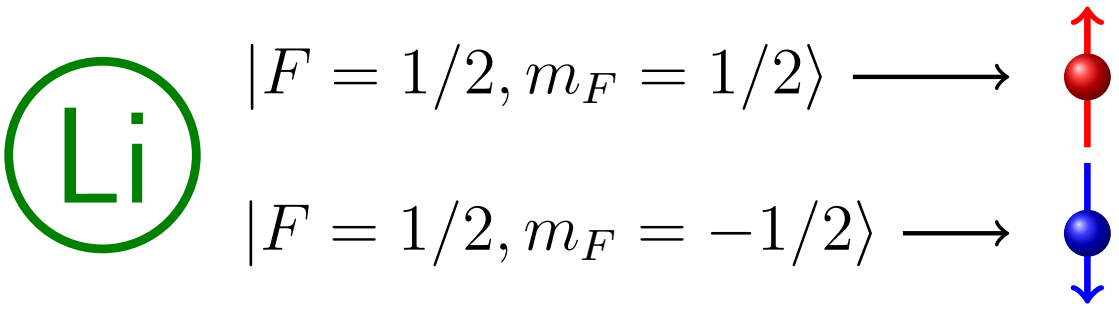
Spins, superfluids and ultracold atomic gases

Pascal Bugnion and Gareth Conduit

QMC in the Apuan Alps 2013

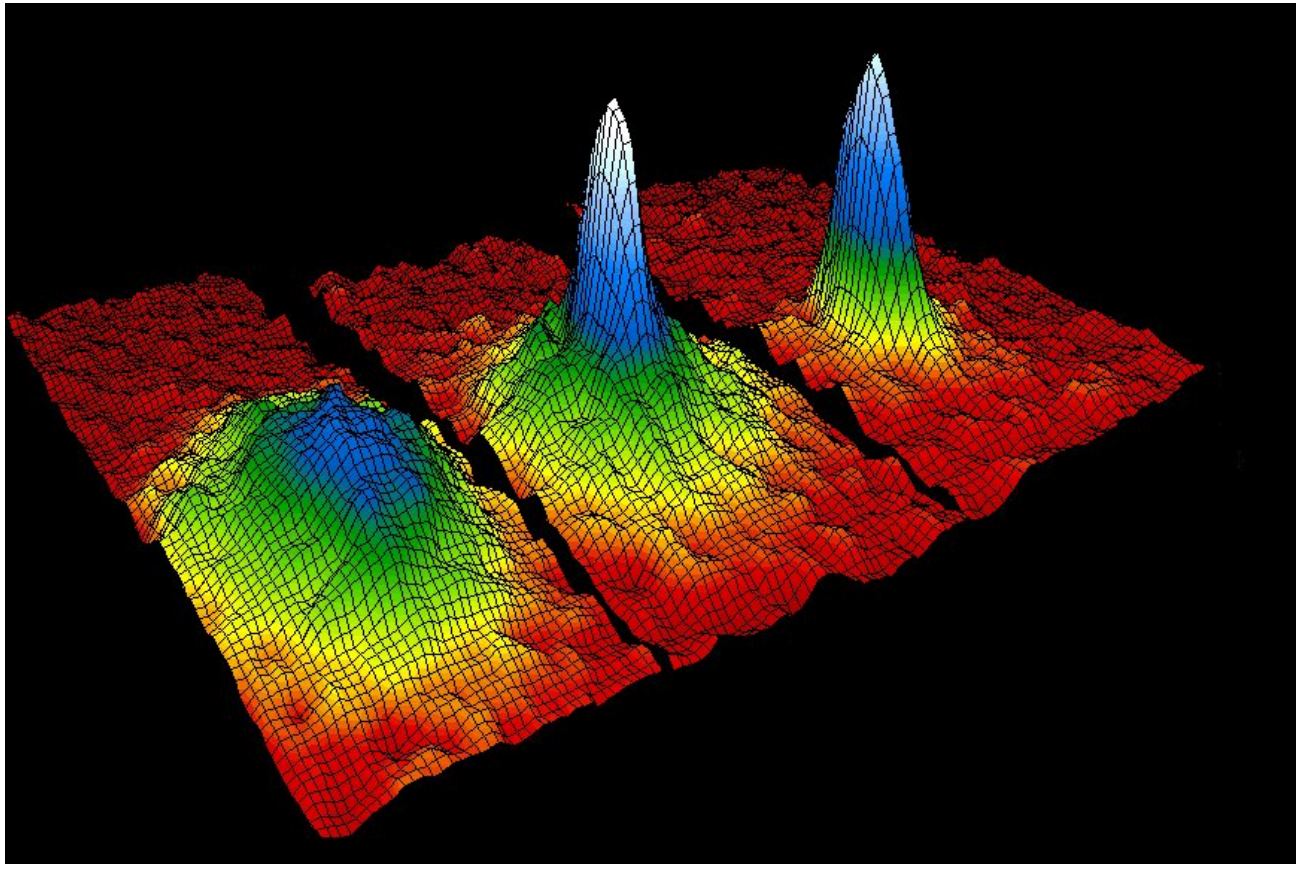
Why cold atoms?

Simulate complex many body systems that are often difficult to observe in the solid state.

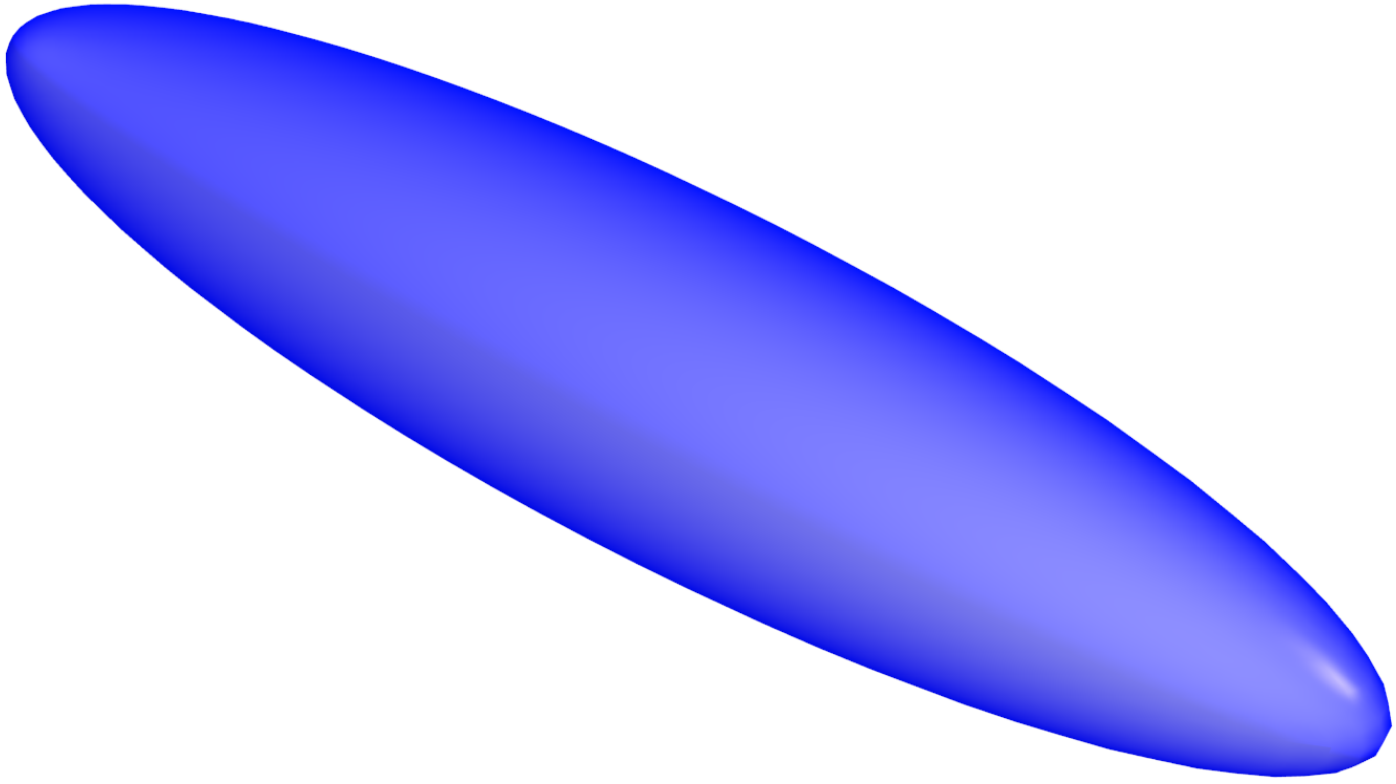


The inter-atom interactions can be tuned by changing an external magnetic field.

Attractive interactions

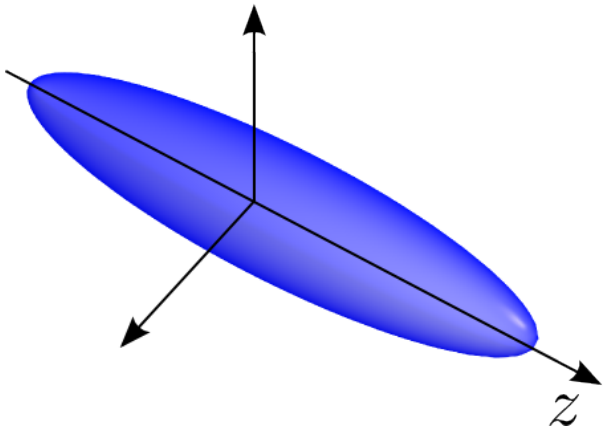


Experimental setup

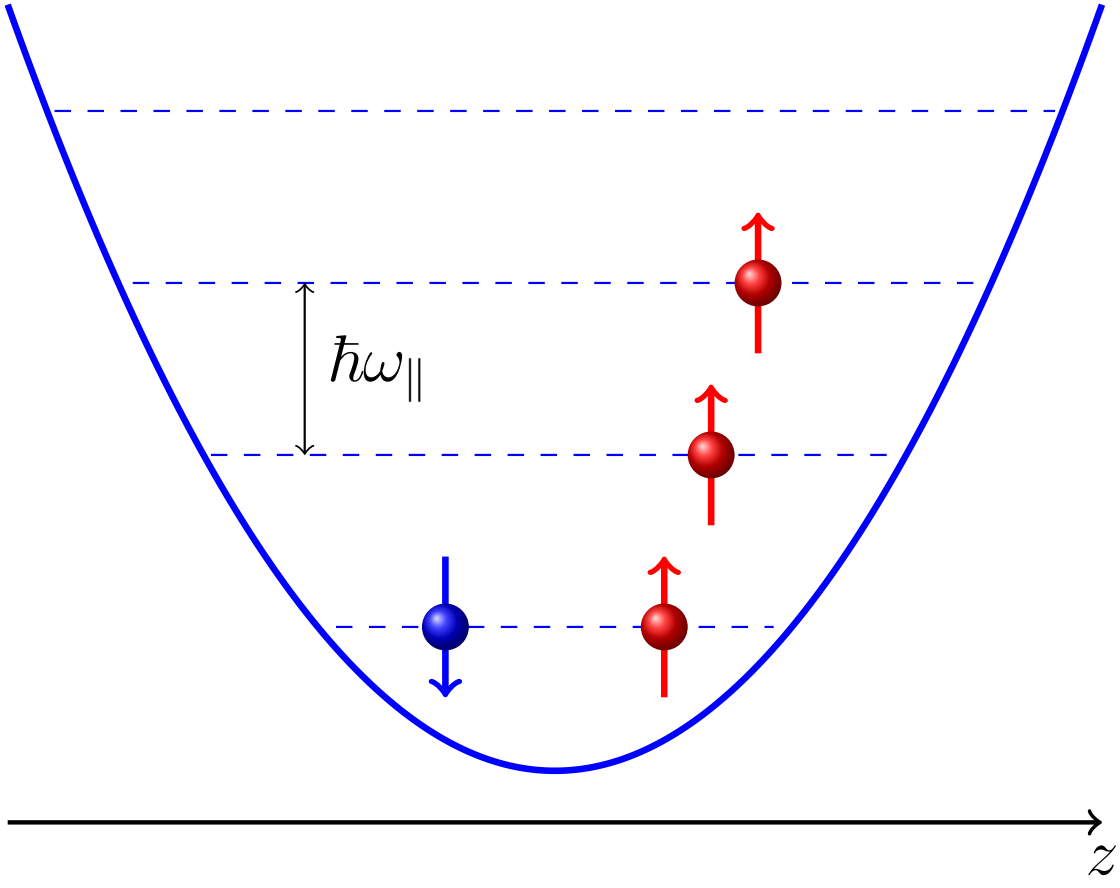


F. Serwane et al., Science **332**, 336 (2011)

Experimental setup



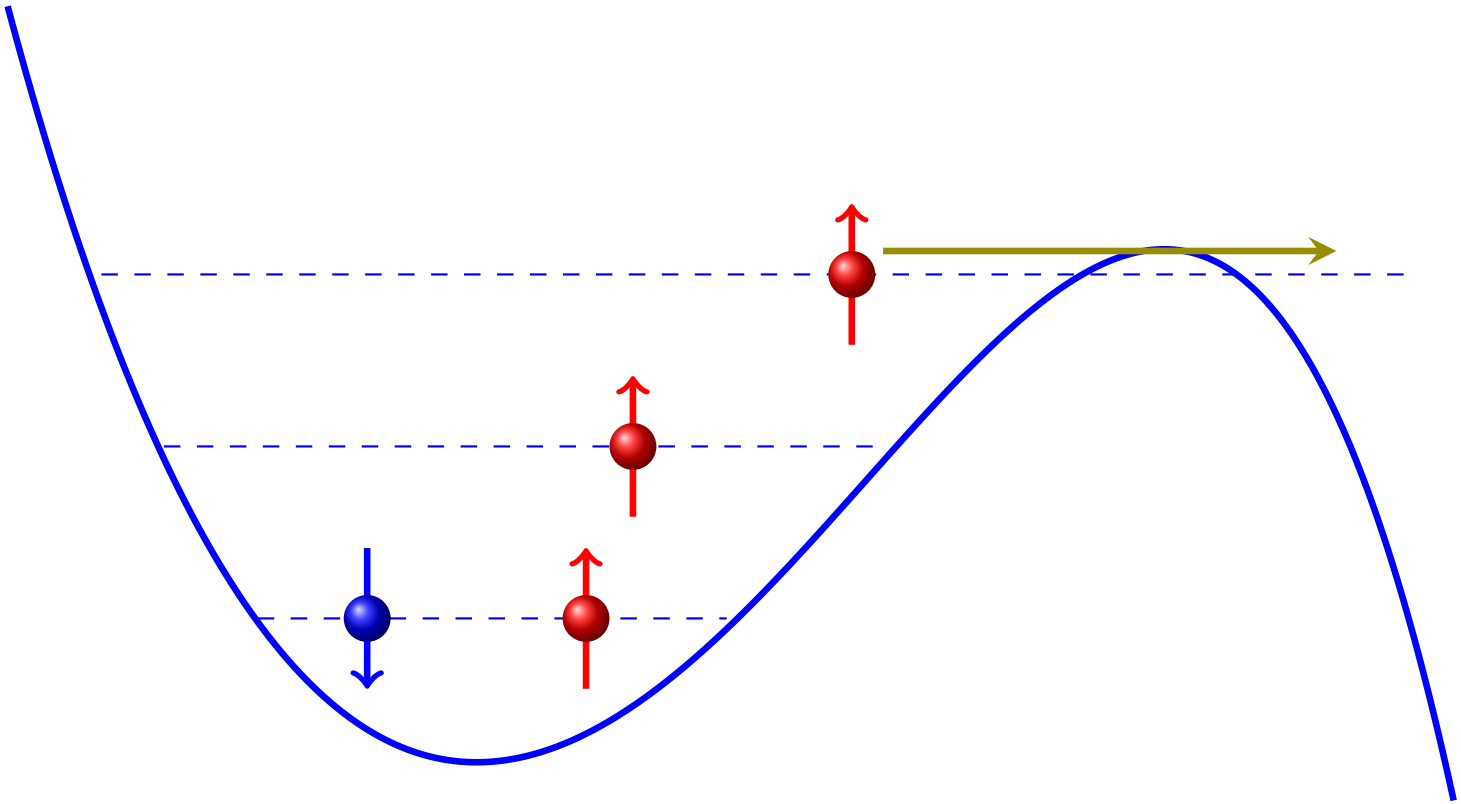
$\omega_{\parallel} \approx 10\omega_{\perp}$



F. Serwane et al., Science **332**, 336 (2011)

Experimental setup

Measure energy by tilting the trap until an atom tunnels out.



Hamiltonian

$$V(\mathbf{r}_i - \mathbf{r}_j) = \begin{cases} g\delta(\mathbf{r}_i - \mathbf{r}_j) & \text{anti-parallel spins} \\ 0 & \text{parallel spins} \end{cases}$$

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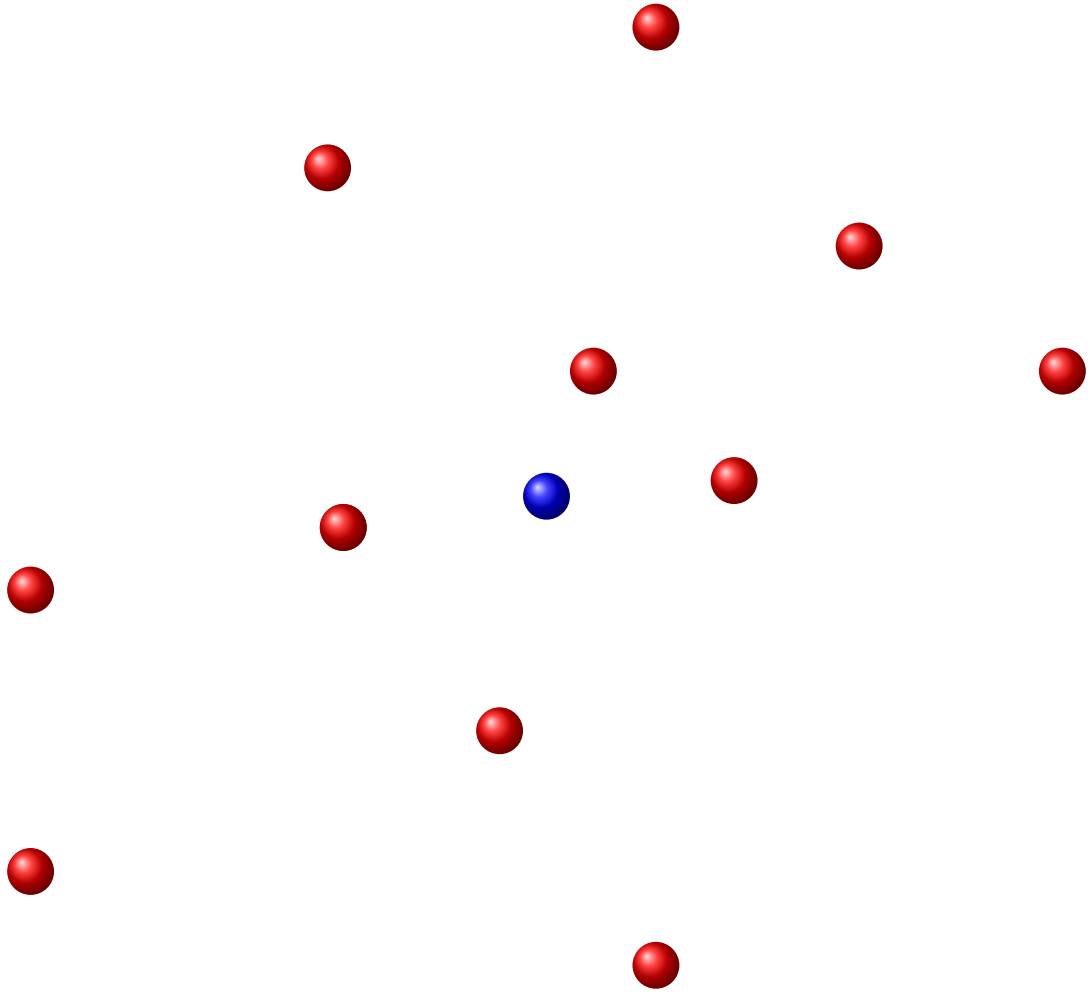
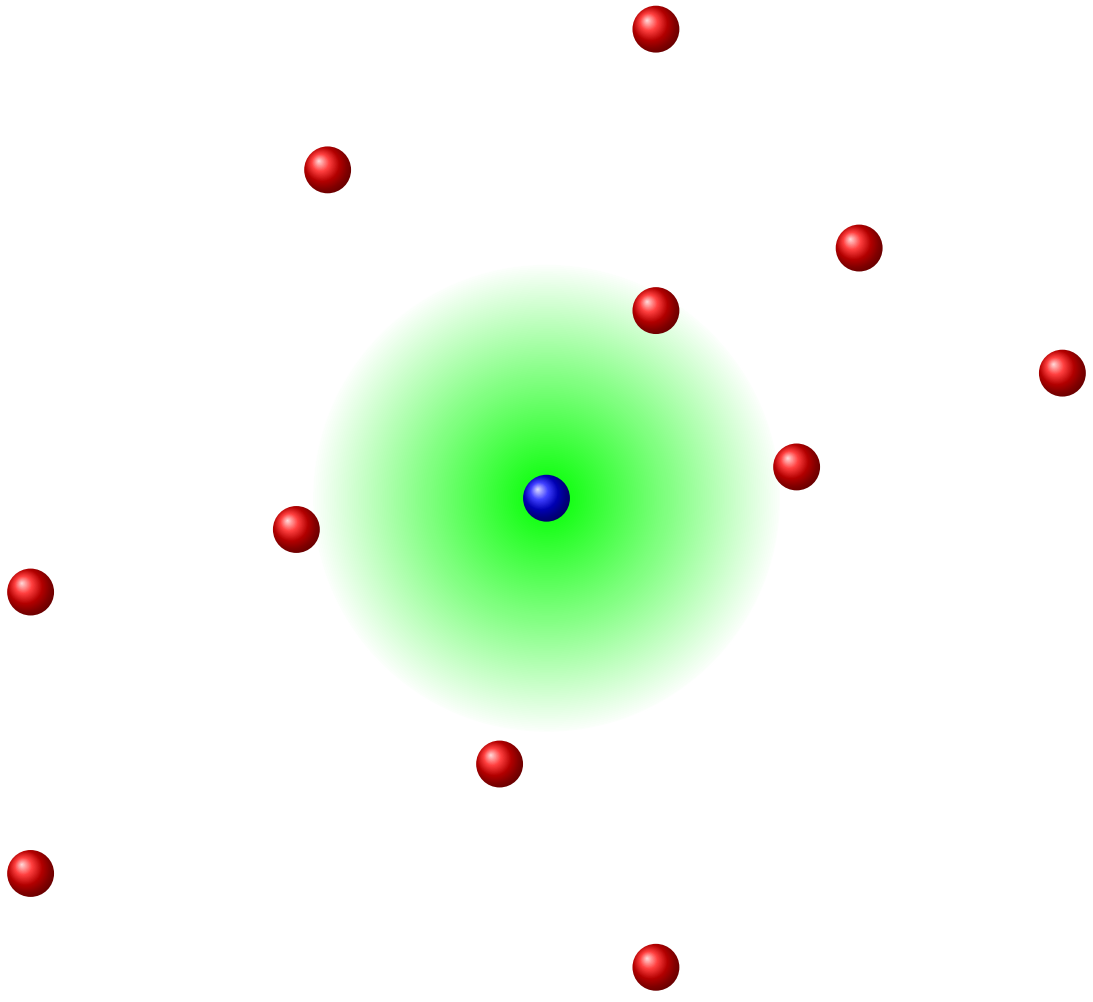
$$\hat{\mathcal{H}} = \sum_{i \in \{\uparrow,\downarrow\}} \hat{h}(\mathbf{r}_i) + g \sum_{i \in \uparrow, j \in \downarrow} \delta(\mathbf{r}_i - \mathbf{r}_j)$$

