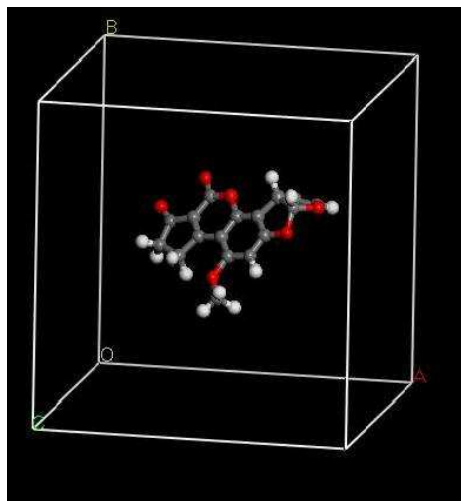
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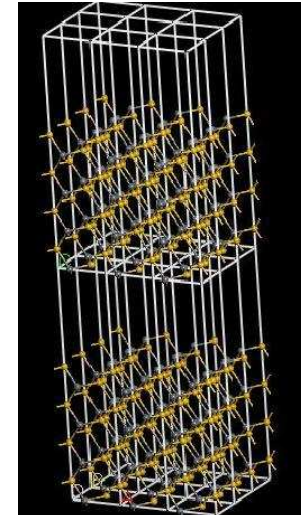
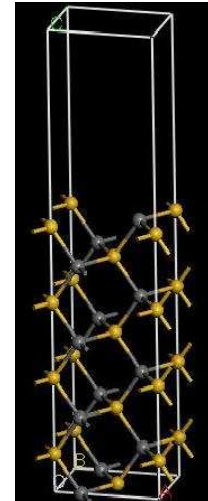
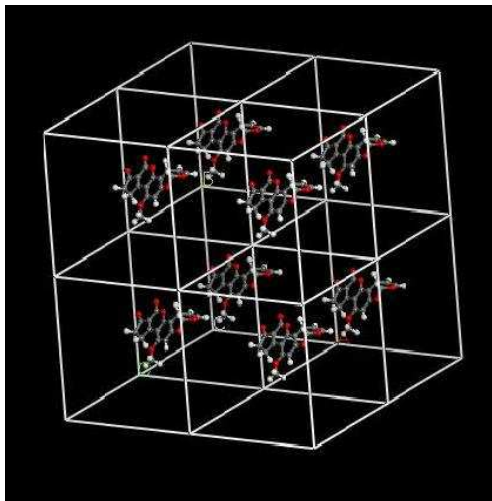
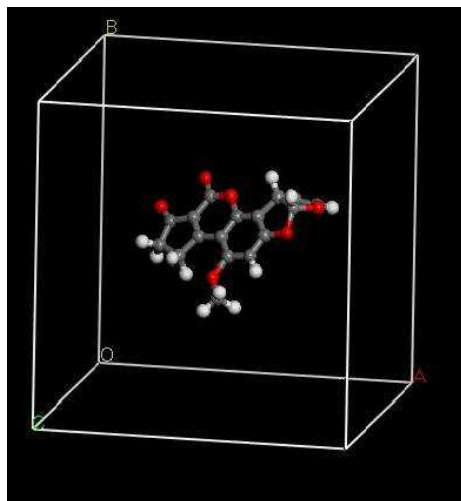
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- When calculating surface energy, try to use same cell for bulk and slab.

