Energy levels of Bloch electrons in magnetic fields

Douglas R. Hofstadter

Douglas Hofstadter

"Goedel, Escher, Bach: An Eternal Golden Braid"



No magnetic field:

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r})$$

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

No magnetic field:

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r})$$

With magnetic field:

$$\hat{H} = \frac{1}{2m} \left(\mathbf{p} - \frac{q}{c} \mathbf{A} \right)^2 + V(\mathbf{r})$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Magnetic fields and Bloch electrons

Peierls substitution:

$$\hbar \mathbf{k} \to \hbar \mathbf{k} - \frac{q}{c} \mathbf{A}$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Lattice



▲□▶ ▲圖▶ ▲≣▶ ▲≣▶ = のへで

Landau Gauge

$\mathbf{B} = B\hat{\mathbf{z}}$ $\mathbf{B} = \nabla \times \mathbf{A}$



Landau Gauge



◆□▶ ◆□▶ ◆三▶ ◆三▶ ●□ ● ●

Landau Gauge





Tight-Binding model (no field)

Each lattice site carries a localised orbital, denoted by $|n,m\rangle$. The Hamiltonian induces transitions between neighbouring lattice points.

Tight-Binding model (no field)

Each lattice site carries a localised orbital, denoted by $|n, m\rangle$. The Hamiltonian induces transitions between neighbouring lattice points.

$$\hat{H} |n, m\rangle = E_0 |n, m\rangle +$$
$$t(|n+1, m\rangle + |n-1, m\rangle +$$
$$+ |n, m+1\rangle + |n, m-1\rangle)$$

Transition matrix element gains a path dependence:

$$t \to t \cdot \exp\left(-i\frac{q}{\hbar c}\mathbf{A}\cdot\mathbf{l}\right)$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Effect of magnetic field

Using
$$\mathbf{A} = Bx\hat{\mathbf{y}}$$
:

Transition amplitude along x unaffected:

 $t \rightarrow t$

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

Effect of magnetic field

Using
$$\mathbf{A} = Bx\hat{\mathbf{y}}$$
:

Transition amplitude along x unaffected:

$$t \to t$$

Transition amplitude along y gains x-dependence:

$$t \to t \cdot \exp\left(-i\frac{q}{\hbar c}Bx \cdot (\pm a)\right)$$

▲□▶ ▲圖▶ ▲臣▶ ▲臣▶ 三臣 - のへ⊙

Dimensionless parameters...

$$\alpha = a^2 B \cdot \frac{q}{hc}$$

Dimensionless parameters...

$$\alpha = a^2 B \cdot \frac{q}{hc}$$

Then, along $\hat{\mathbf{y}}$:

$$t \to t \cdot \exp\left(-i \cdot 2\pi\alpha n\right)$$



Tight-Binding Hamiltonian (with field)

$$\hat{H} |n,m\rangle = E_0 |n,m\rangle + t(|n+1,m\rangle + |n-1,m\rangle)$$

$$+te^{-i2\pi\alpha n}|n,m+1
angle+te^{i2\pi\alpha n}|n,m-1
angle$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ





▲□▶ ▲圖▶ ▲国▶ ▲国▶ - 国 - のへ⊙



<ロト < 団 > < 国 > < 国 > < 国 > < 国 > < 国 > < 国 > < 国 > の < G</p>



▲□▶ ▲□▶ ▲目▶ ▲目▶ 三日 - のへ⊙



◆□▶ ◆□▶ ◆目▶ ◆目▶ 目 のへぐ



◆□▶ ◆□▶ ◆ □▶ ◆ □▶ ● □ ● ● ●



◆□▶ ◆□▶ ◆目▶ ◆目▶ ▲□▶ ◆□◆



▲□▶ ▲圖▶ ▲国▶ ▲国▶ - 国 - のへ⊙