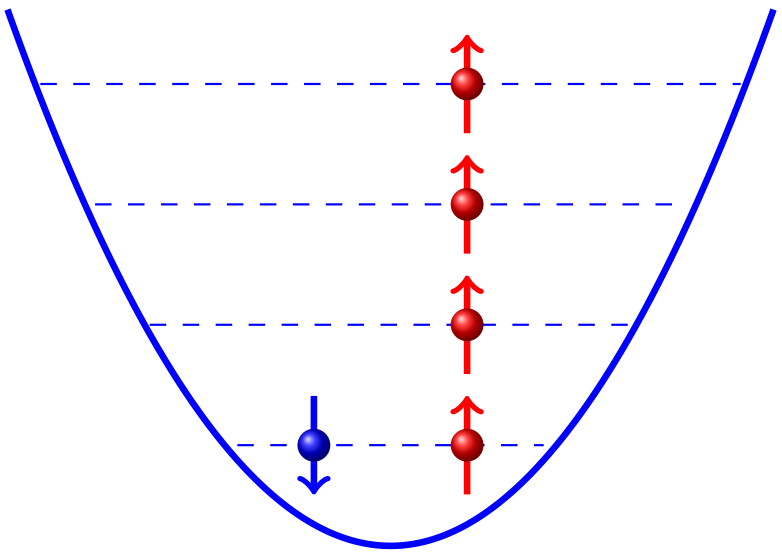
Polarons



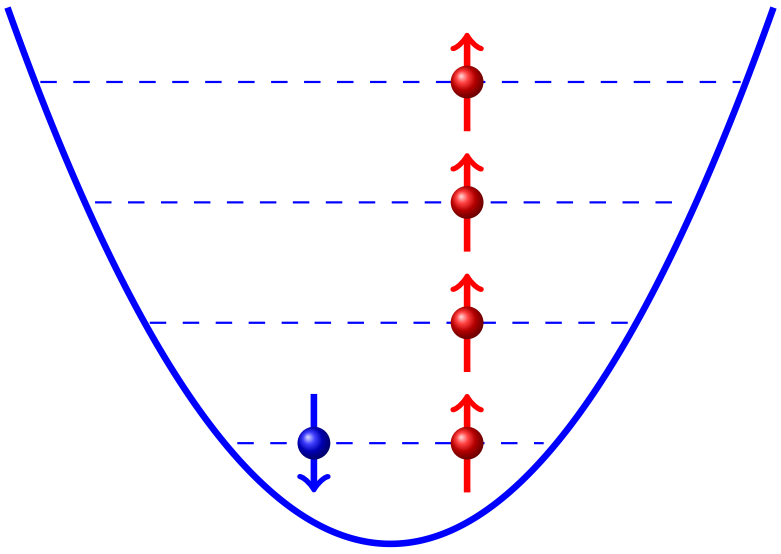
Polarons

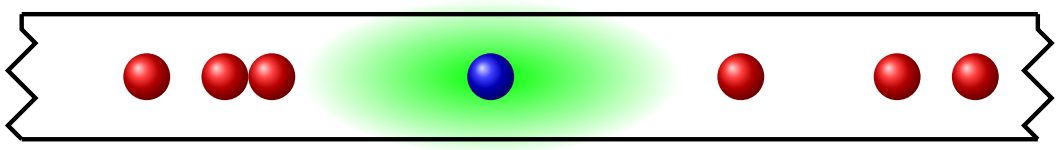


Polarons in a trap

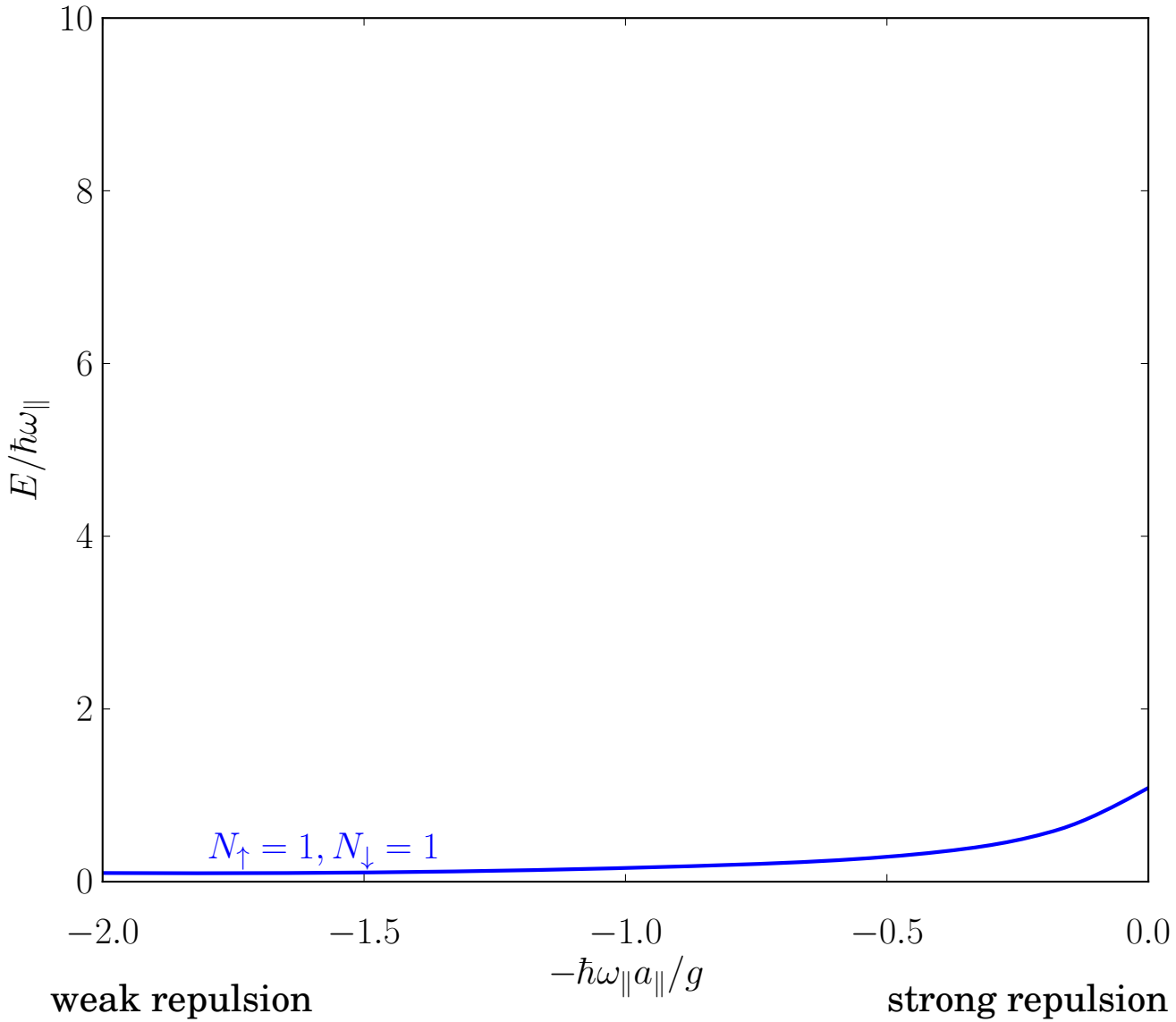


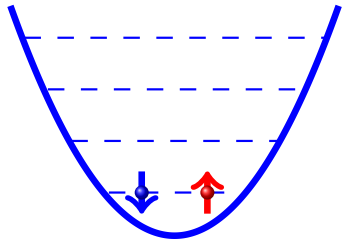
Polarons in a trap



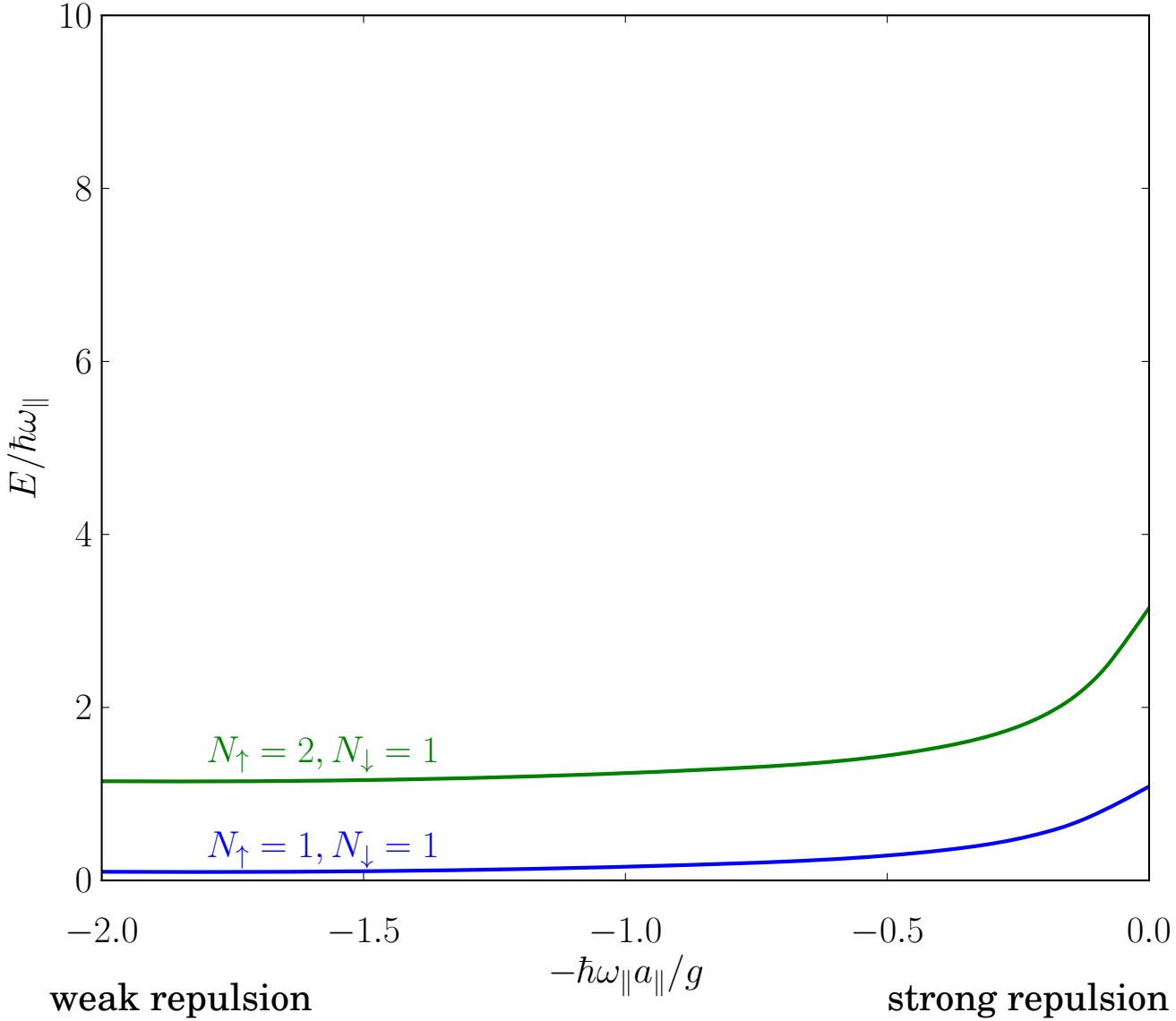


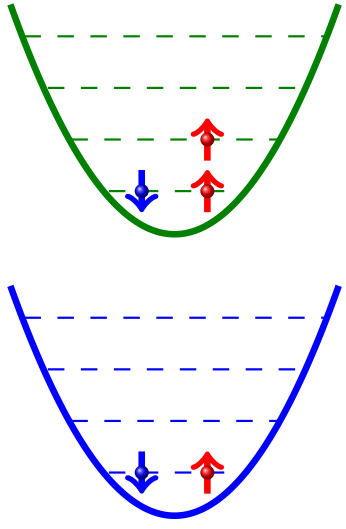
Polaron results





Polaron results





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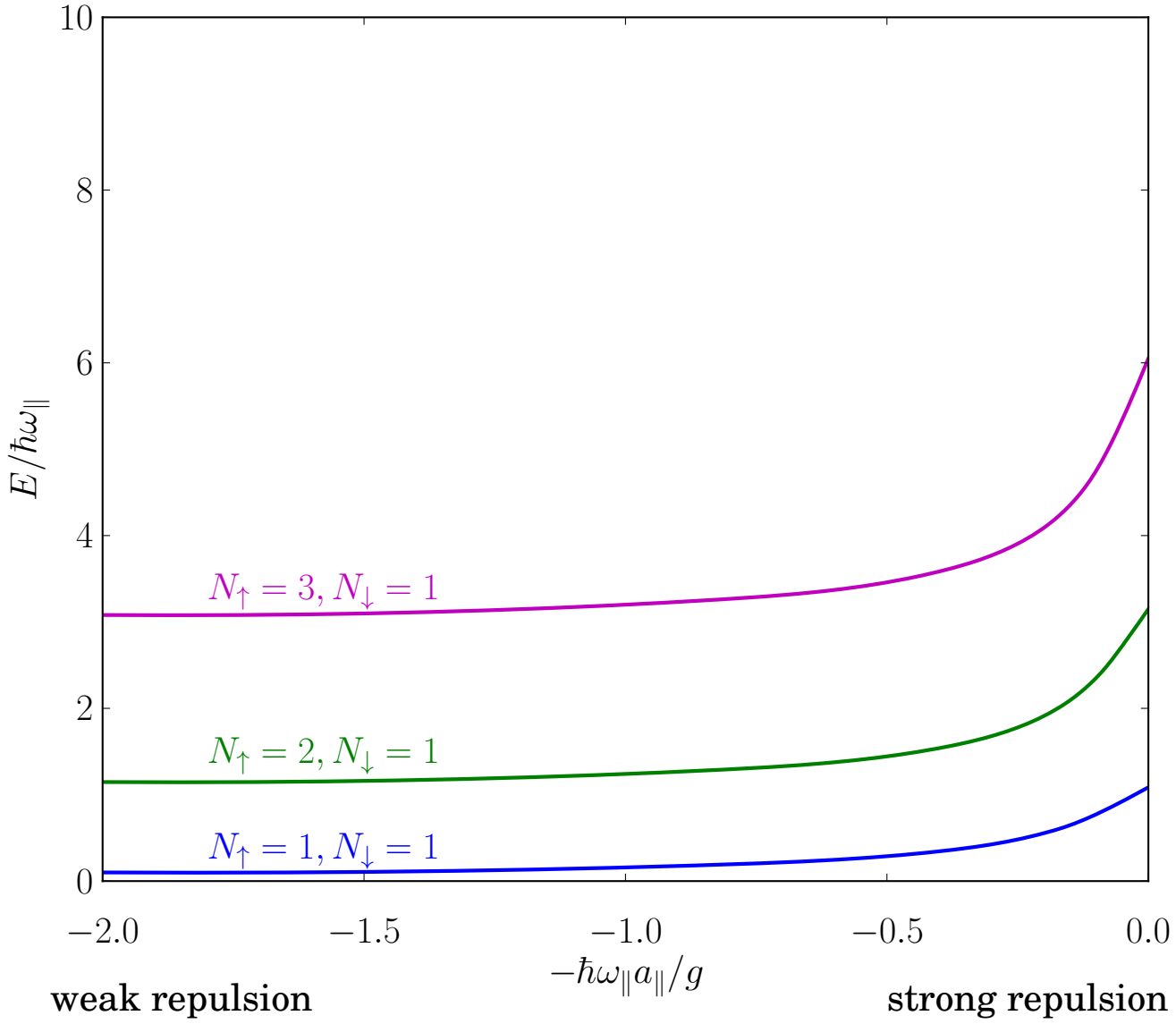

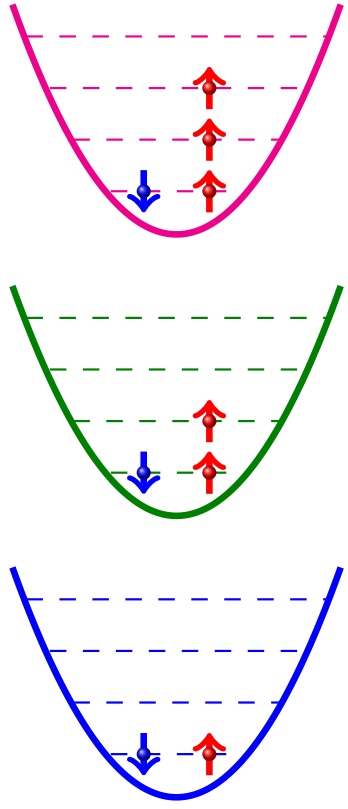

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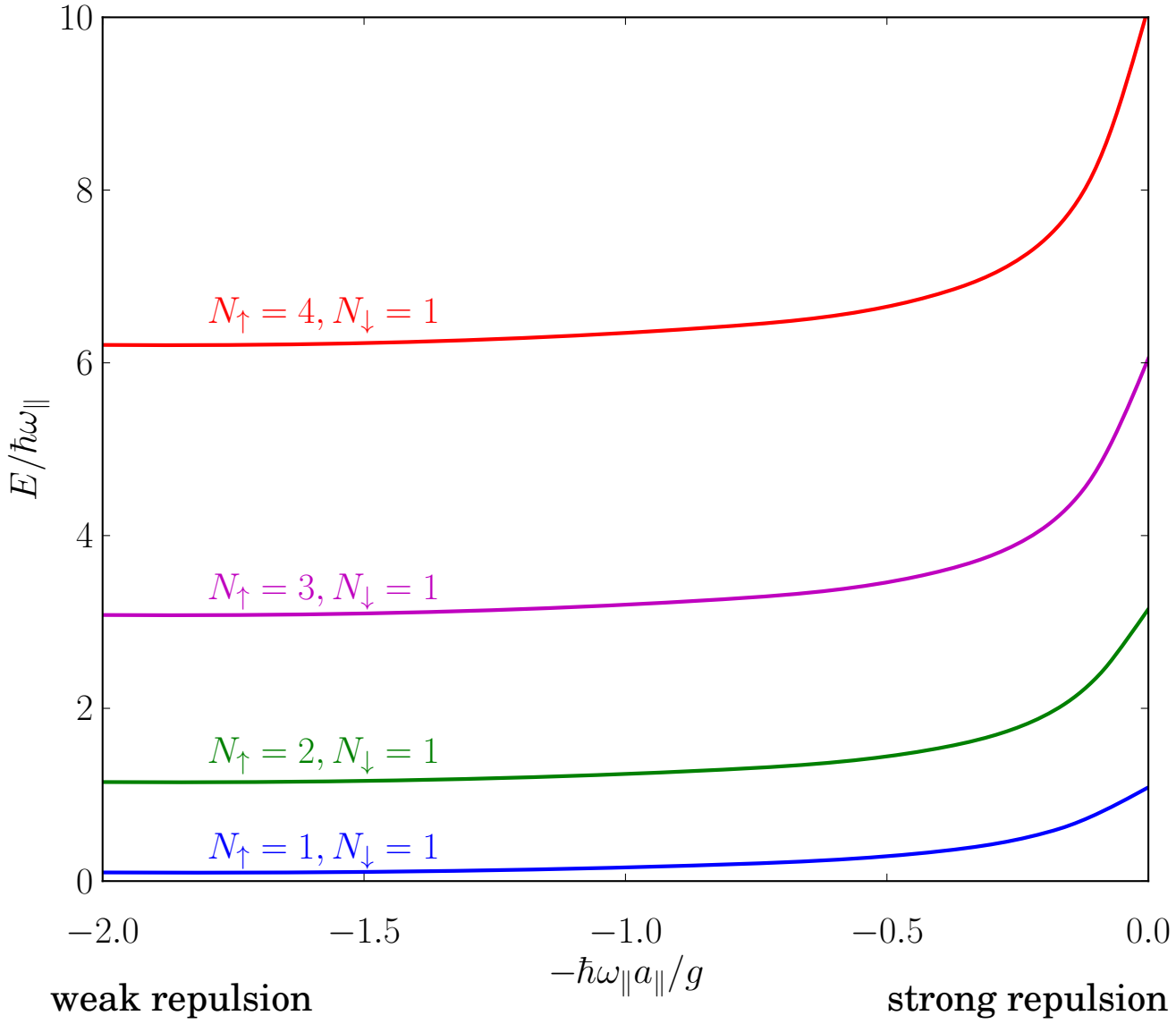

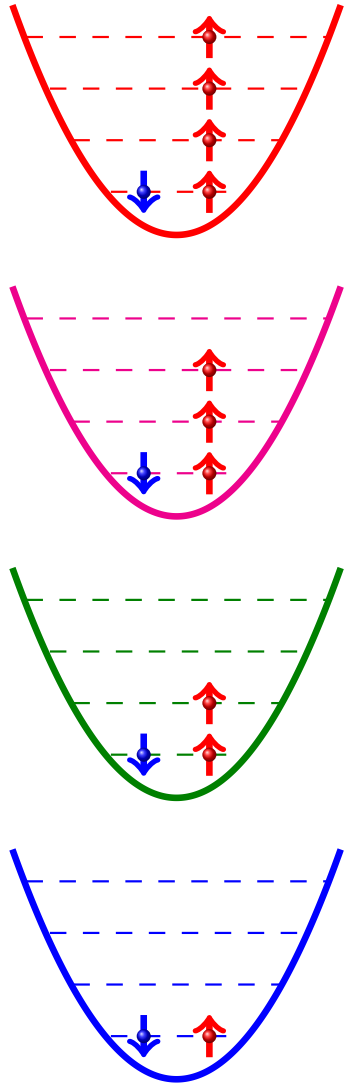

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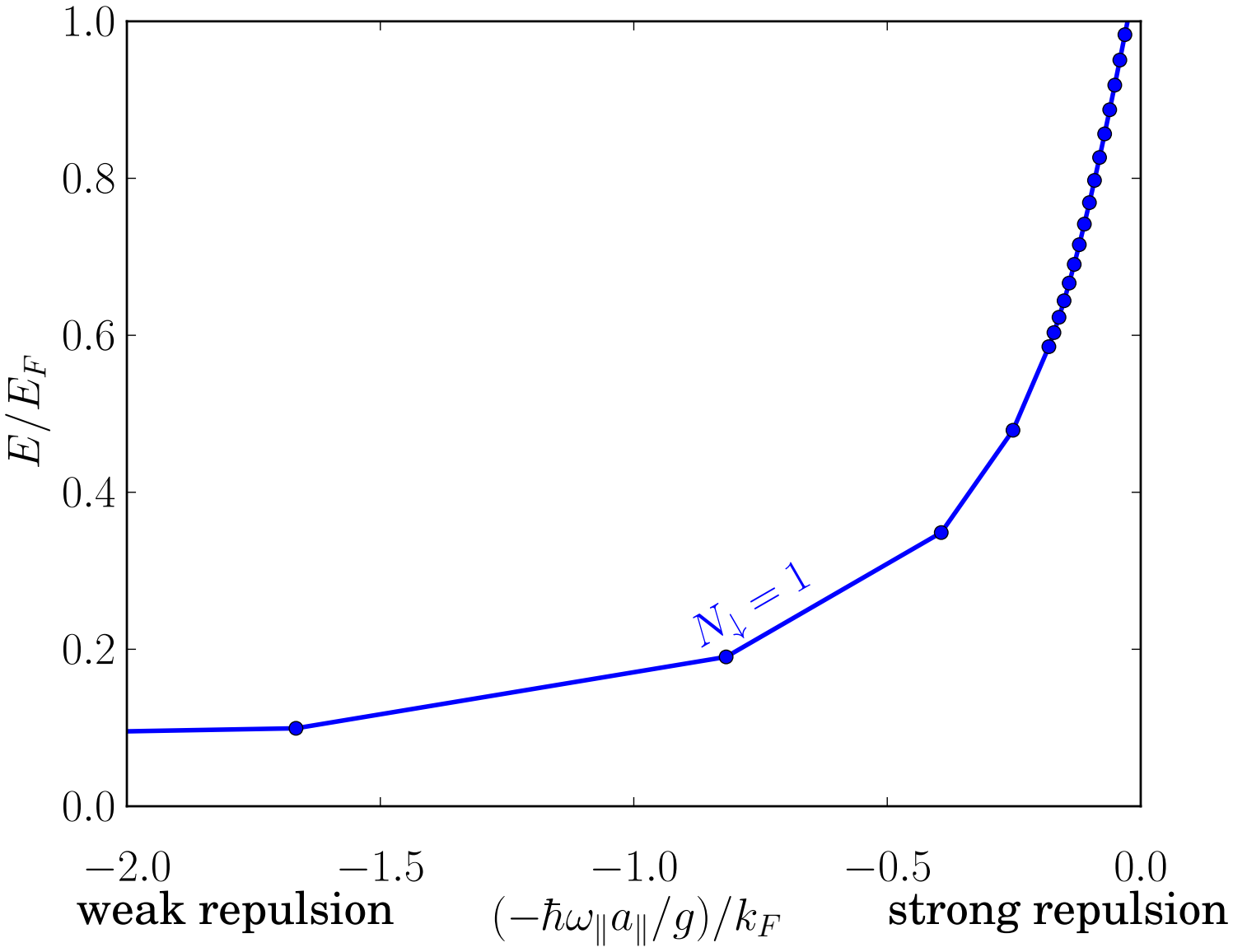

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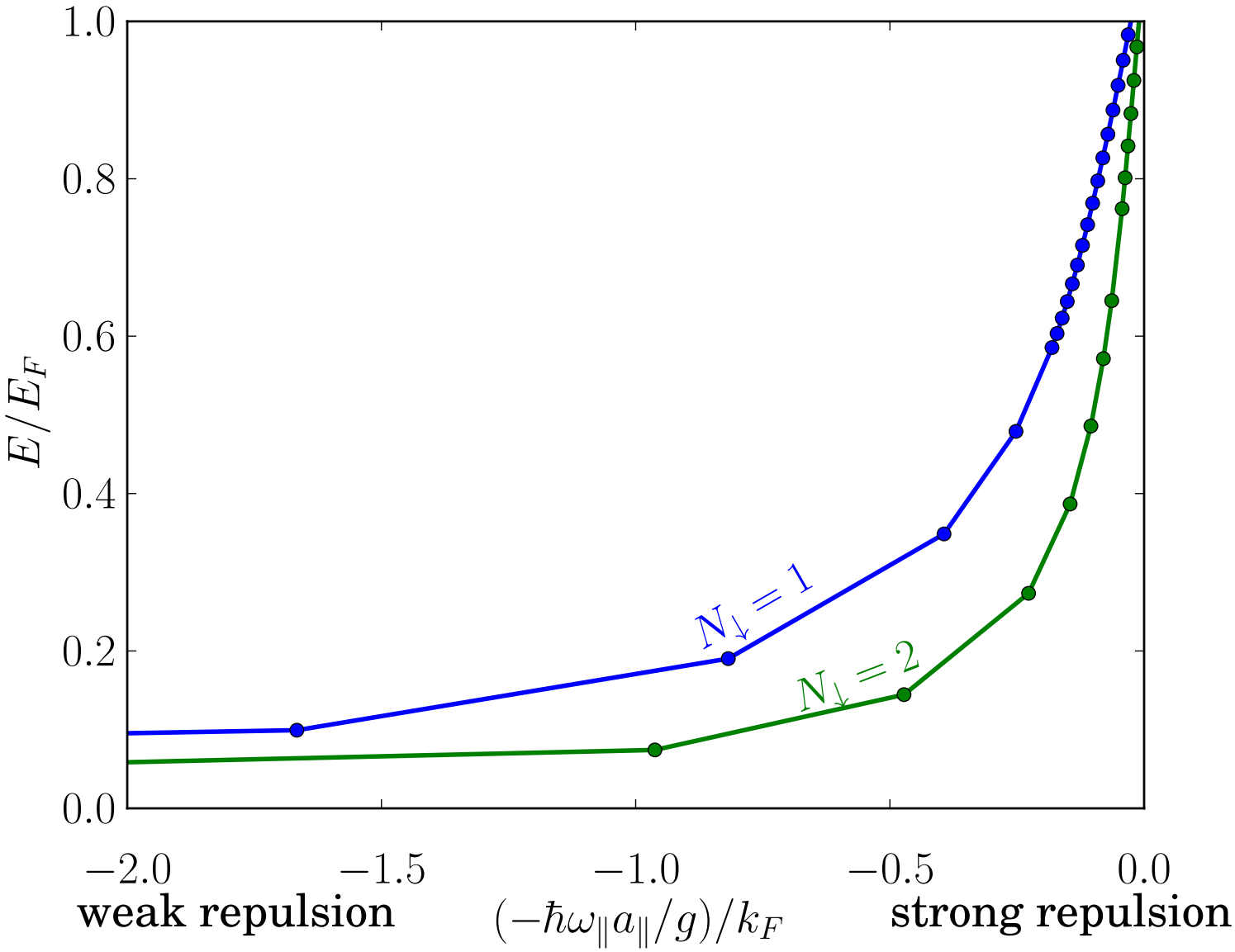