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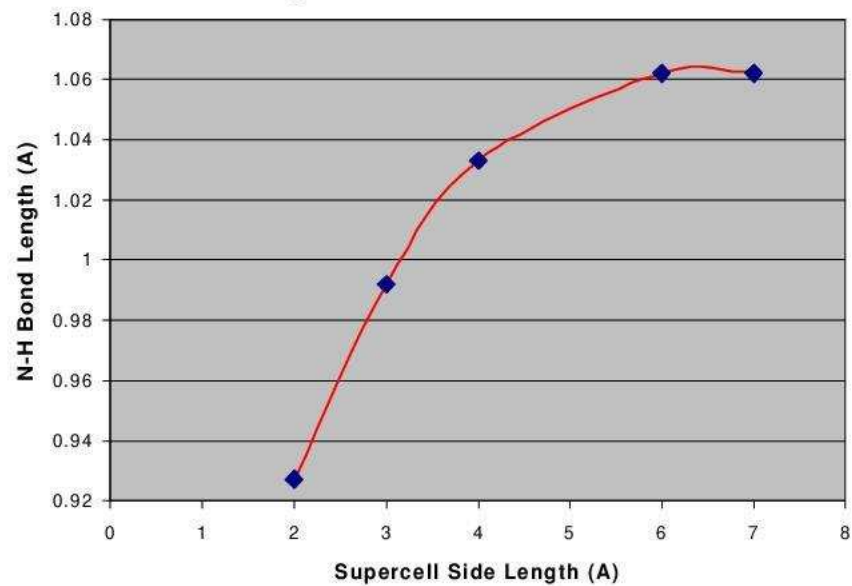
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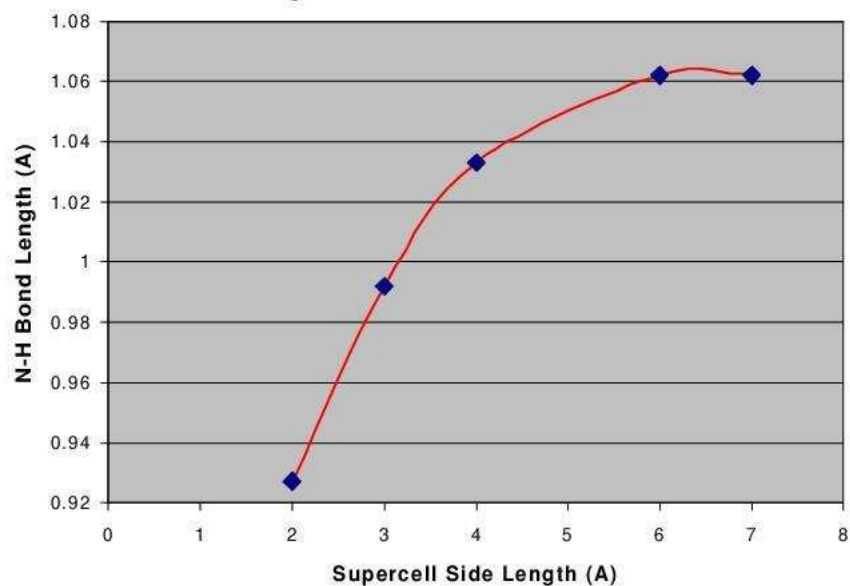
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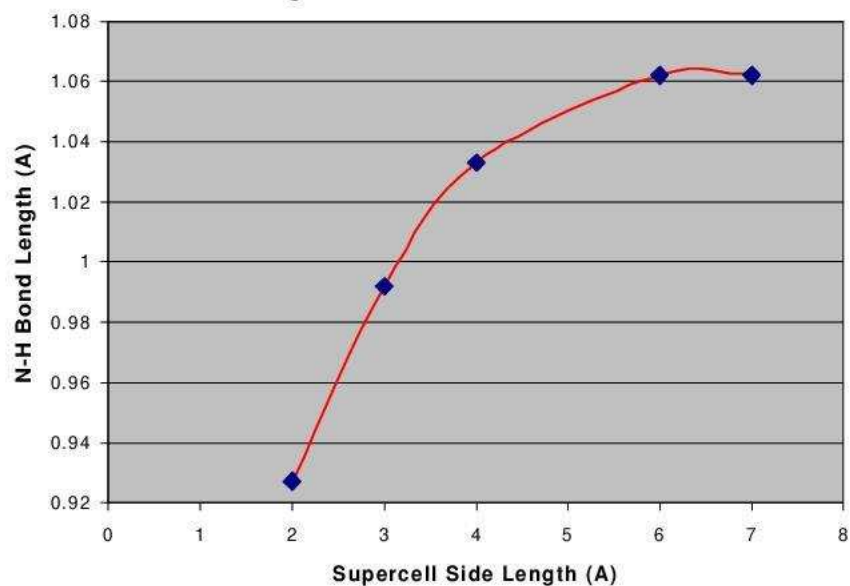
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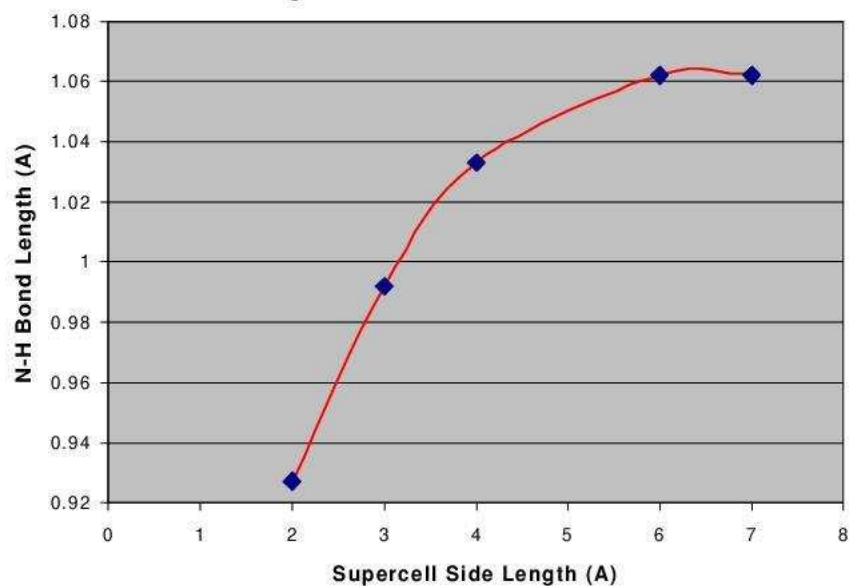
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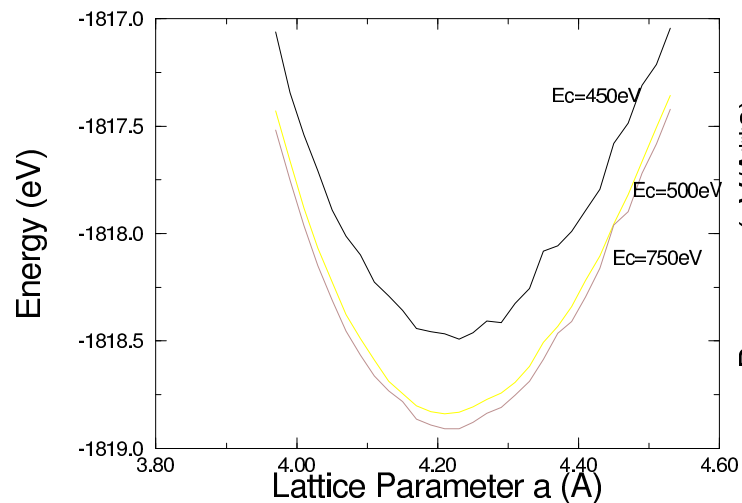