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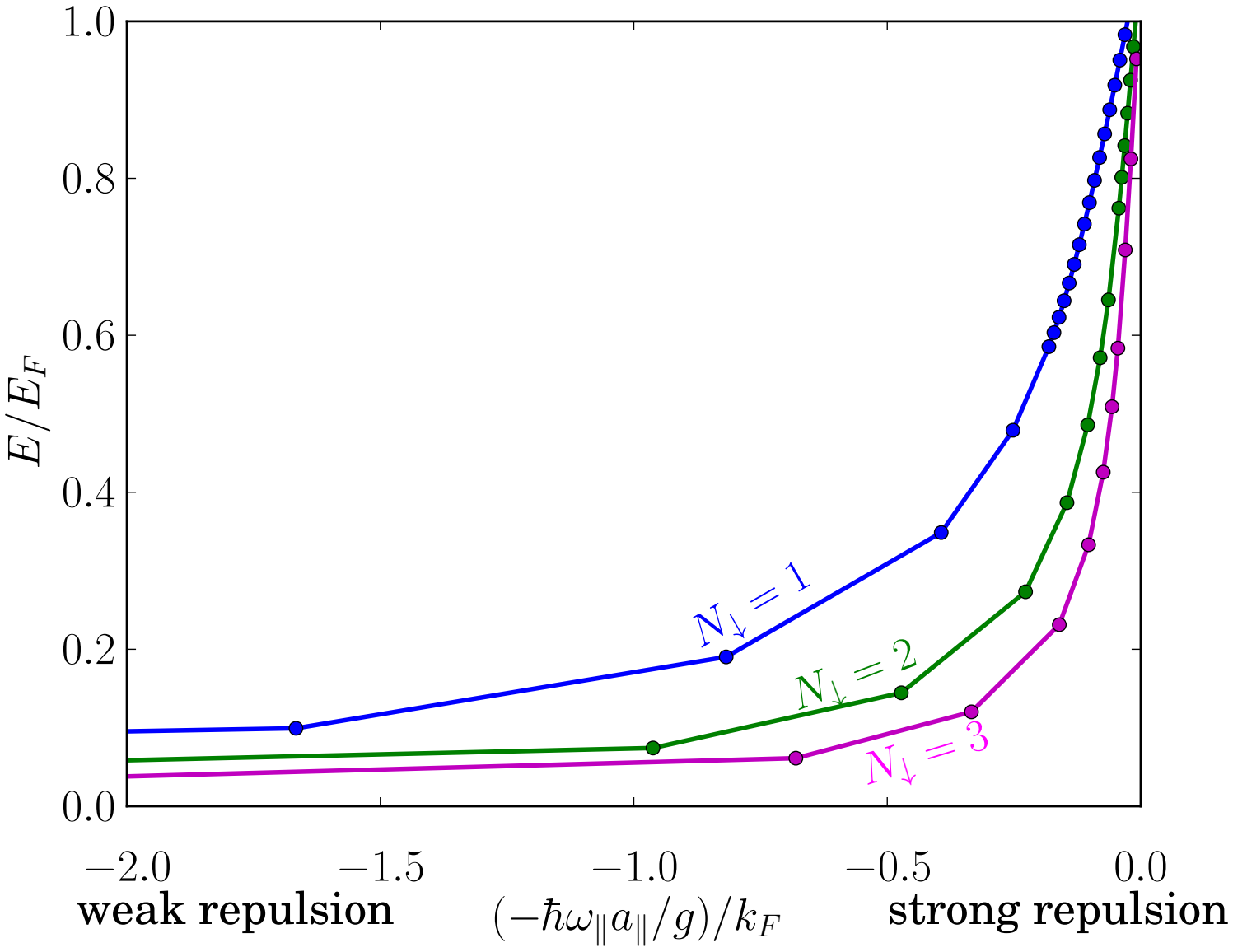

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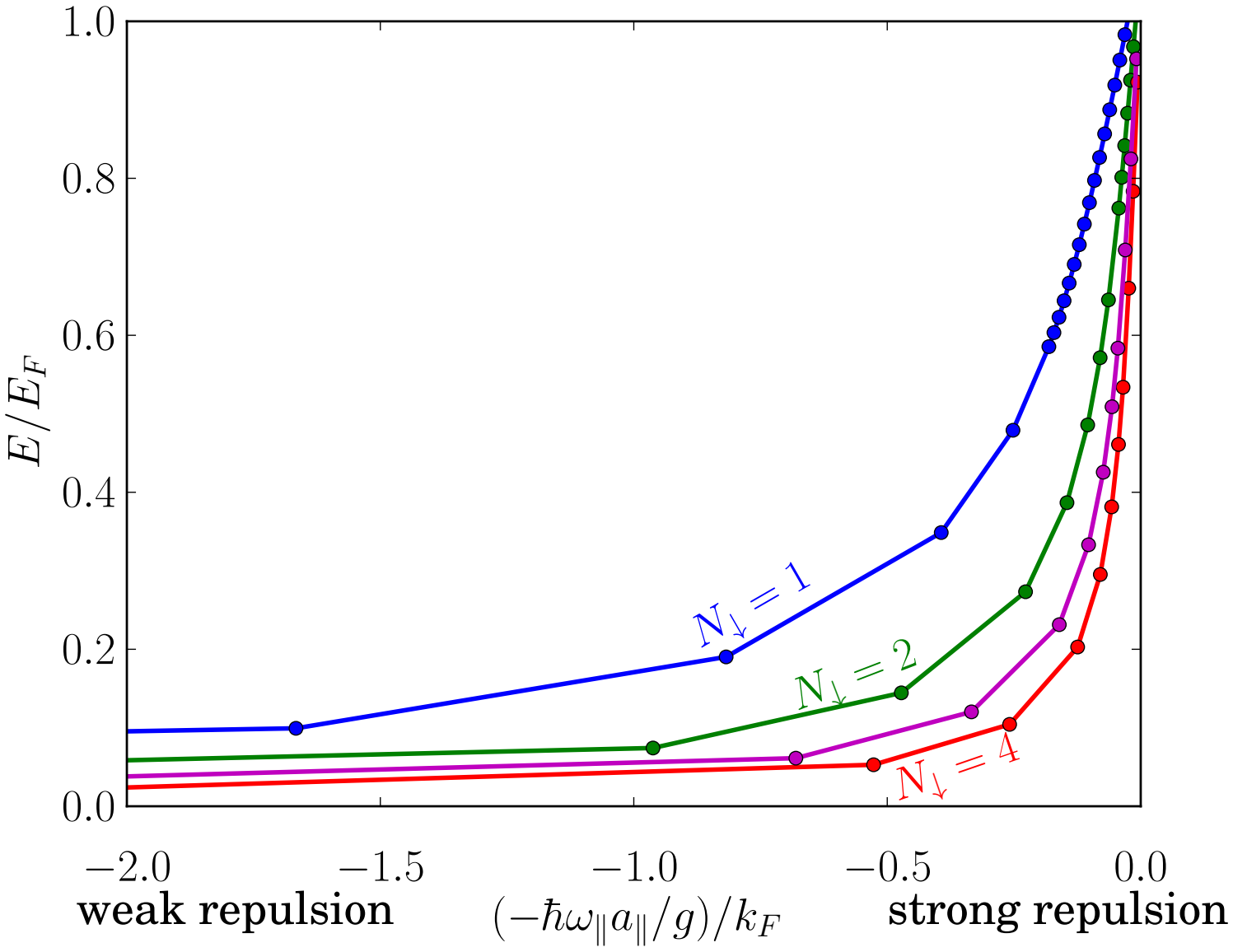

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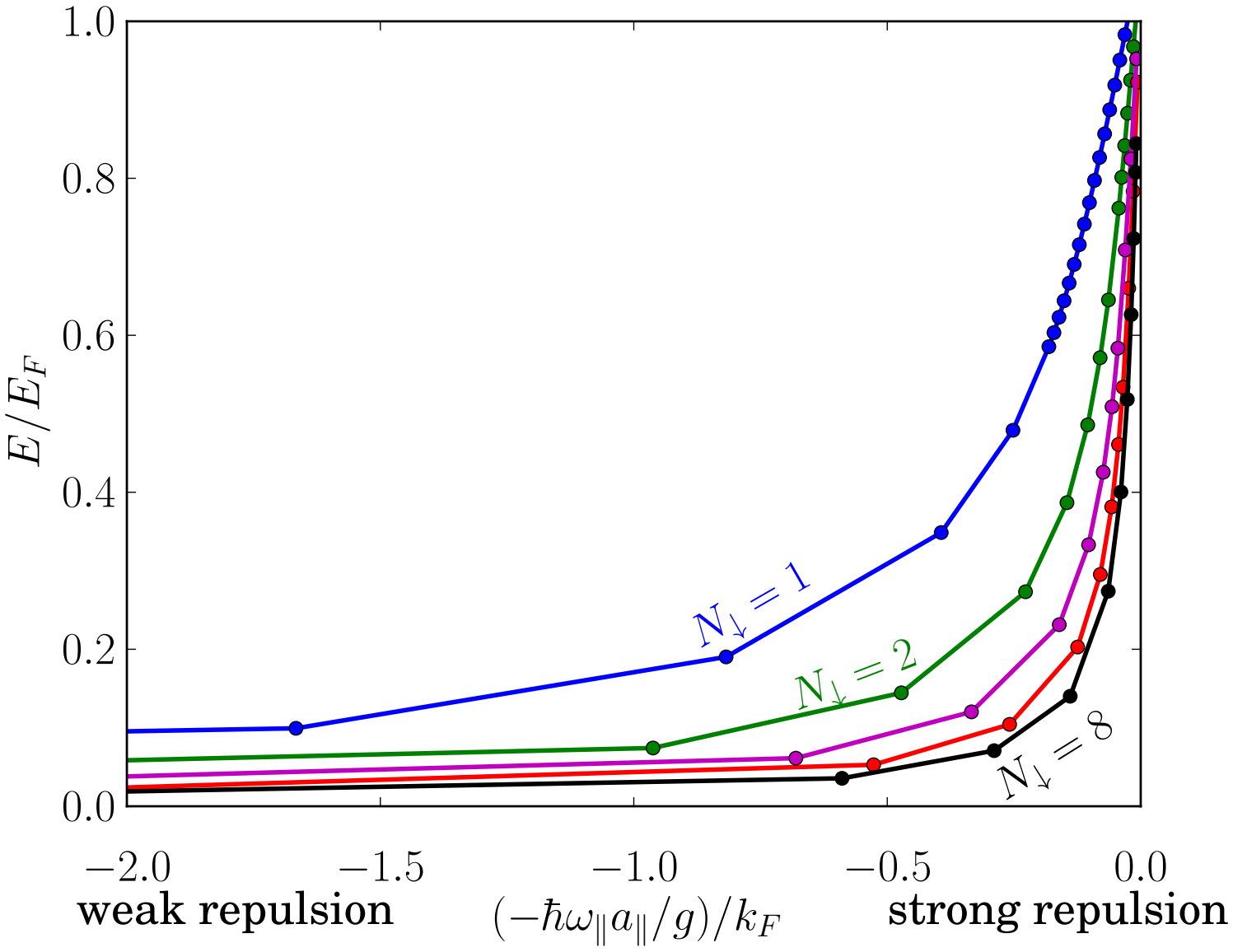
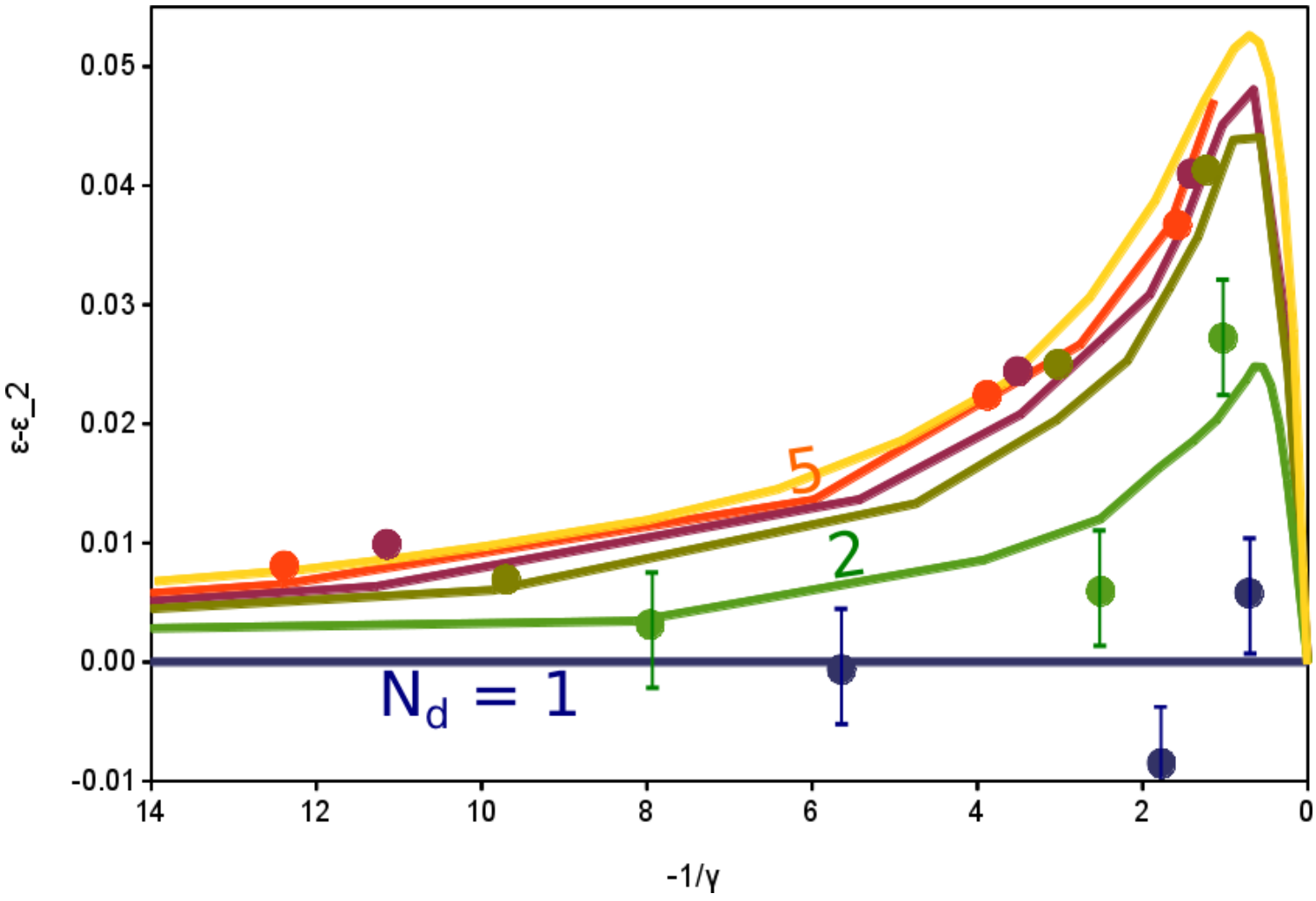
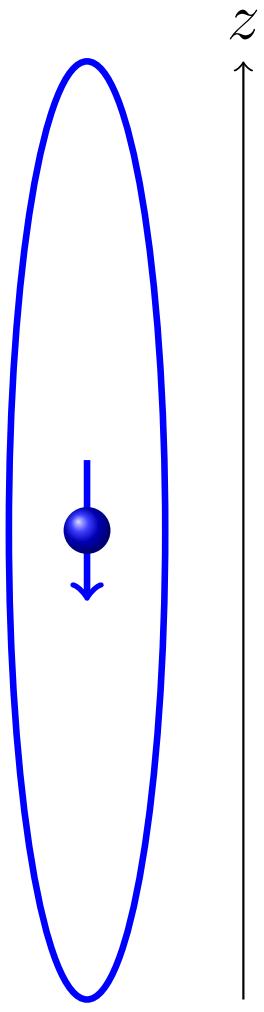


Polaron results: experiment

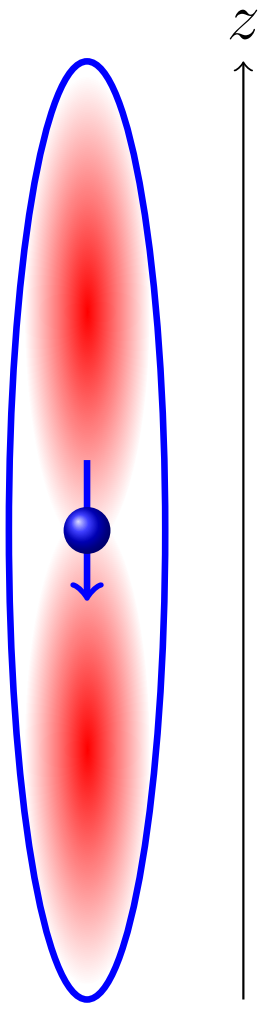


A.N. Wenz et al., arXiv 1307.3343

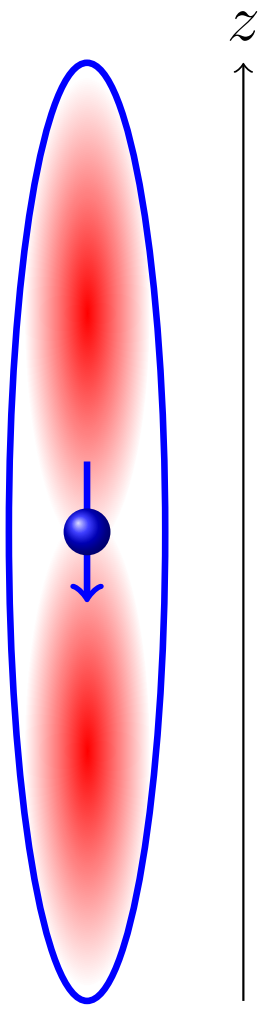
Polaron: density matrices

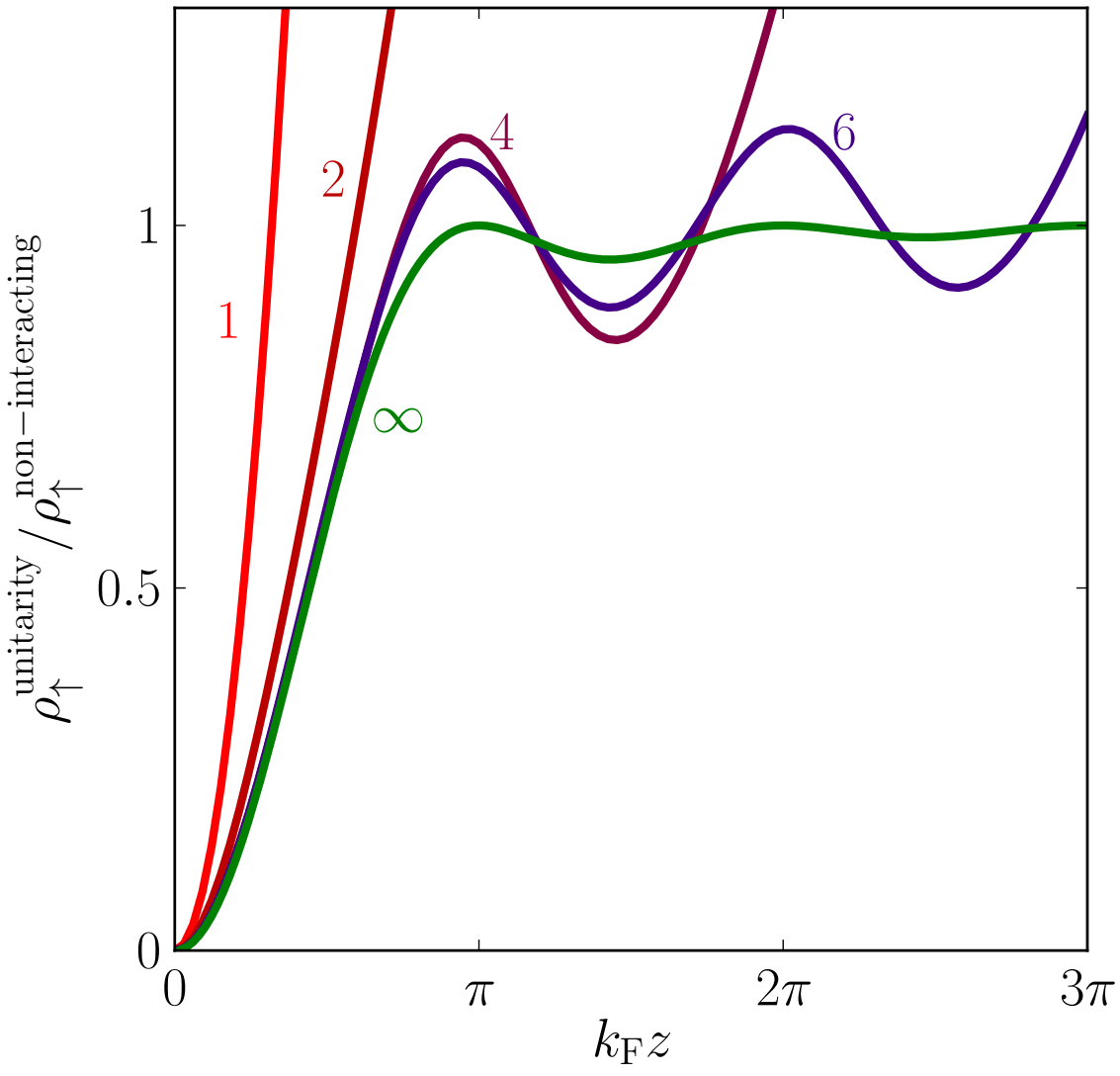


Polaron: density matrices



Polaron: density matrices





Methods

Configuration Interaction

- Full CI or truncated CI for larger systems.
- Need excited states.
- Use the quantum Harmonic oscillator eigenfunctions as basis state.

Quantum Monte-Carlo

Use DMC for larger systems.

Itinerant ferromagnetism?

$$\hat{\mathcal{H}} = \sum_{i \in \{\uparrow,\downarrow\}} \hat{h}(\mathbf{r}_i) + g \sum_{i \in \uparrow, j \in \downarrow} \delta(\mathbf{r}_i - \mathbf{r}_j)$$

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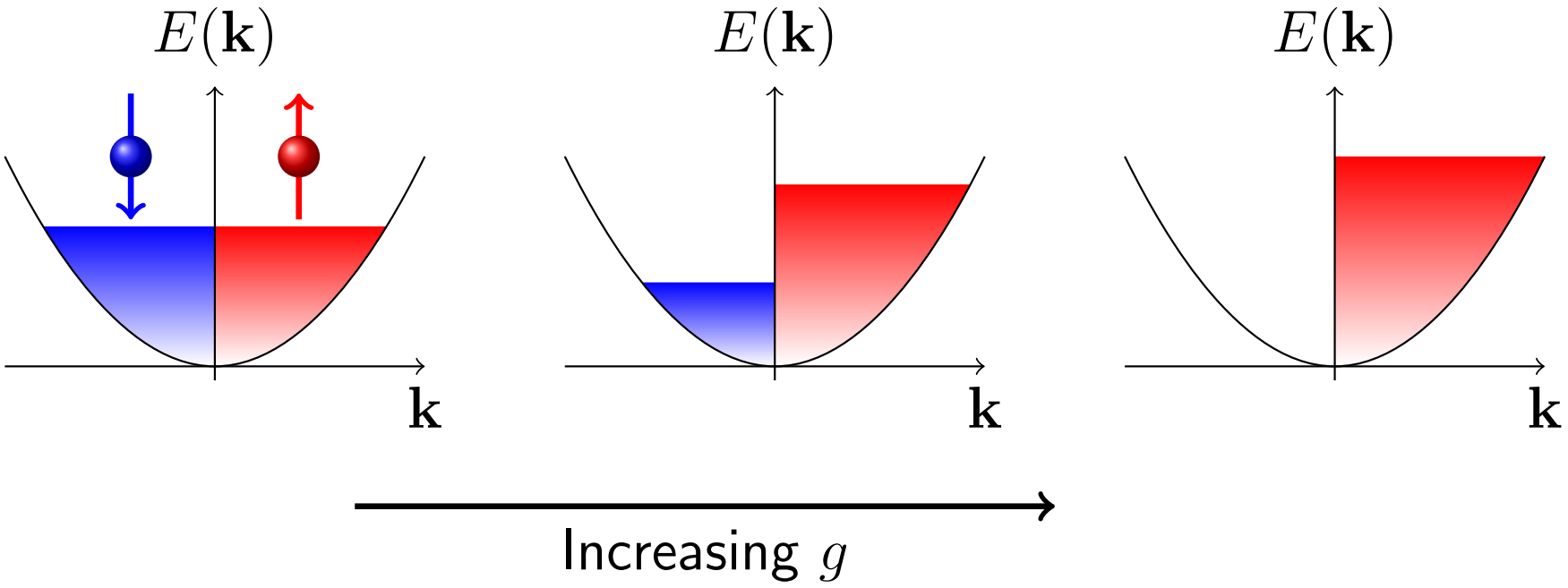
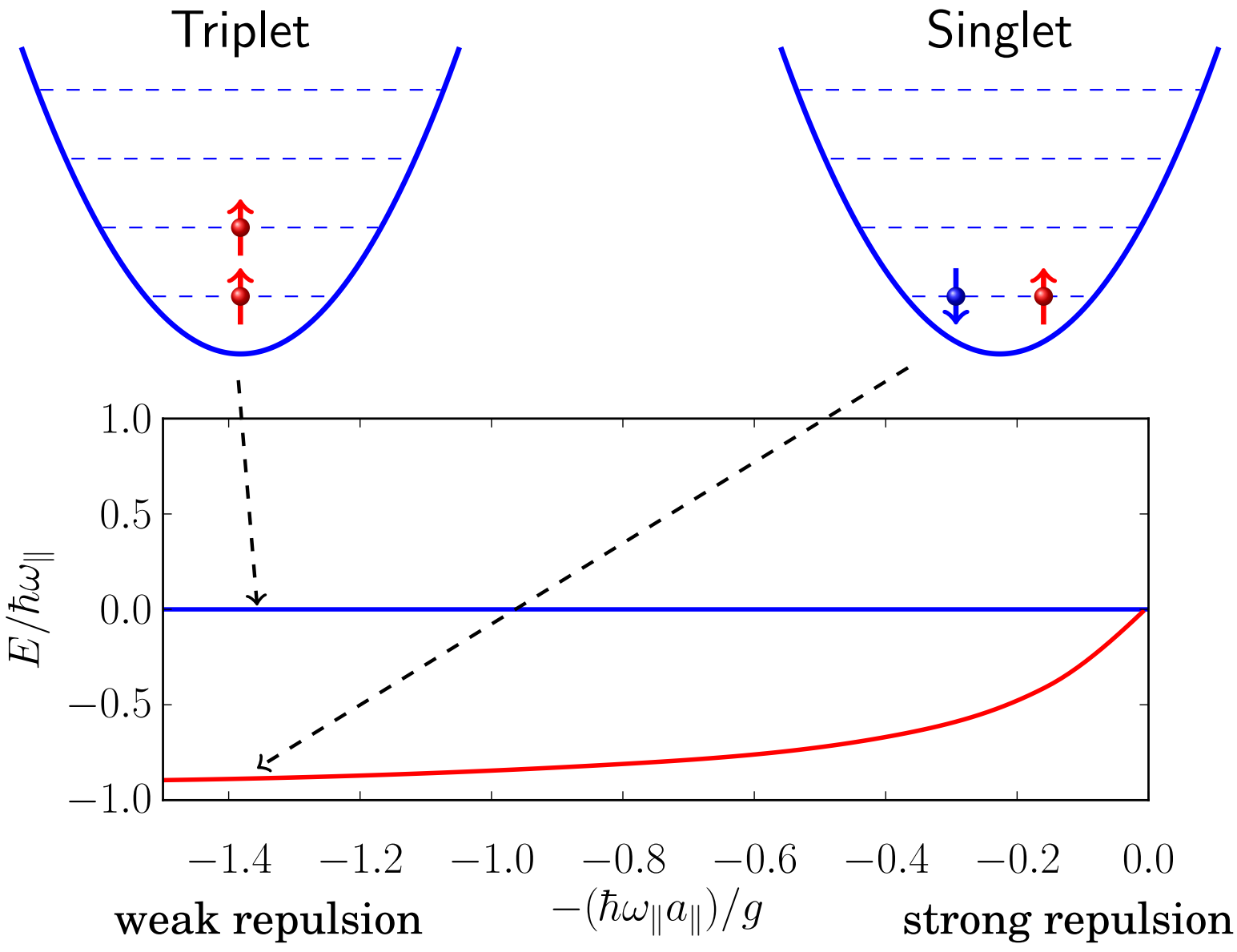
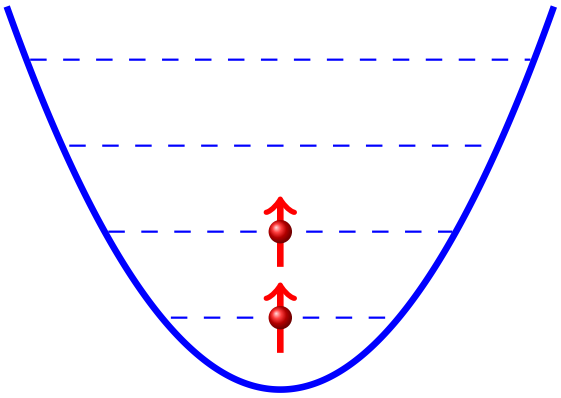


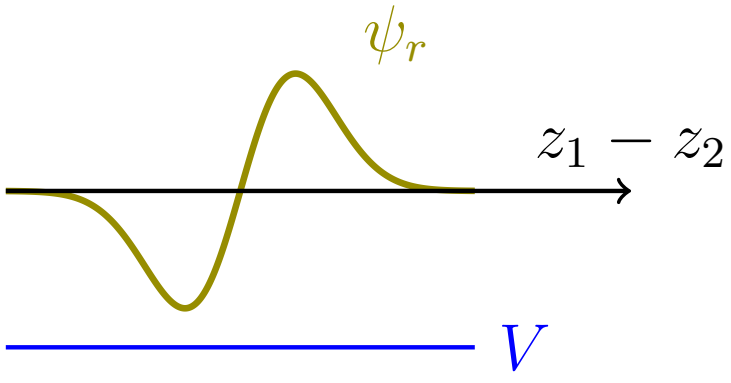
What should we see?



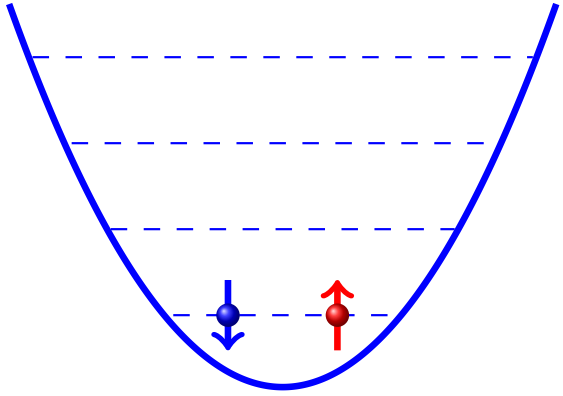
What should we see?

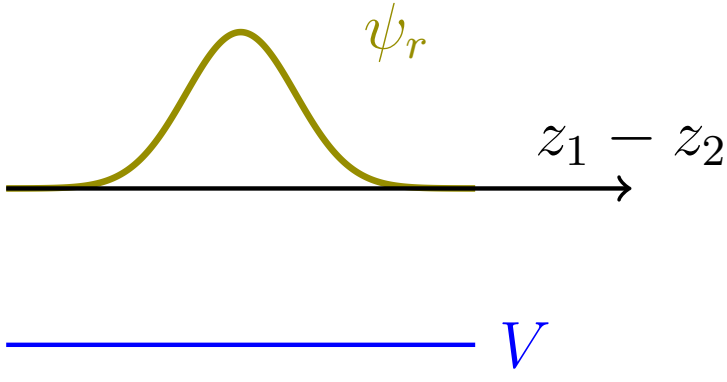
Triplet



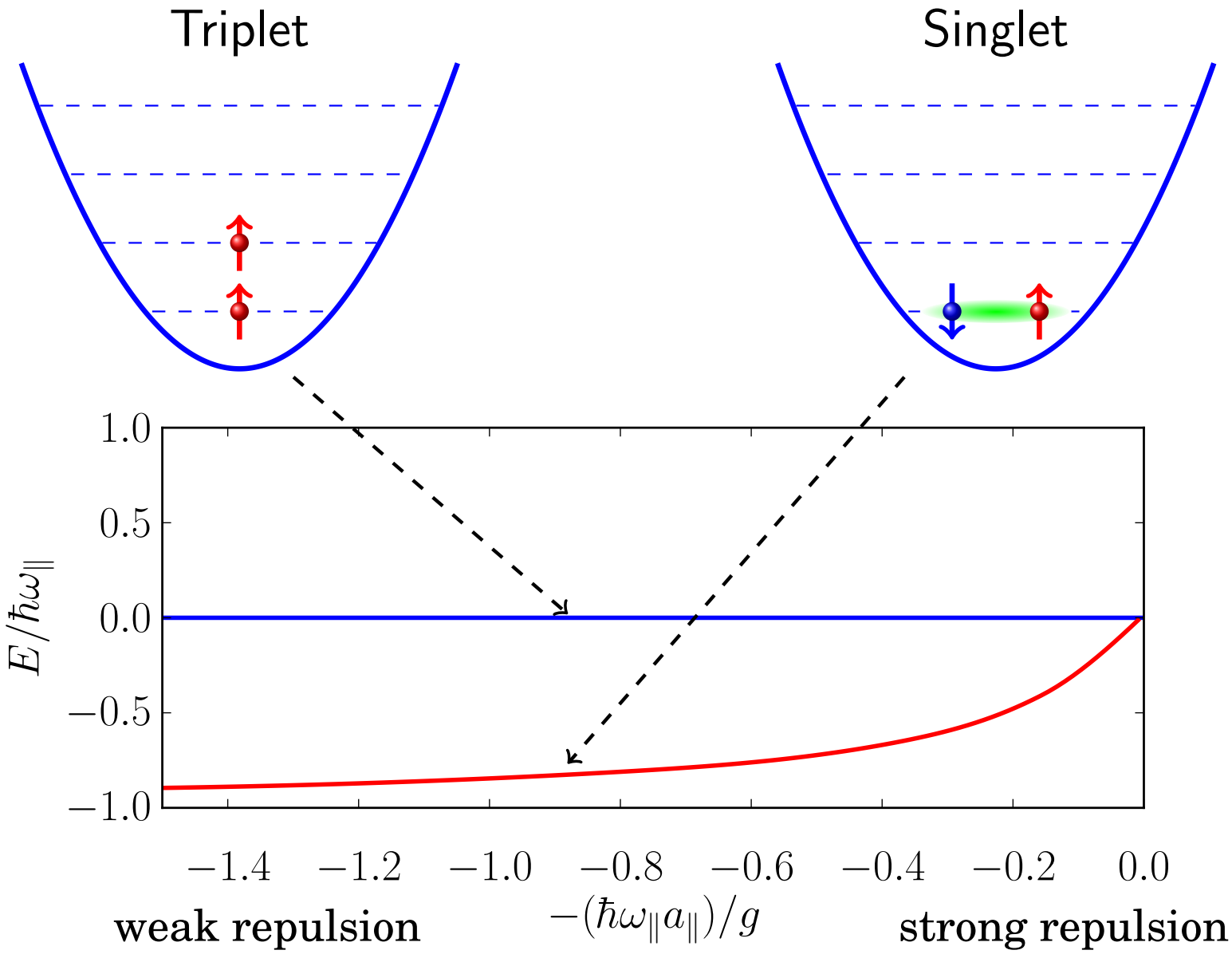


Singlet



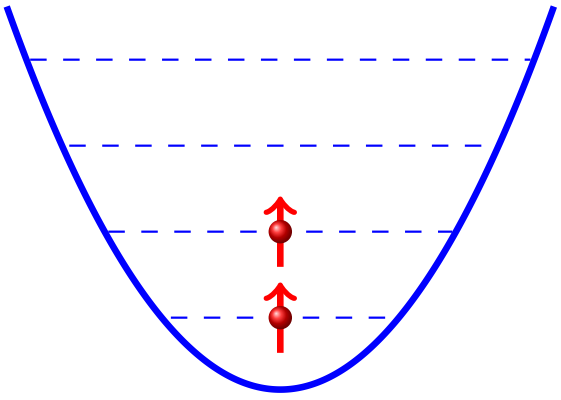


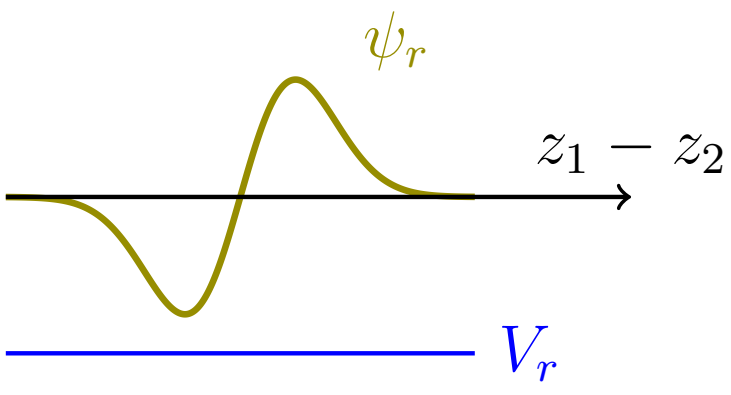
Two atoms at medium interactions



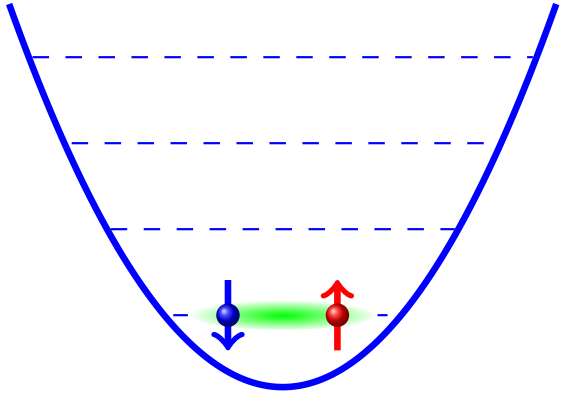
What should we see?