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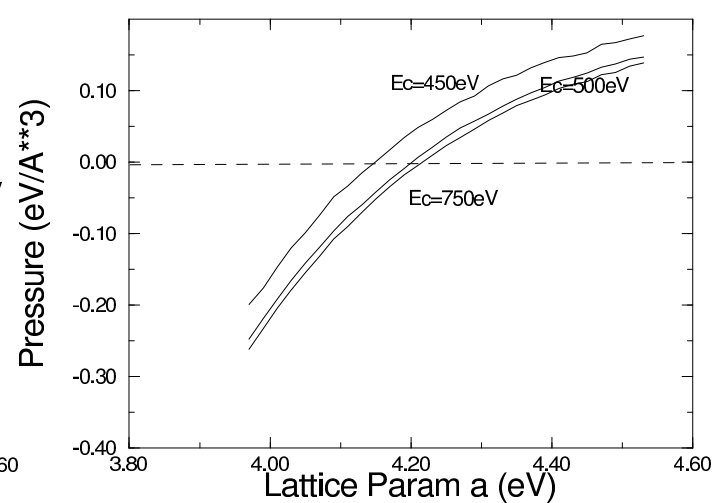
MgO Lattice Param test

O020 and Mg006 pseudopotentials

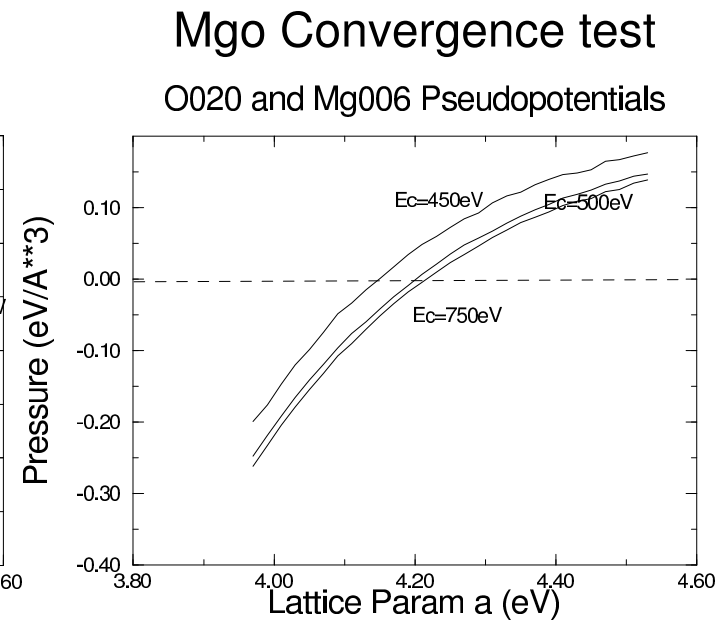
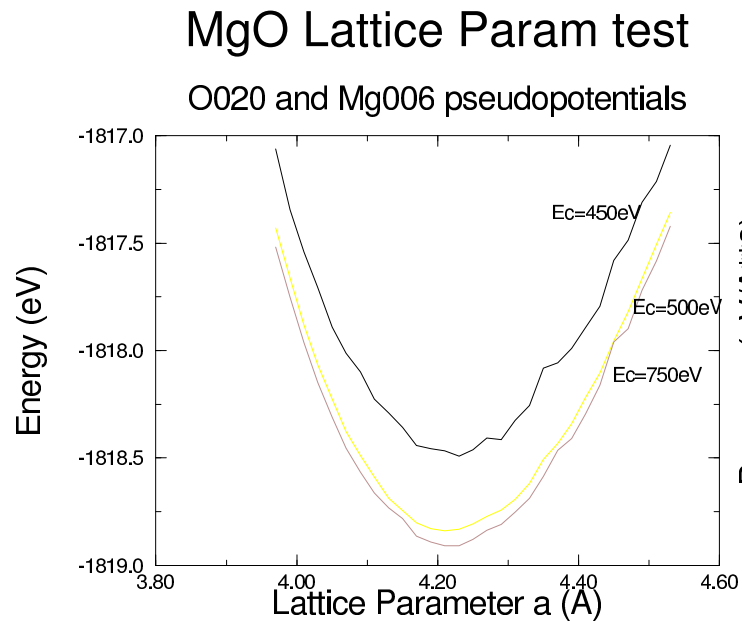


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- Incomplete basis error in stress approximated by **Pulay stress** correction.
- “Jagged” E vs V curve due to discreteness of N_{PW} . Can be corrected using Francis-Payne method (J. Phys. Conden. Matt **2**, 4395 (1990))
- Finite-size basis corrections for energy and stress automatically computed by CASTEP.

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- *fixed cutoff* calculations reset basis for each volume, changing N_{PW} but keeping G_{max} and E_c fixed.
- This is almost always the correct method to use.

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- A poorly converged calculation is of little scientific value.