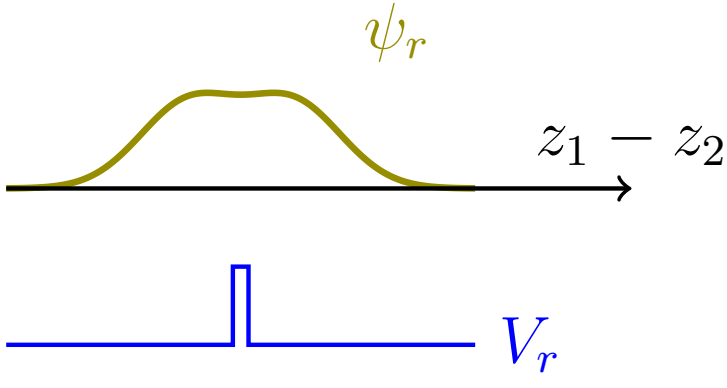
Triplet



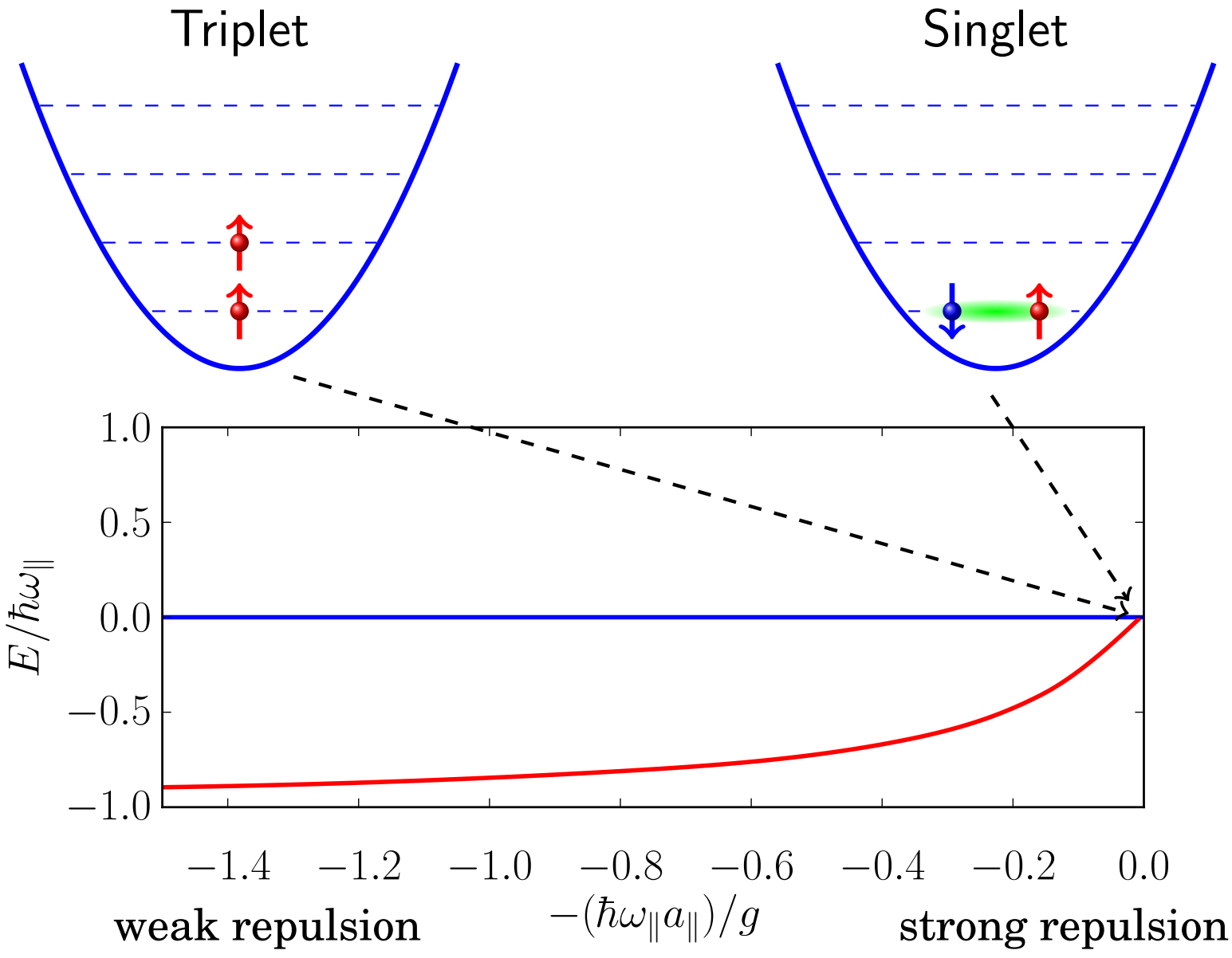


Singlet



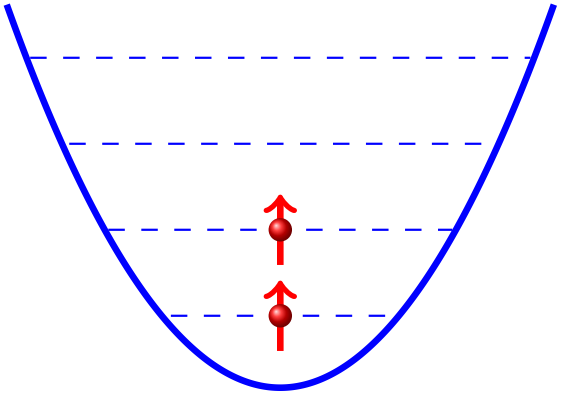


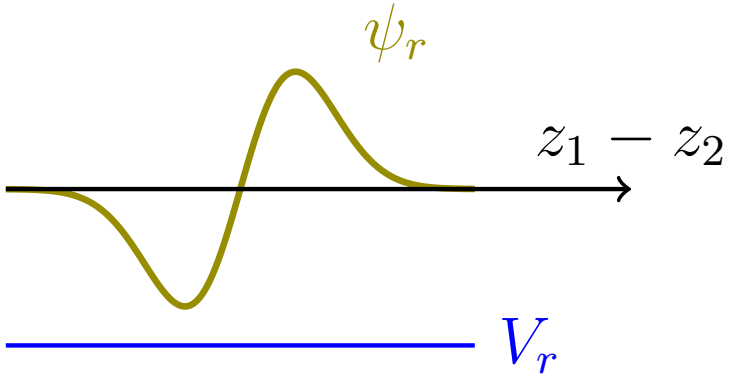
Fermionization: $g \to \infty$



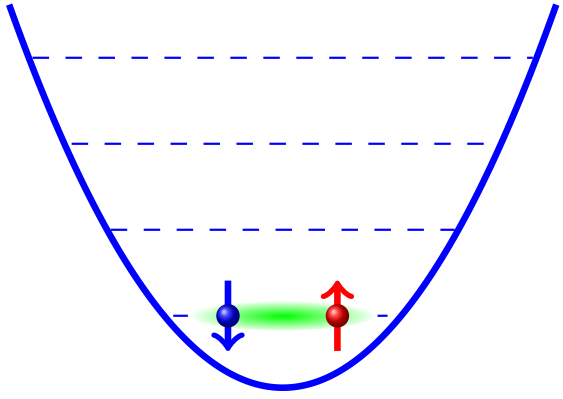
Fermionization: wavefunctions

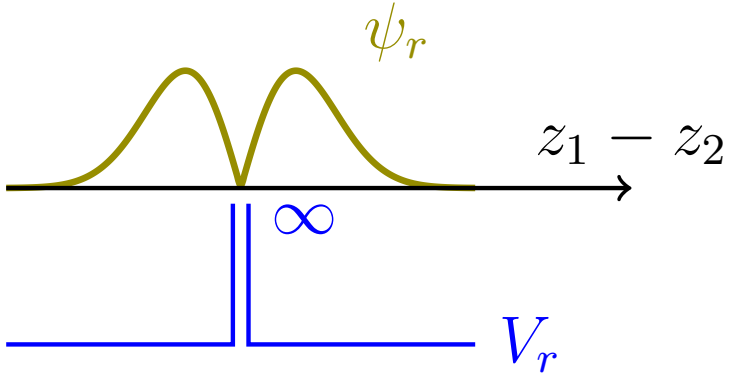
Triplet



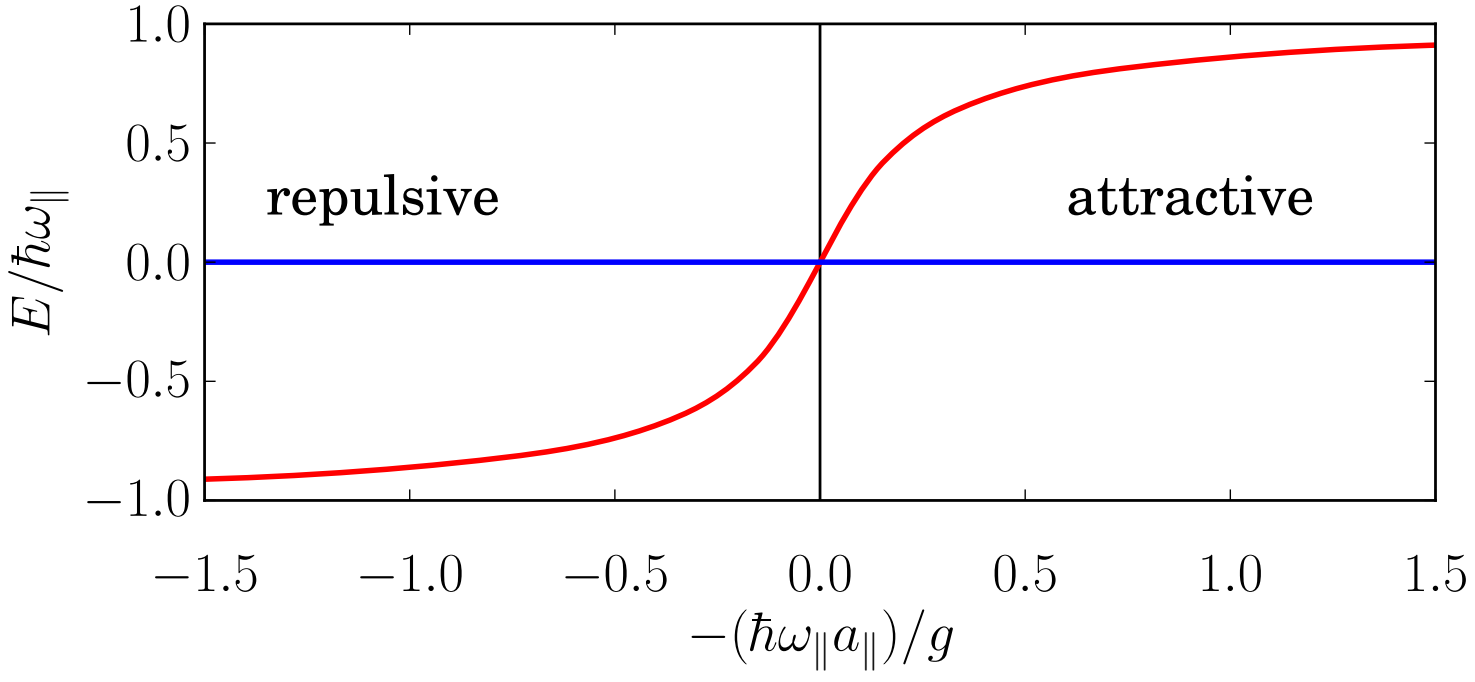


Singlet

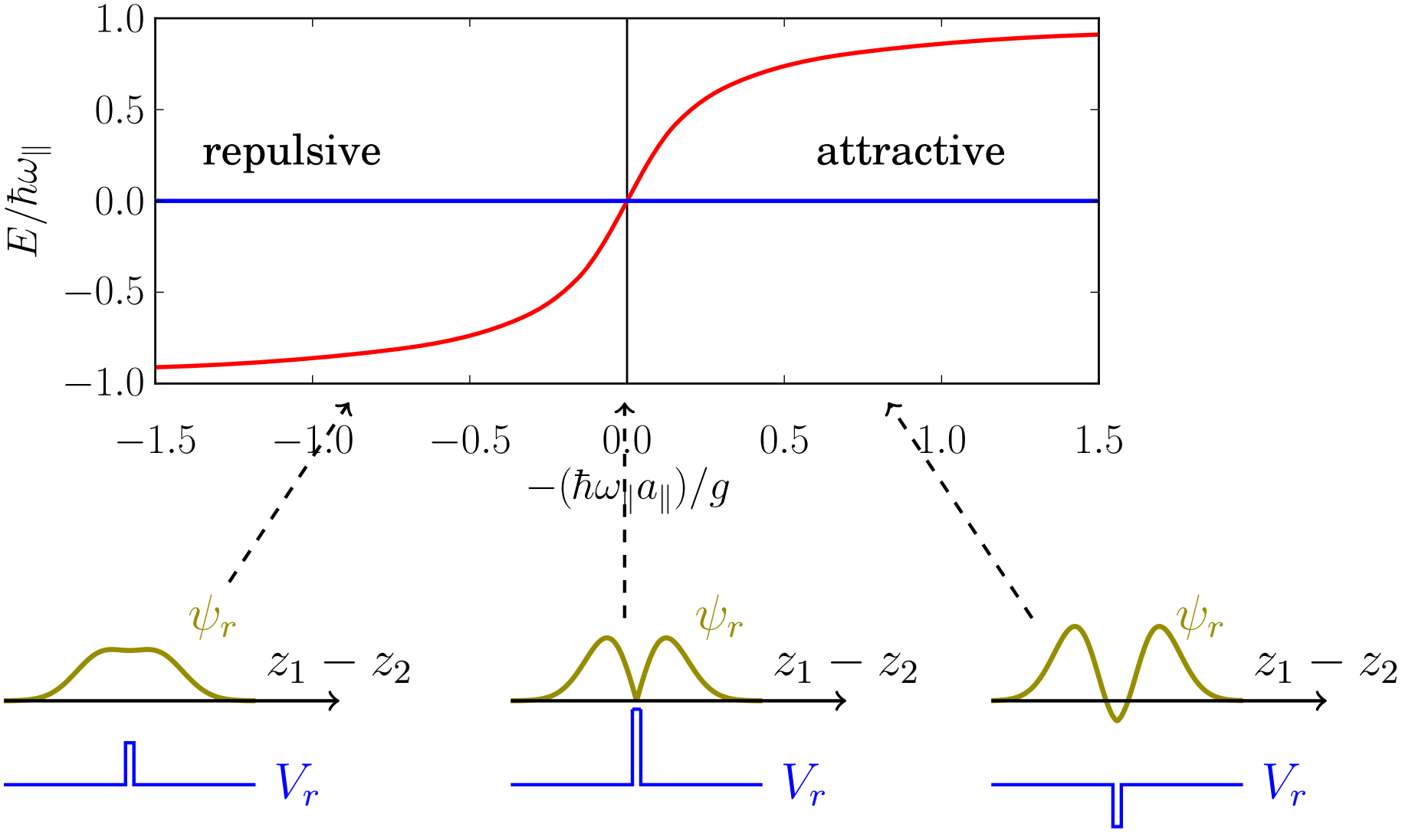




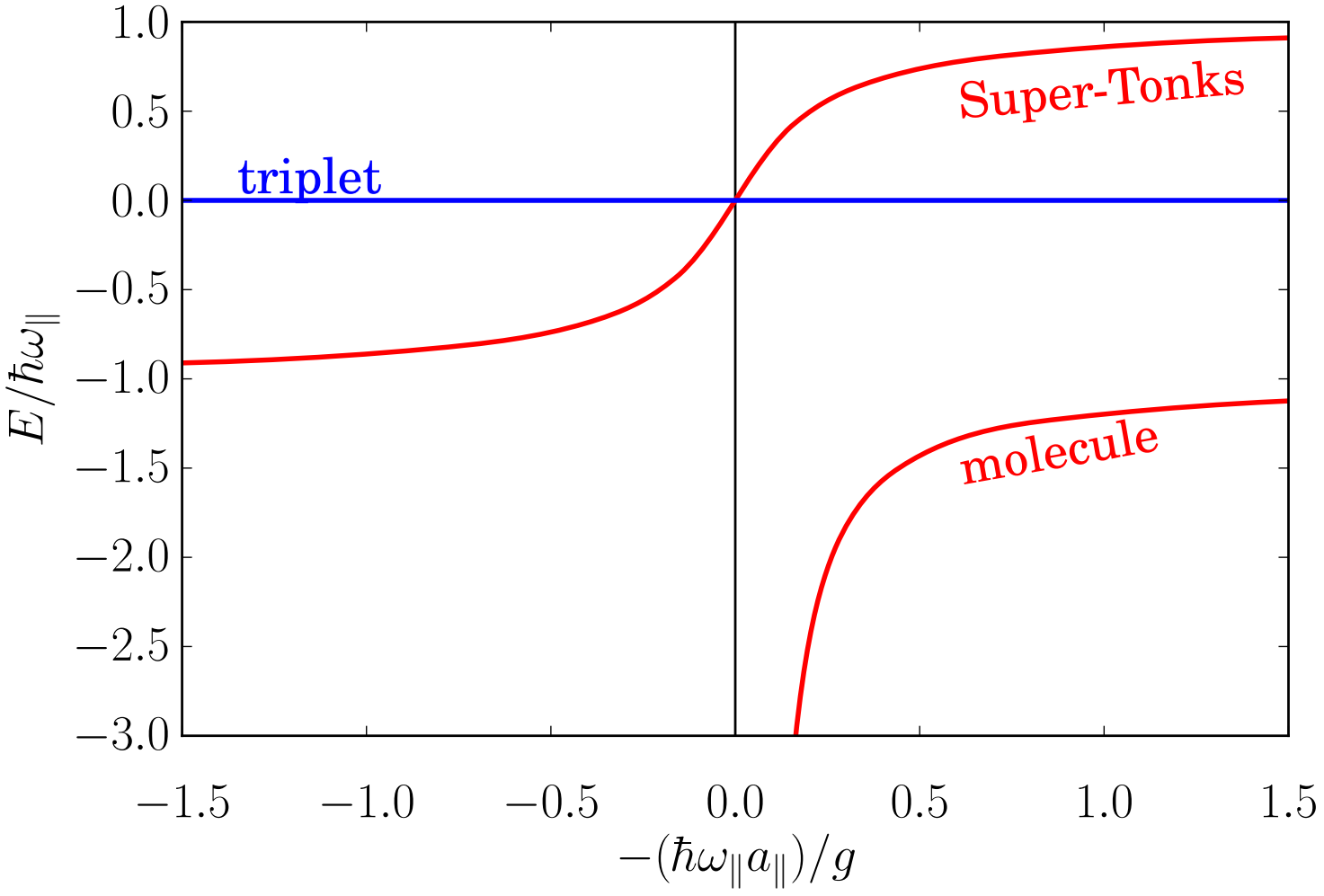
Beyond unitarity: Super-Tonks regime



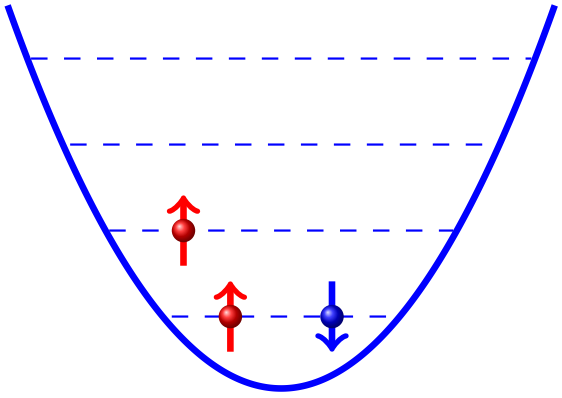
Beyond unitarity: Super-Tonks regime



Molecular bands



Observable: tunnelling experiment



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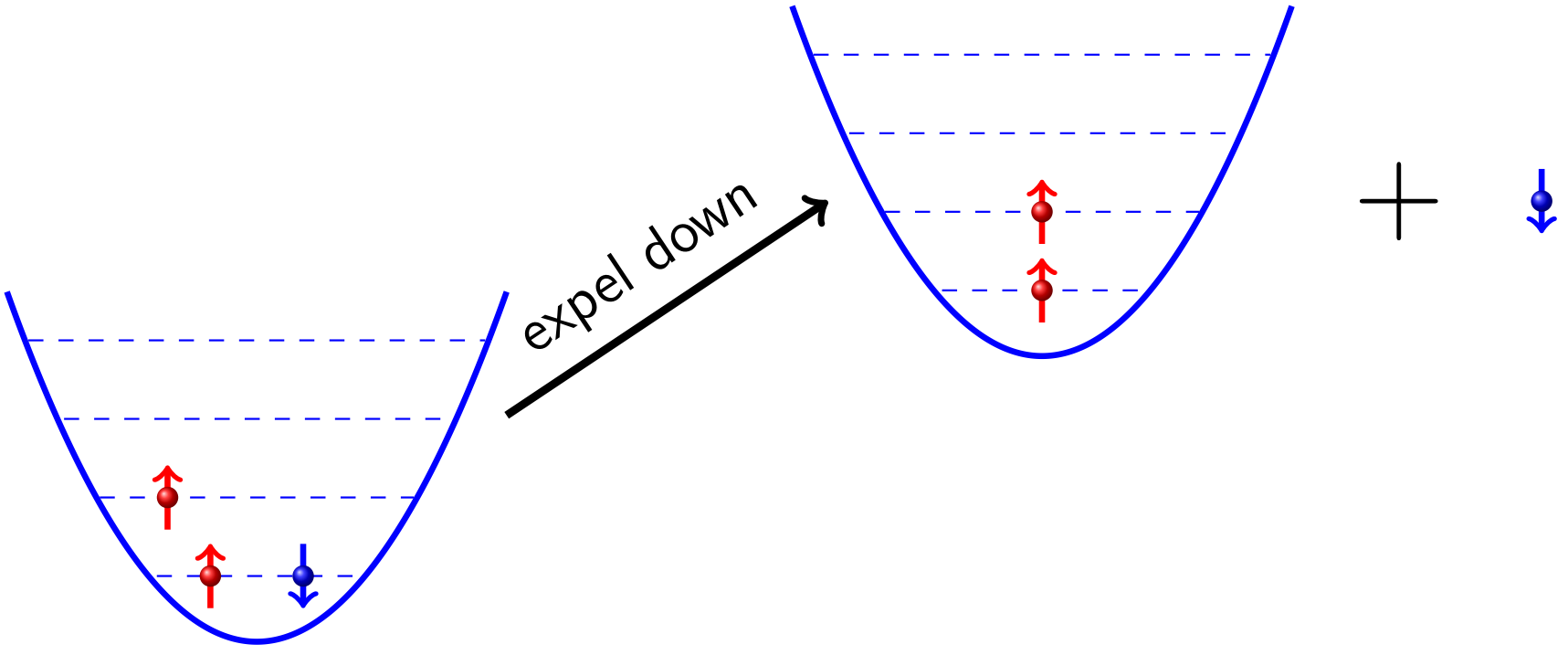

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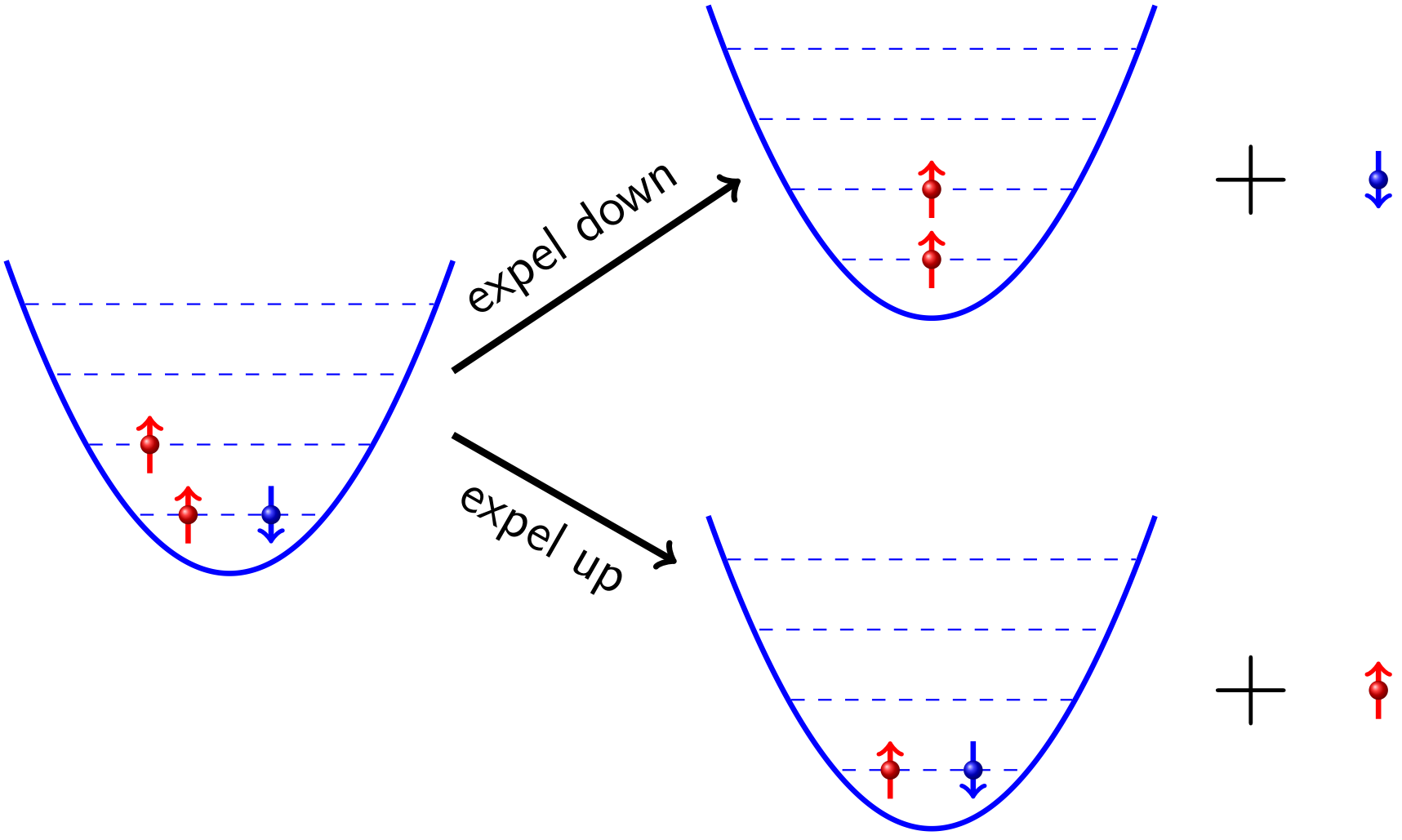
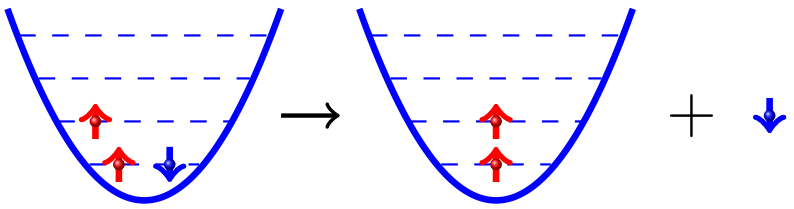
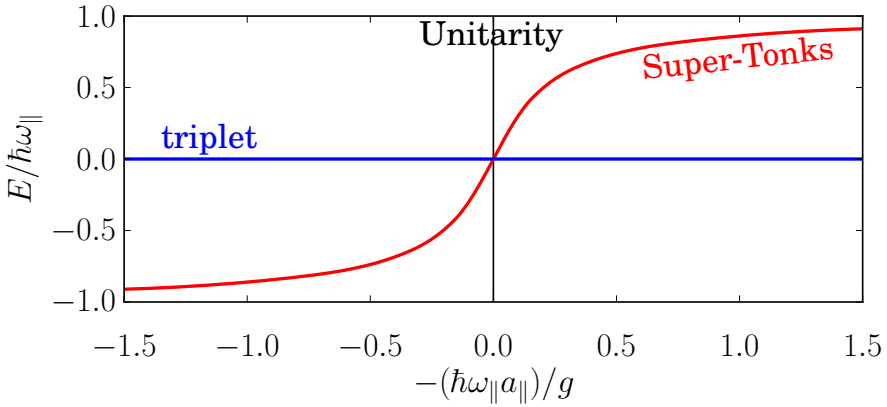
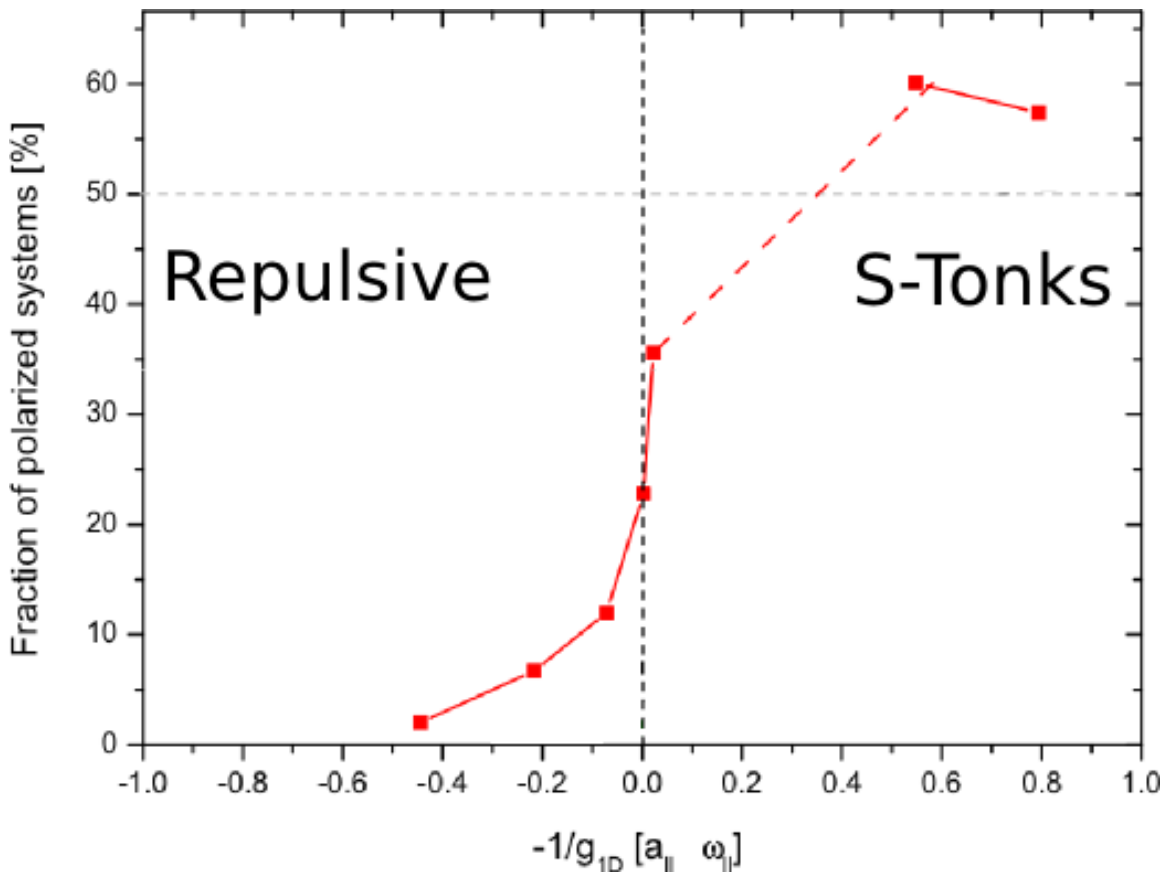


Tunnelling results







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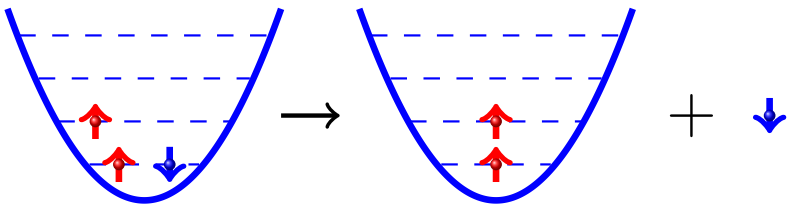
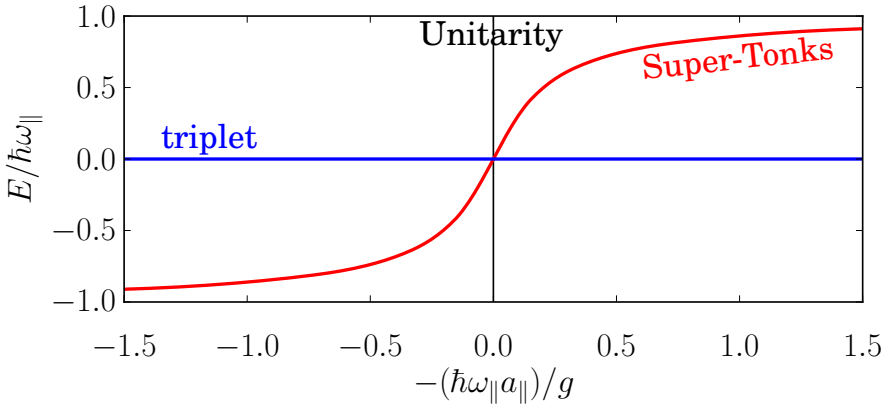
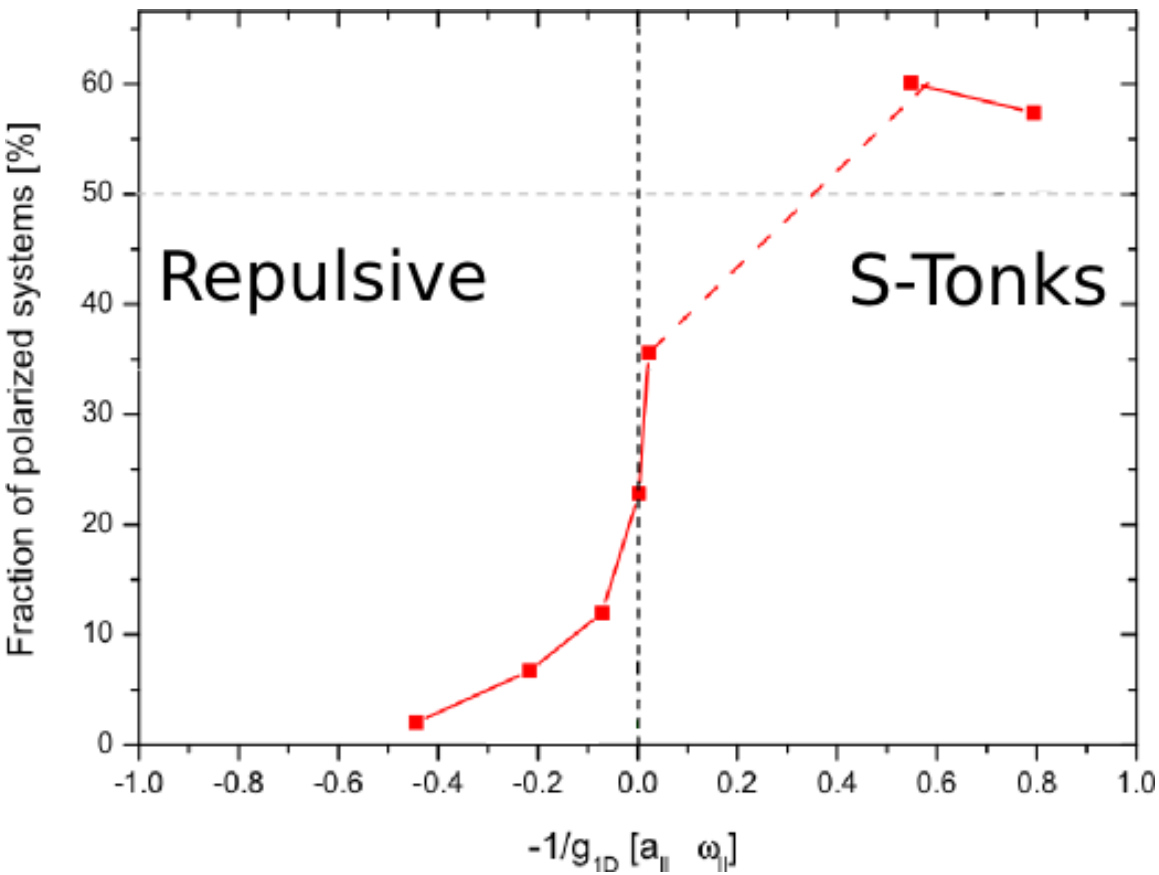


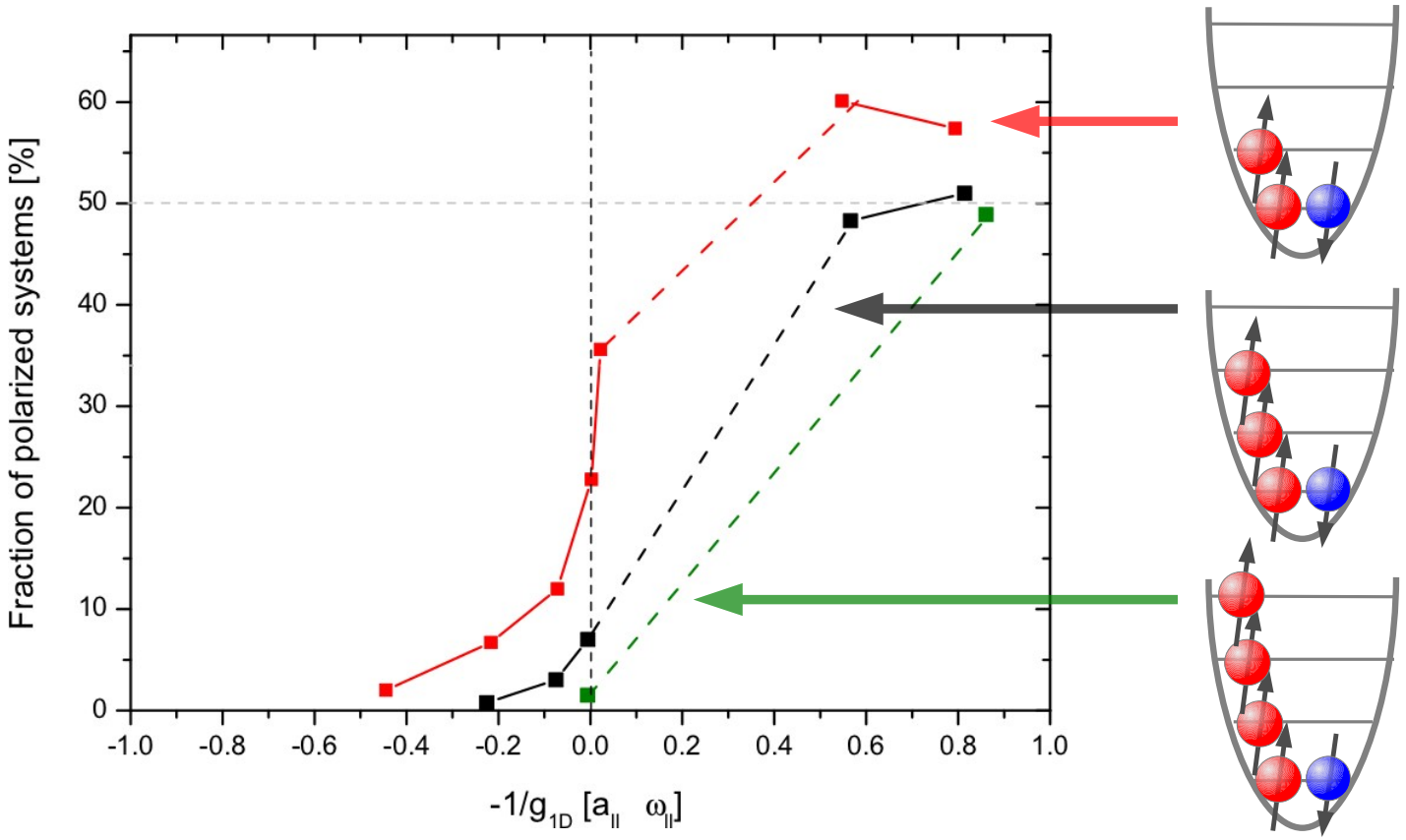




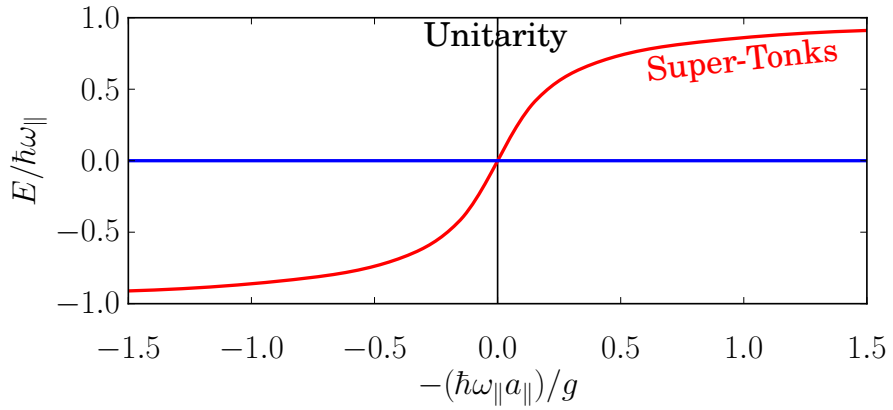
$$|\uparrow\uparrow\downarrow\rangle \rightarrow |\uparrow\uparrow\rangle\,|\downarrow\rangle$$

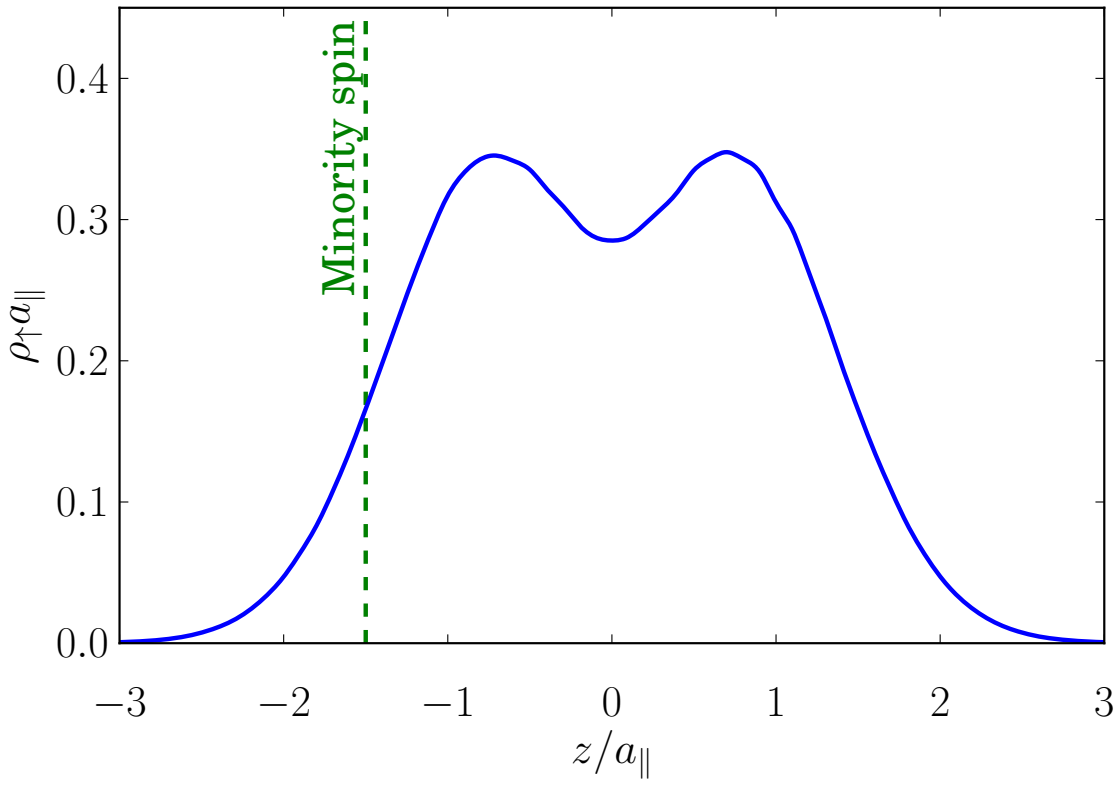
$$|\uparrow\uparrow\downarrow\rangle \rightarrow (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\,|\uparrow\rangle$$

Tunnelling results

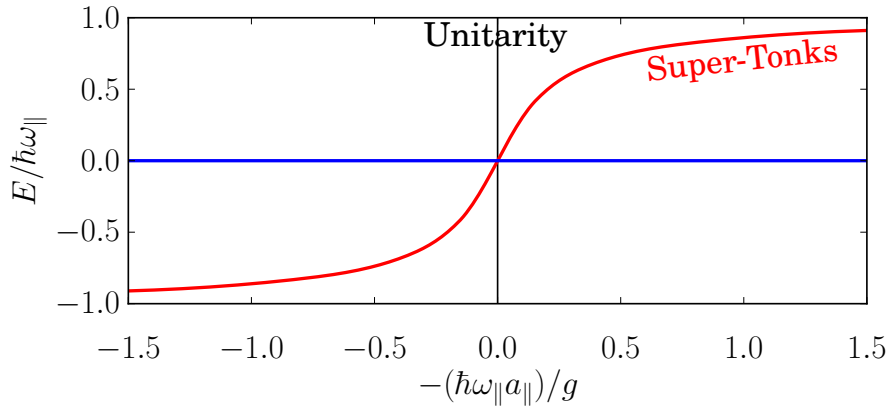


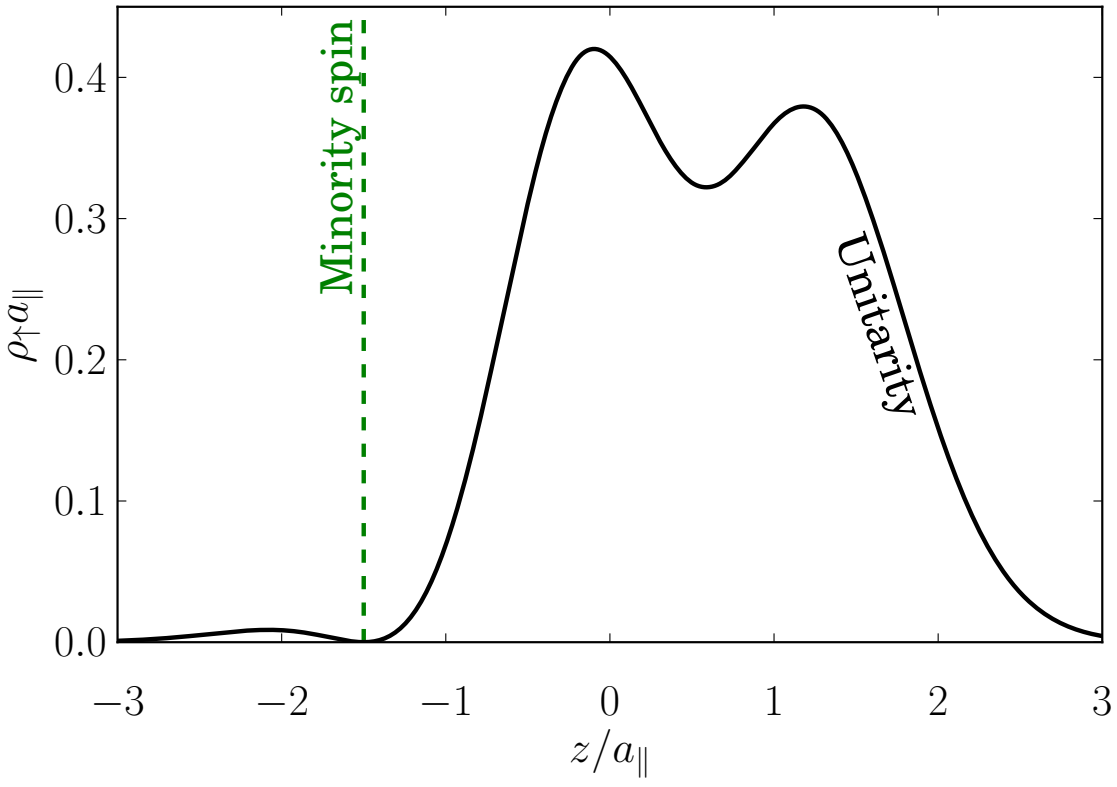
Magnetic correlations? $N_\uparrow = 2, N_\downarrow = 1$



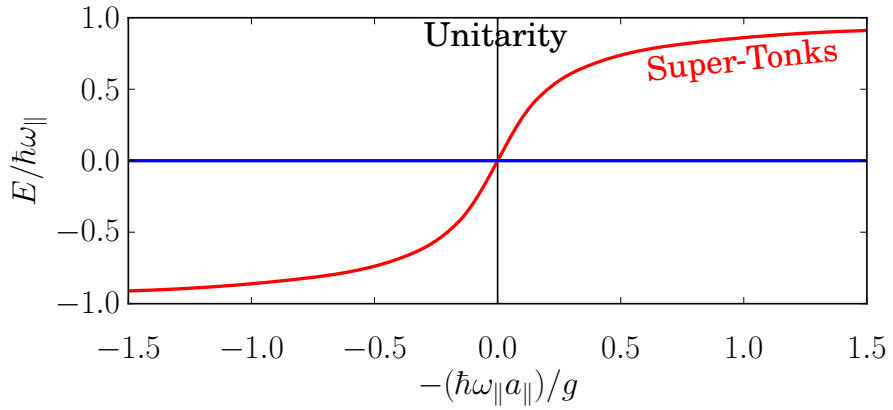


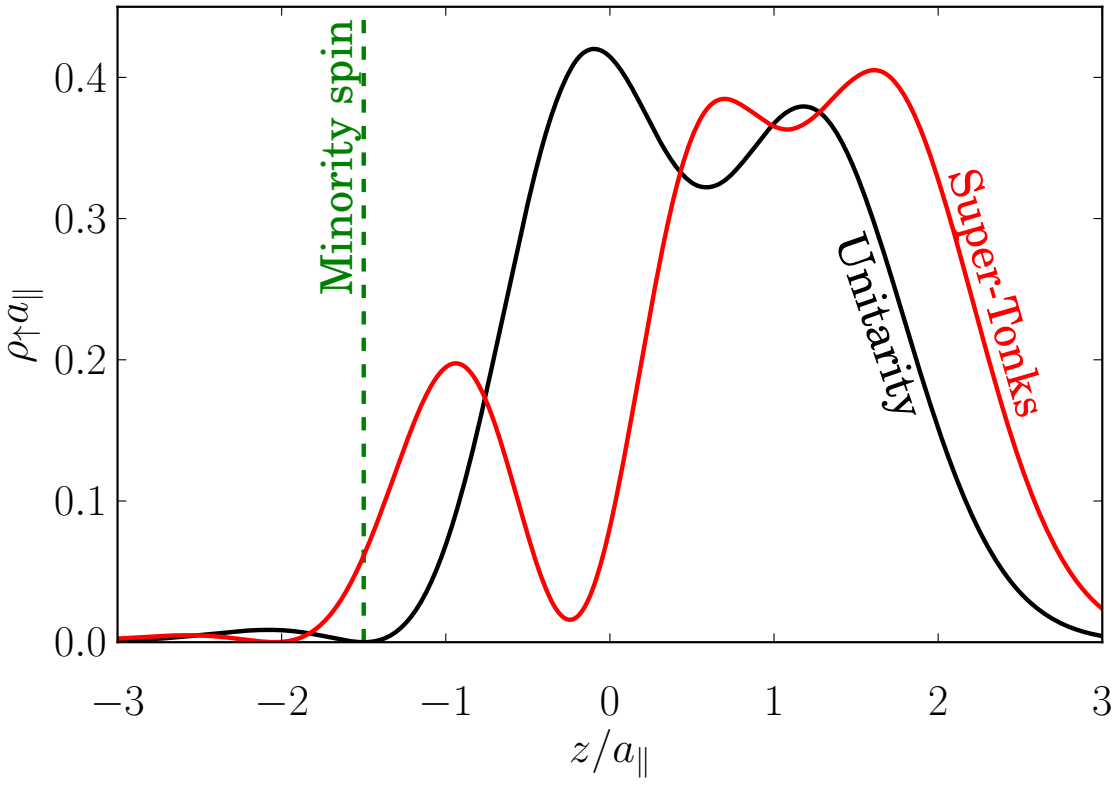
Magnetic correlations? $N_\uparrow = 2, N_\downarrow = 1$



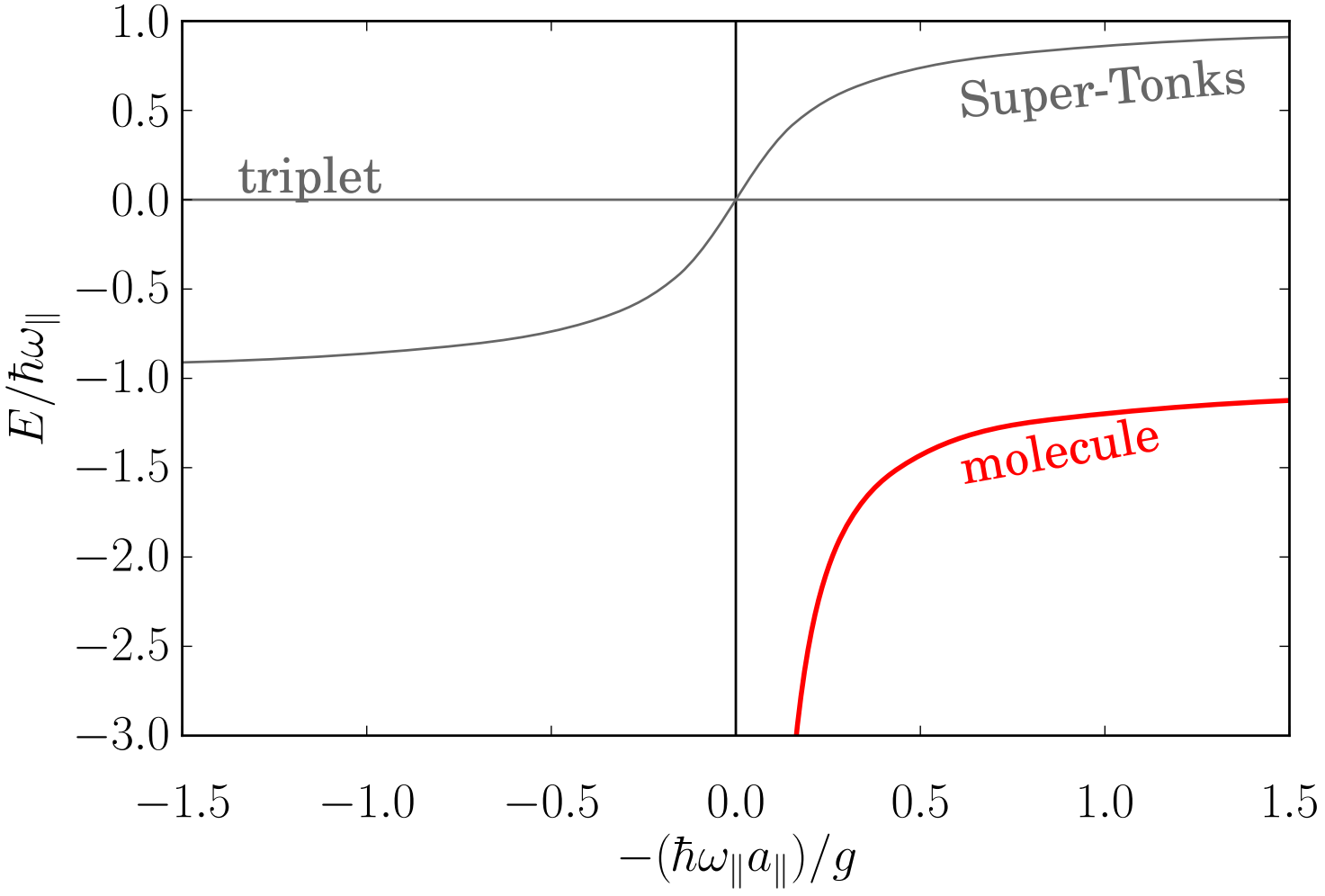


Magnetic correlations? $N_\uparrow = 2, N_\downarrow = 1$

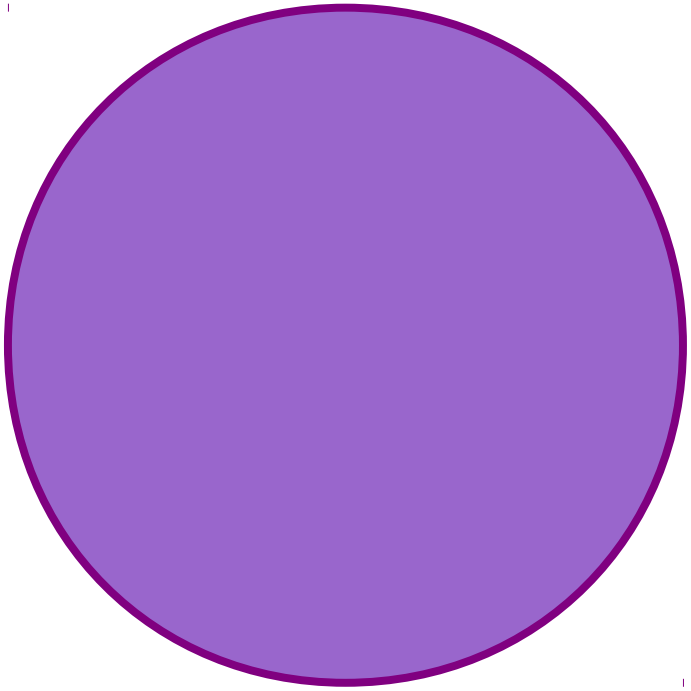




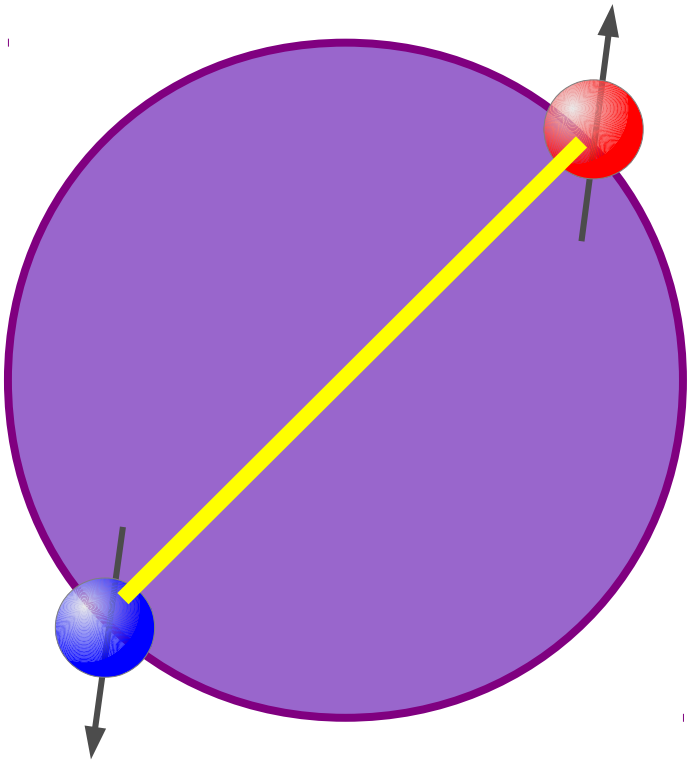
Attractive interactions



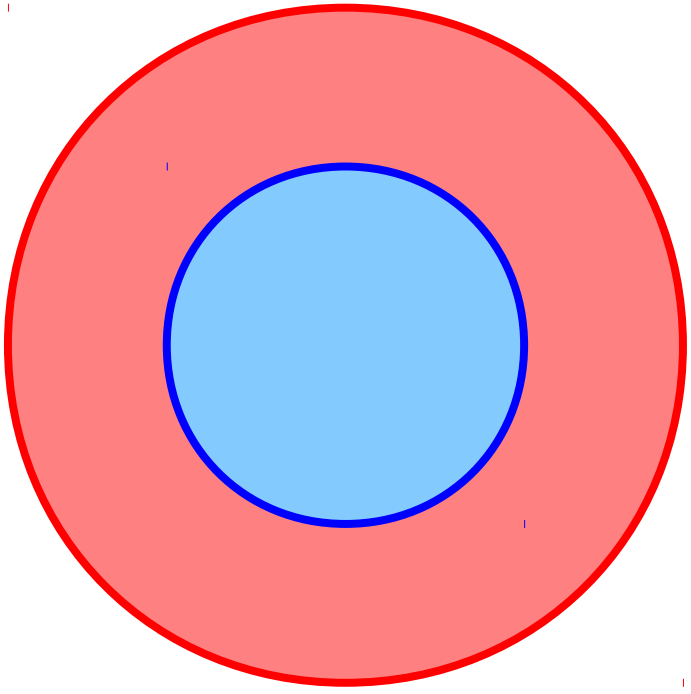
Homogeneous superconductivity



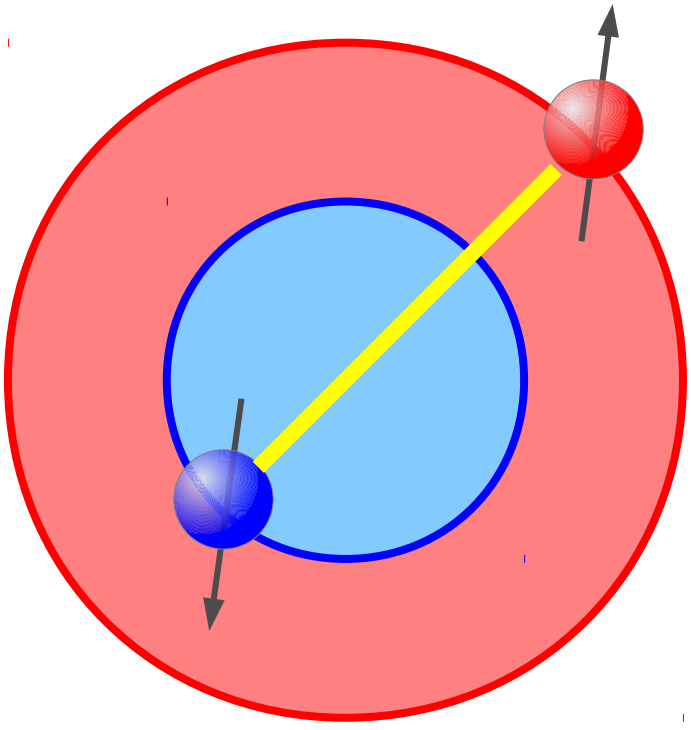
Homogeneous superconductivity



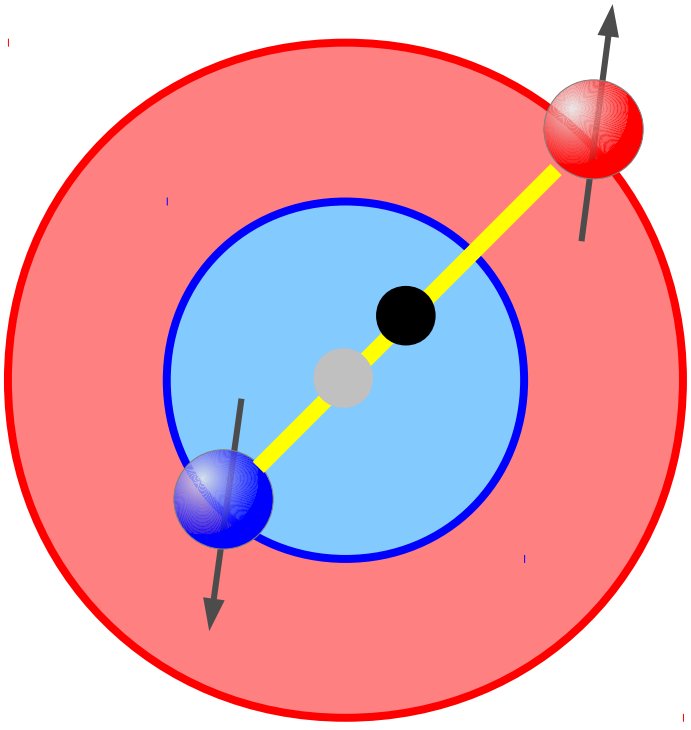
Spin imbalanced superconductors



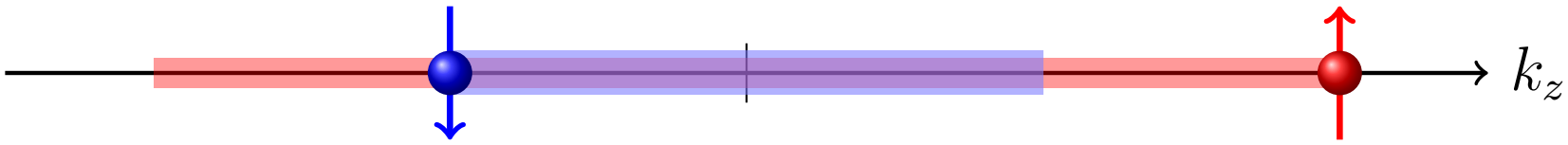
Spin imbalanced superconductors



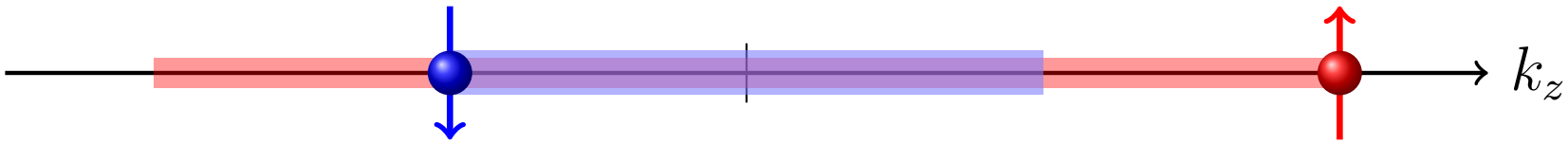
Spin imbalanced superconductors



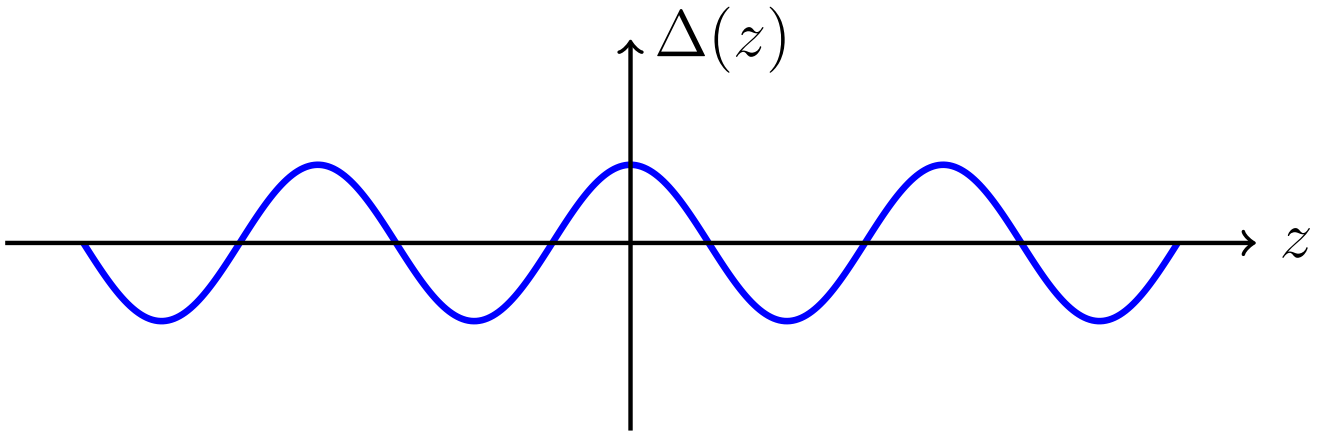
One-dimensional case



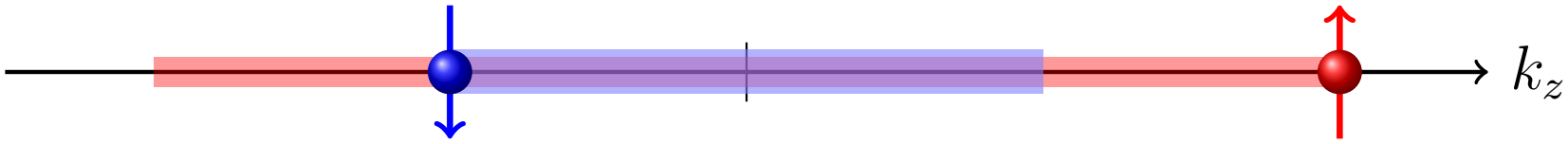
One-dimensional case



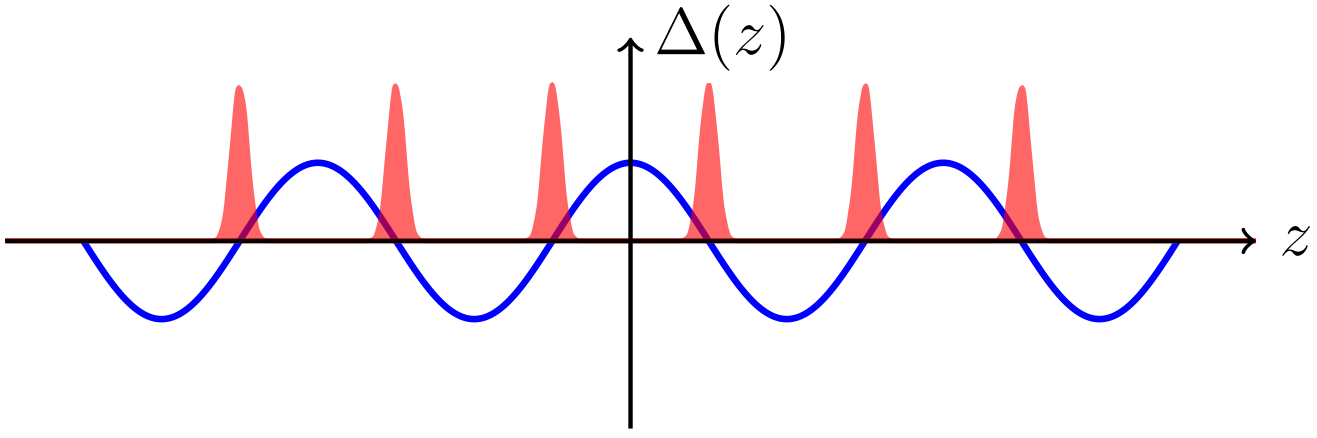
$$\Delta(z) = \Delta_0 \cos(q_z z)$$



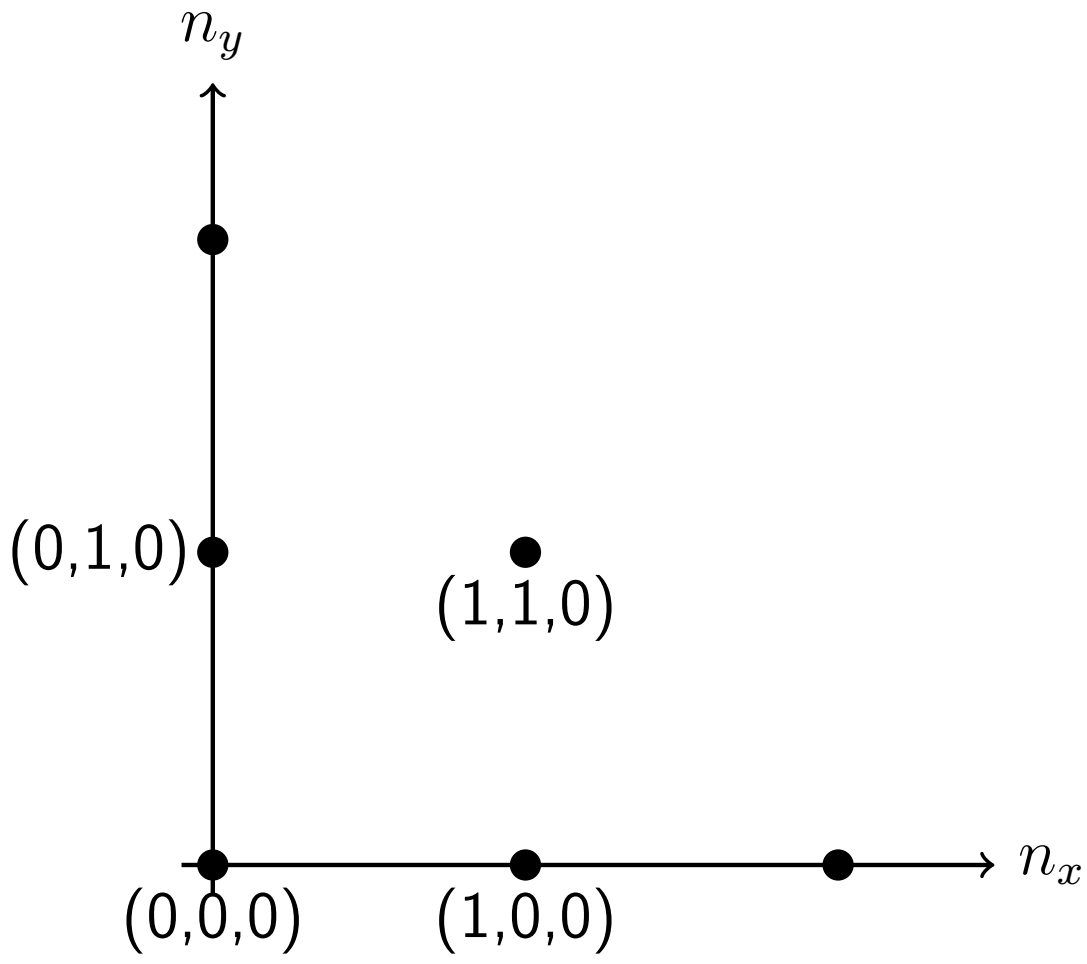
One-dimensional case



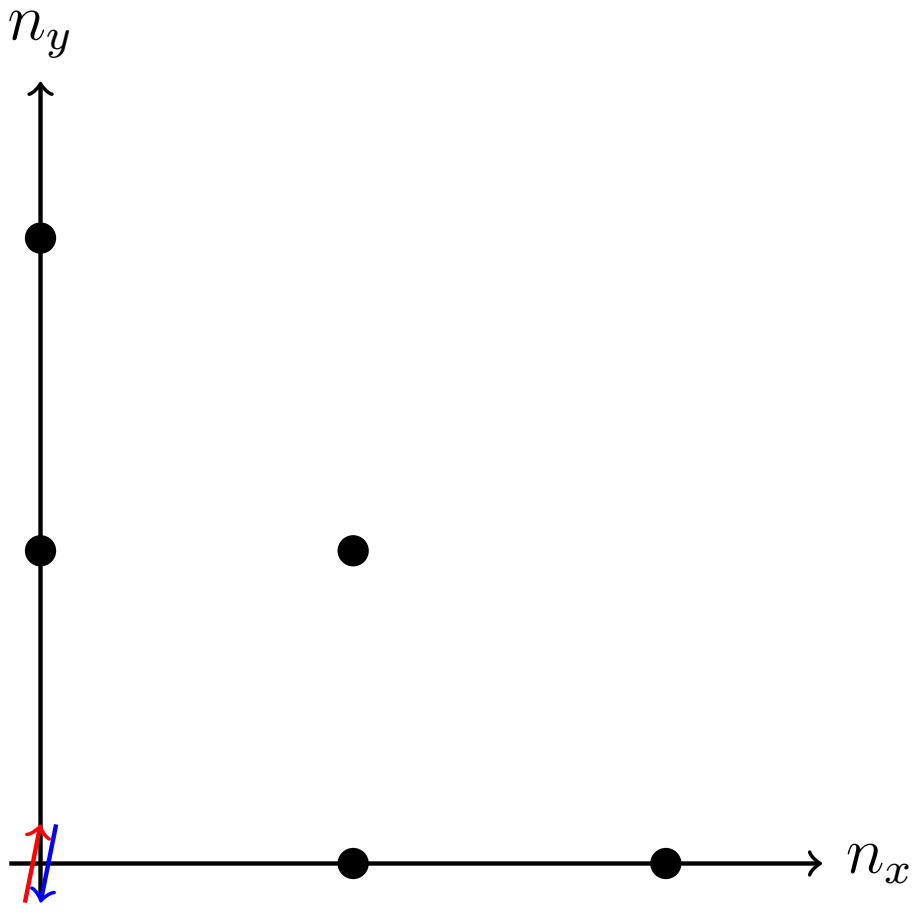
$$\Delta(z) = \Delta_0 \cos(q_z z)$$



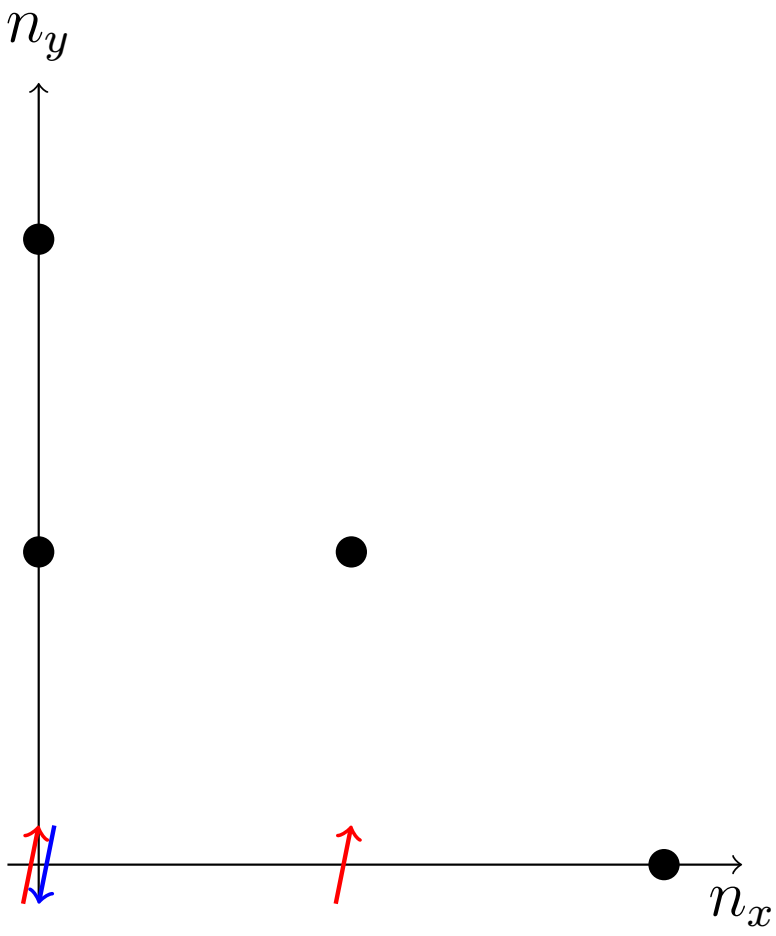
Spherical trap



Spherical trap: two atoms

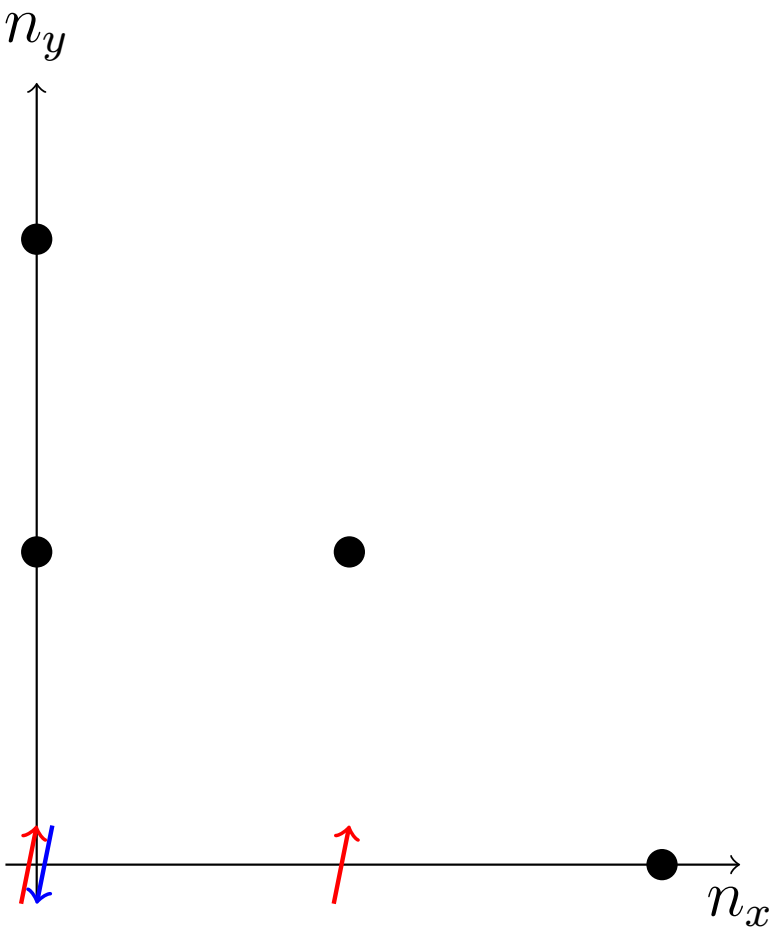


Spherical trap: three atoms

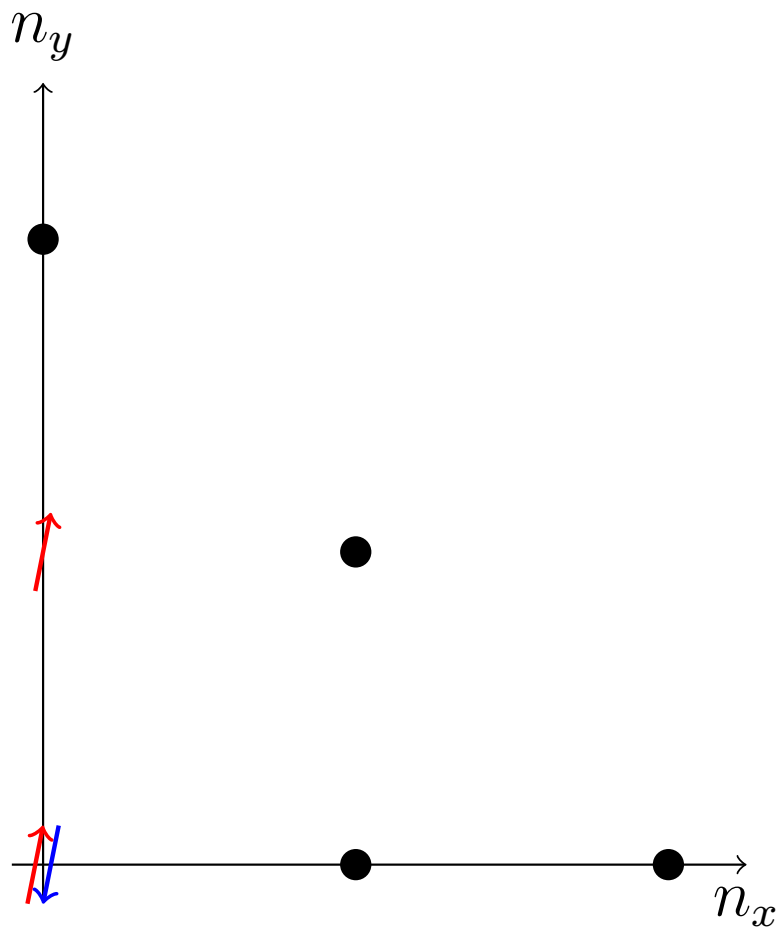


Excess up-spin in (1,0,0).

Spherical trap: three atoms

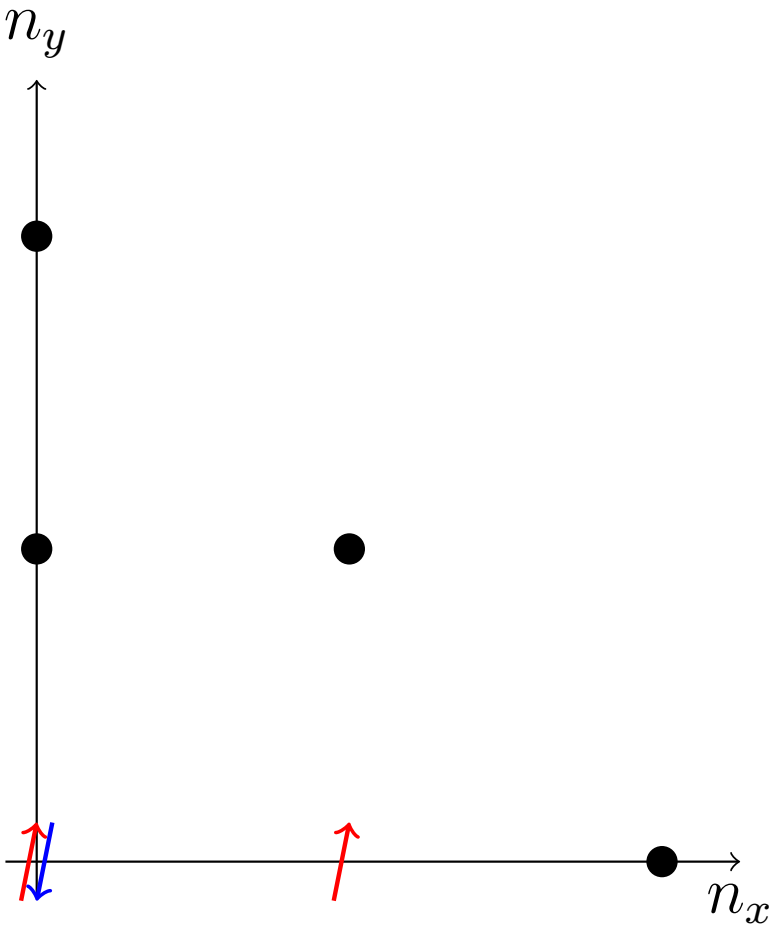


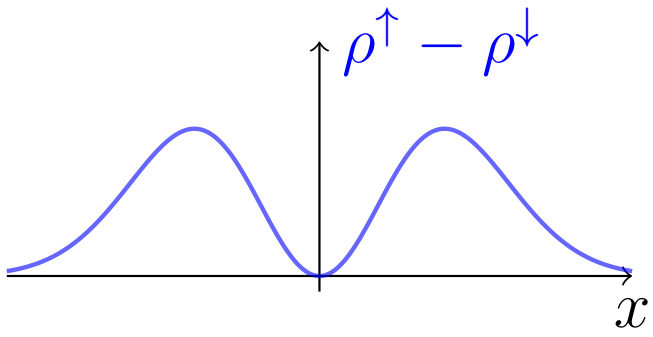
Excess up-spin in (1,0,0).



Excess up-spin in (0,1,0).

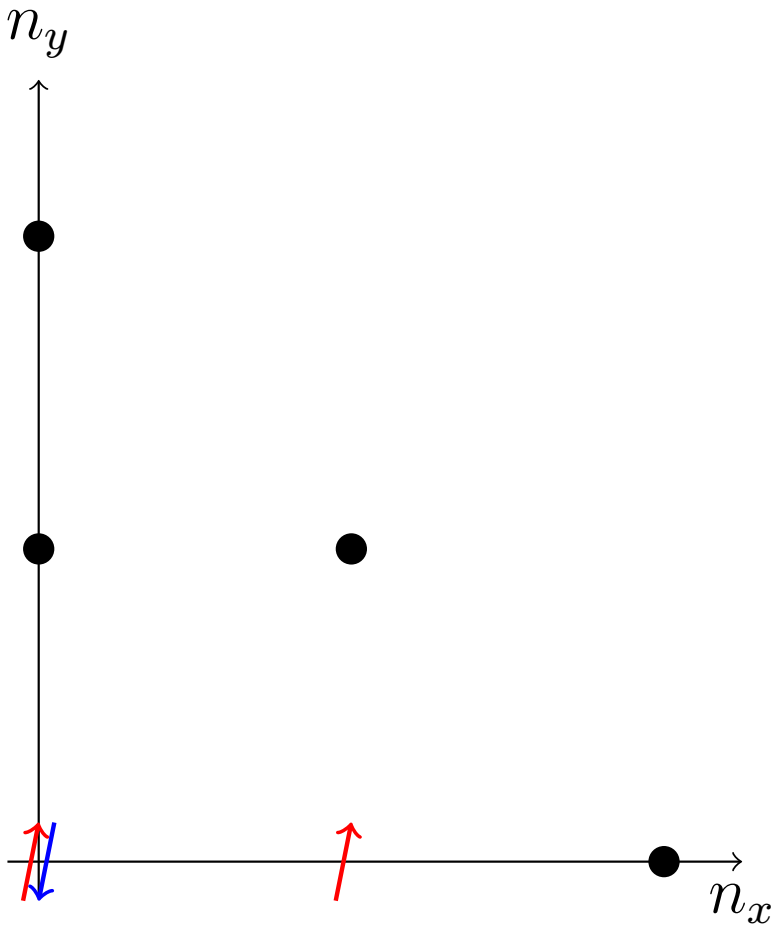
Three atoms: magnetisation

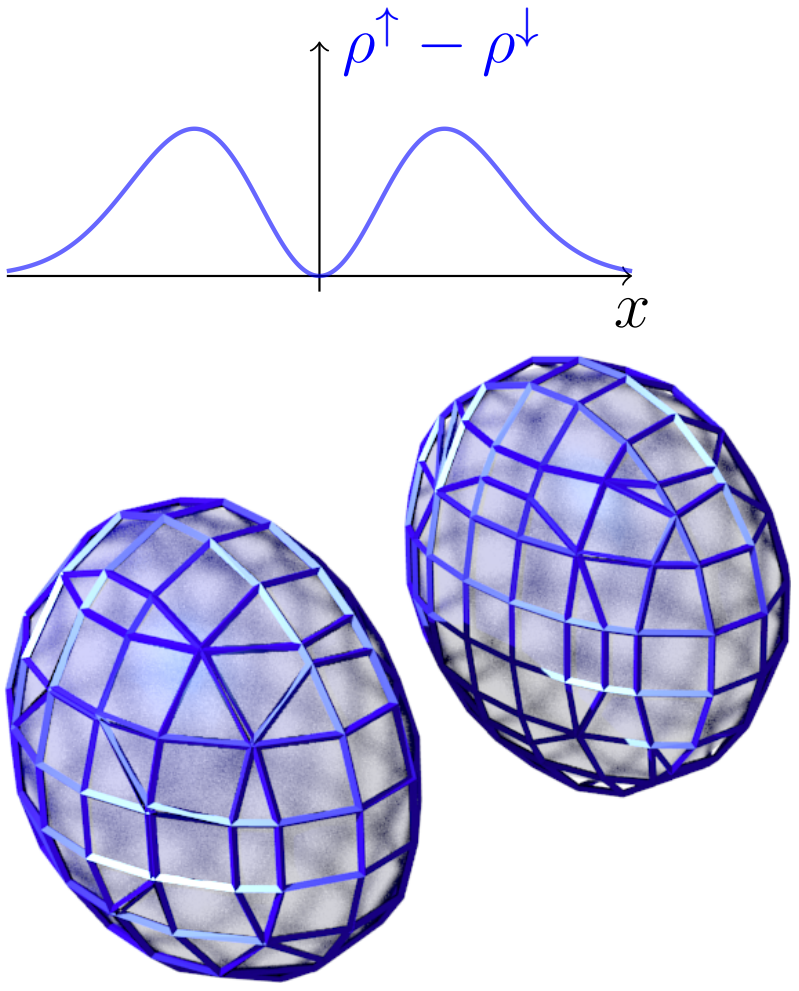




Excess up-spin in (1,0,0).

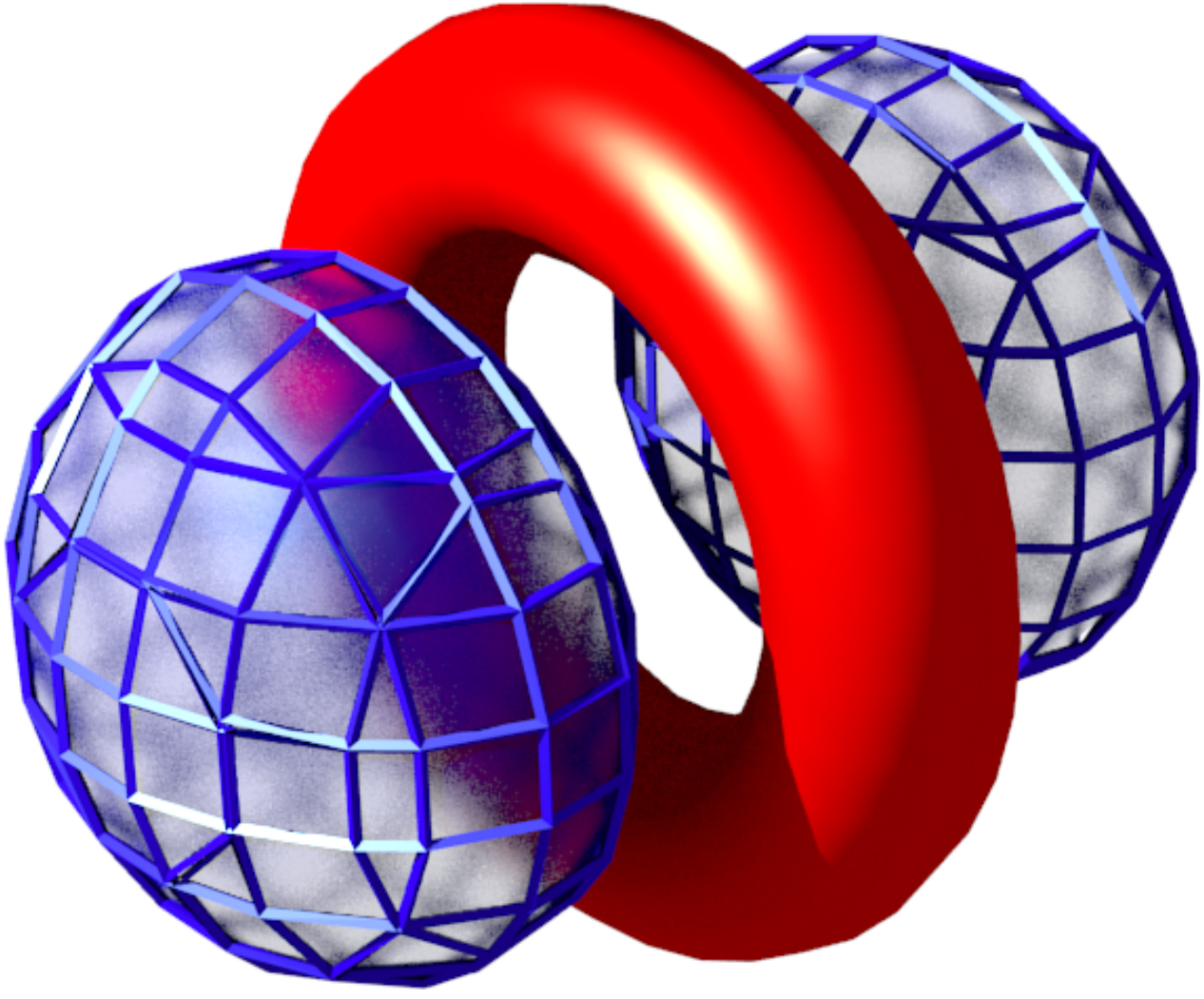
Three atoms: magnetisation



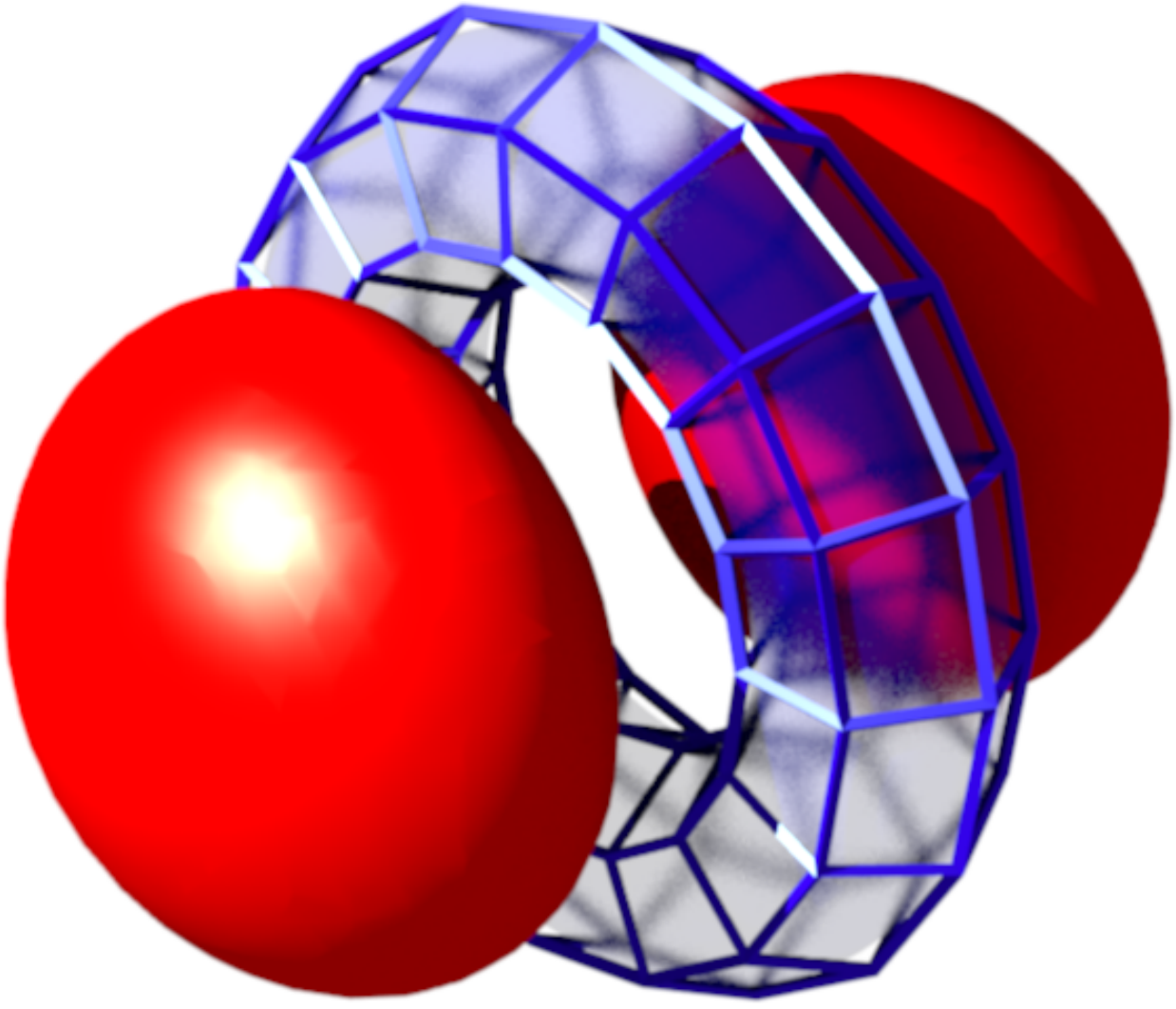


Excess up-spin in (1,0,0).

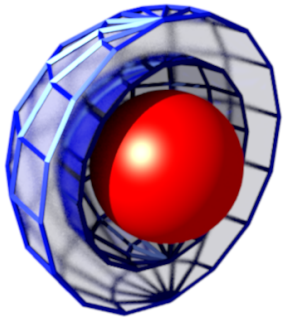
$N_\uparrow = 2, N_\downarrow = 1$



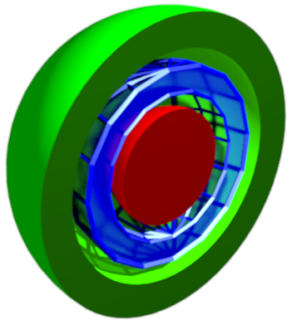
$N_\uparrow = 3, N_\downarrow = 1$



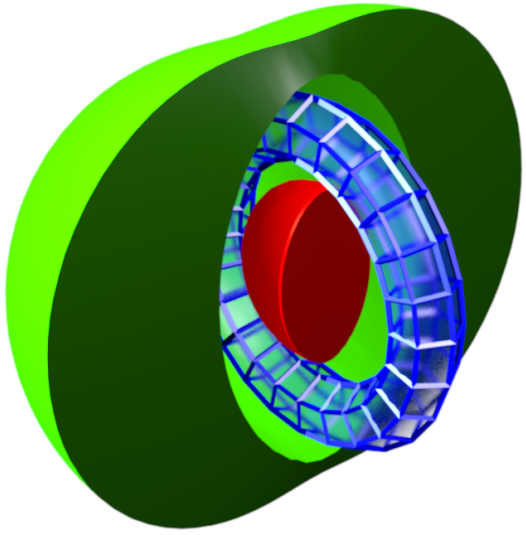
More atoms



$N_\uparrow = 4, N_\downarrow = 1$



$N_\uparrow = 5, N_\downarrow = 1$



$N_\uparrow = 5, N_\downarrow = 2$

Conclusions

Few fermion systems allow the study of the emergence of complex many-body phenomena in a theoretically and experimentally tractable setting.

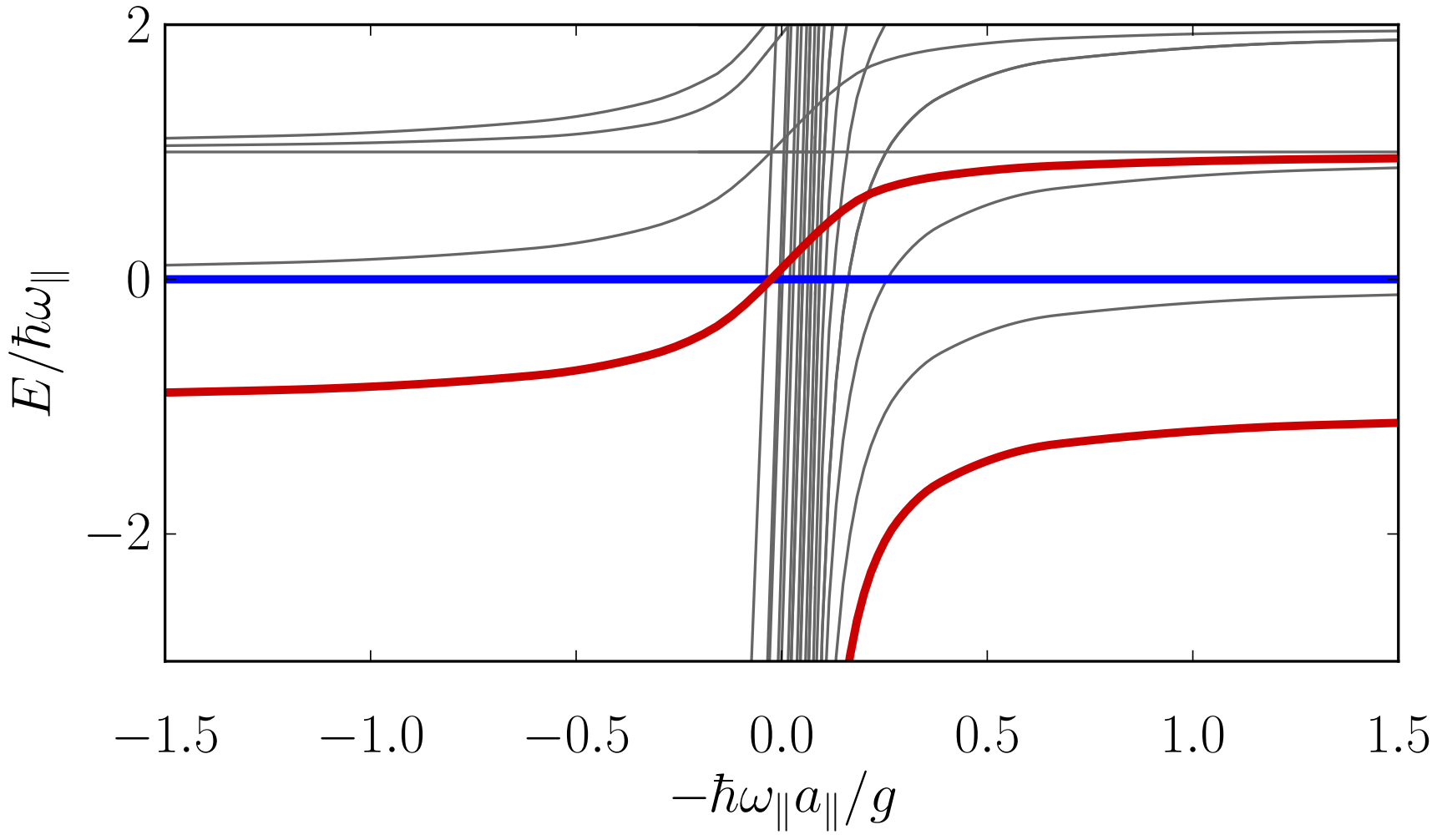
Repulsive interactions

- Repulsive polaron
- Emergence of itinerant magnetic correlations in the Super-Tonks regime

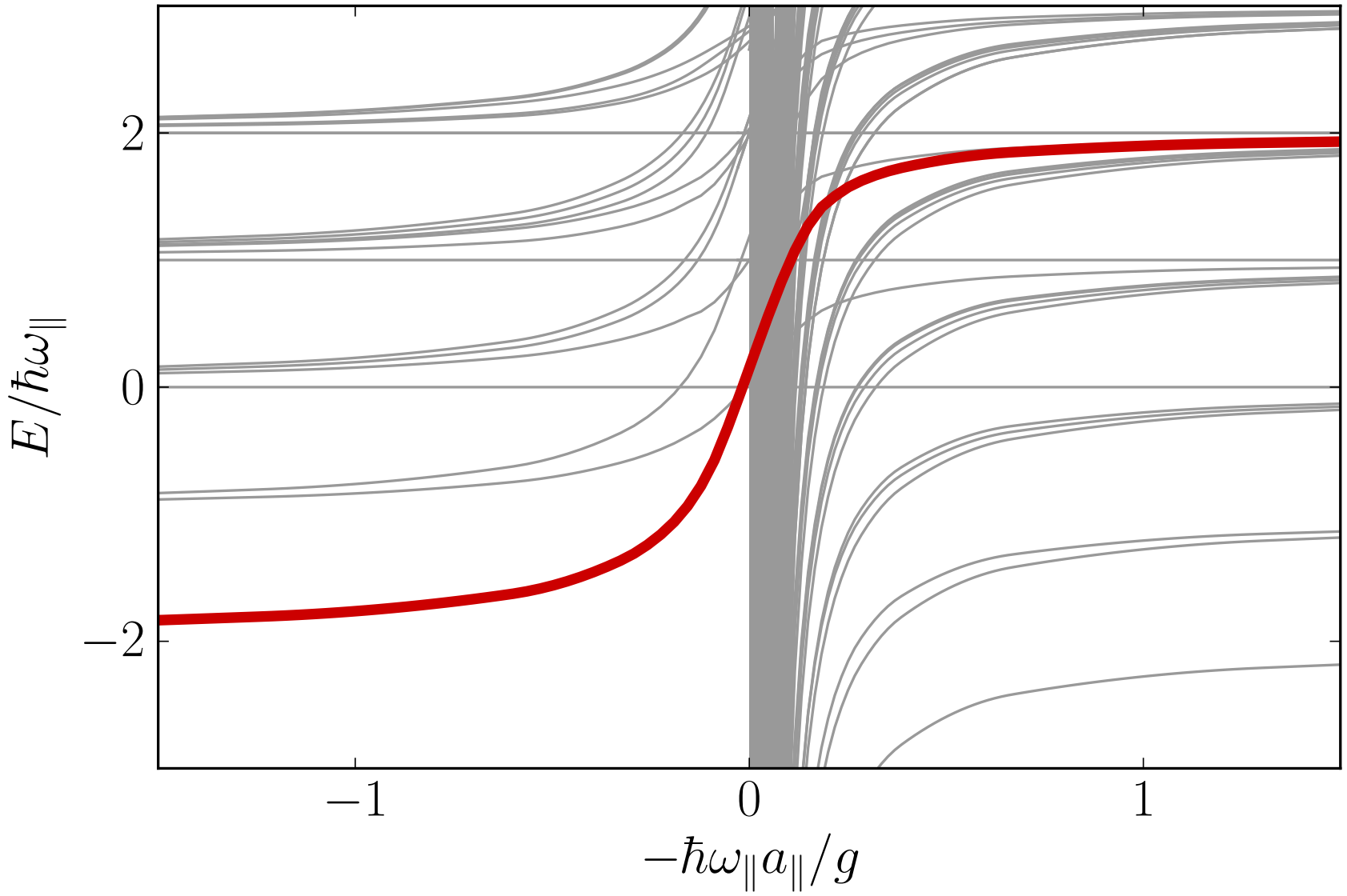
Attractive interactions

- Inhomogeneous pairing

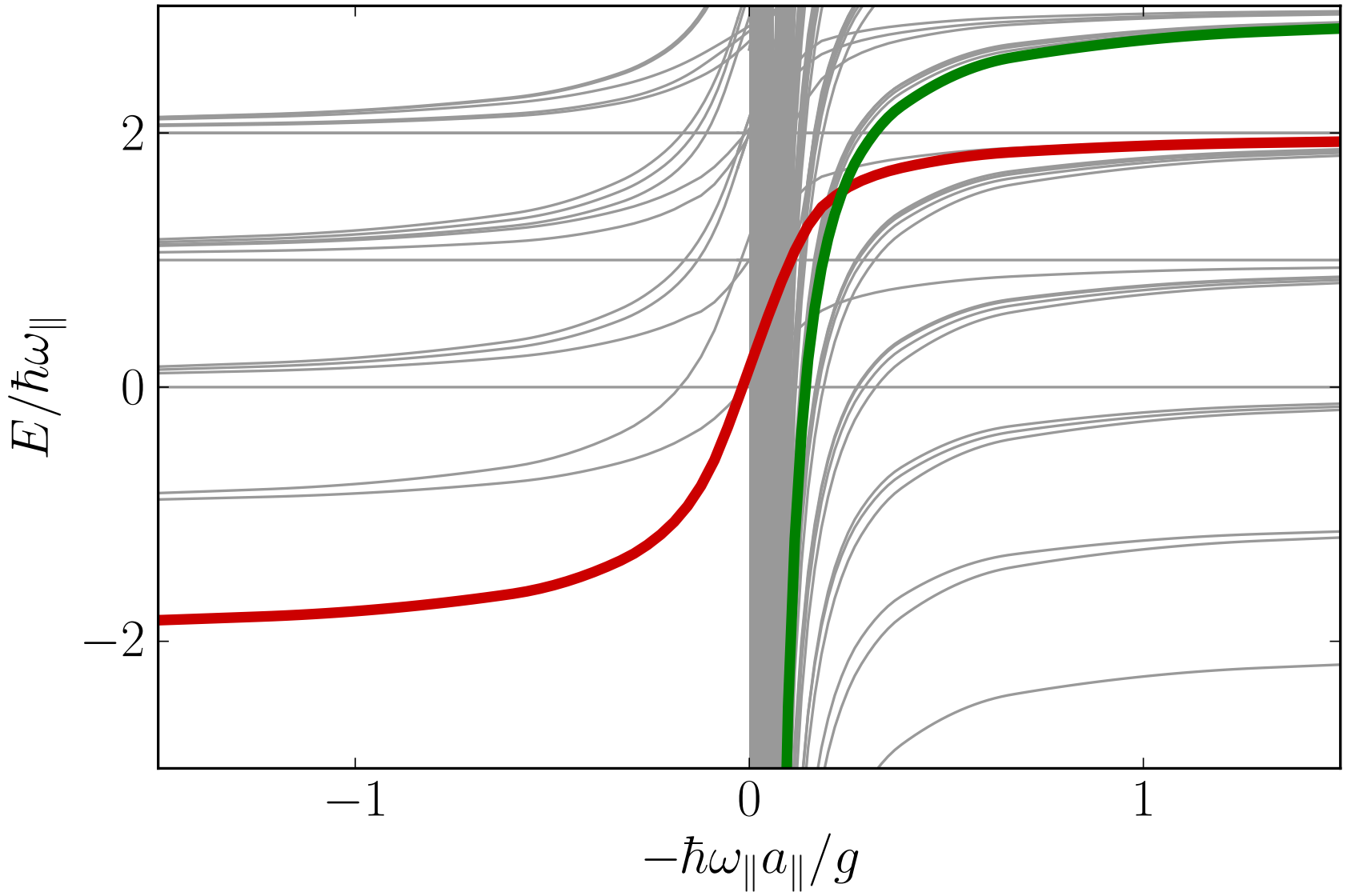
Band diagrams: $N_\uparrow = N_\downarrow = 1$



Band diagrams: $N_\uparrow = 2, N_\downarrow = 1$



Band diagrams: $N_\uparrow = 2, N_\downarrow = 1$



Band diagrams: $N_\uparrow = 3, N_\downarrow = 2$