Propagation of Polariton Wavepackets

Fabrice P. Laussy, fabrice.laussy@gmail.com

Daniele Sanvitto, Alberto Amo, Elena del Valle,

Dario Ballarini, Carlos Tejedor & Luis Viña.

Thanks to Iacopo Carusotto for helpful comments.

Experiments also involving M. D. Martin, A. Lemaitre, J. Bloch, D. Krizhanovskii & M. Skolnick.

ICSCE4

Cambridge, UK

September 2008

University of Southampton

UAM

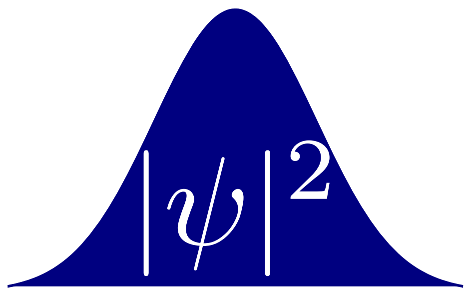
UNIVERSIDAD AUTONOMA DE MADRID

Introduction

Outset: *we study the dynamics of wavepackets*

$$i\hbar\partial_t |\psi\rangle = (D + V + \mathcal{I} + \Gamma) |\psi\rangle + F$$

$$\psi(x,0) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x - x0)^2}{2\sigma_x^2}\right) \exp(ik_0 x)$$



Introduction

$$i\hbar\partial_t \left|\psi\right\rangle = (D + V + \mathcal{I} + \Gamma)\left|\psi\right\rangle + F$$

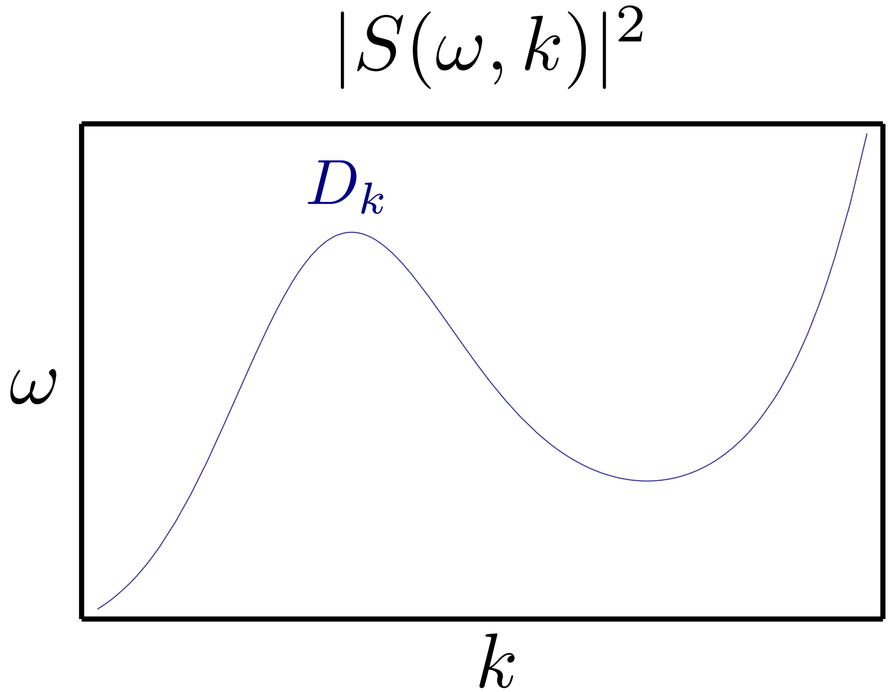
D is the *free propagation* energy

Diagonal in k-space:

$$\left\langle k'\right| D \left|k\right\rangle = D_k \delta(k - k')$$

$$\psi(k,t) = e^{-i\frac{D_k}{\hbar}t}\psi(k,0)$$

$$|S(\omega,k)|^2 = |\psi(k,0)|\delta(\omega - D_k/\hbar)$$



Introduction

$$i\hbar\partial_t \left|\psi\right\rangle = \left(D + V + \mathcal{I} + \Gamma\right)\left|\psi\right\rangle + F$$

V is any potential that gets in the way

Diagonal in real space:

$$\left\langle y\right| V \left|x\right\rangle = V(x)\delta(x-y)$$

Typical, convenient choice:

$$V(x) = \frac{V_0}{\sigma_V\sqrt{2\pi}} \exp\left(-\frac{(x-x_V)^2}{2\sigma_V^2}\right)$$

Introduction

$$i\hbar\partial_t \left|\psi\right\rangle = (D + V + \mathcal{I} + \Gamma)\left|\psi\right\rangle + F$$

F is the coherent excitation of the system:

$$\textit{pump} + \textit{probe}$$

$$F(x,t) = F_{\mathrm{p}} e^{-\frac{(x-x_{\mathrm{p}})^2}{\sigma_{\mathrm{p}}}} \exp(i(k_{\mathrm{p}}x - \omega_{\mathrm{p}}t))$$
$$+ F_{\mathrm{i}} e^{-\frac{(x-x_{\mathrm{i}})^2}{\sigma_{\mathrm{i}}}} e^{-\frac{(t-t_{\mathrm{i}})^2}{\sigma_{\mathrm{t}}}} \exp(i(k_{\mathrm{i}}x - \omega_{\mathrm{i}}t))$$

Introduction

$$i\hbar\partial_t |\psi\rangle = (D + V + \mathcal{I} + \Gamma) |\psi\rangle + F$$

Γ **is the decay:** $\Gamma = -i\hbar\gamma/2$

Stabilizes the pumping.

Smoothes the spectra:

$$|S(\omega,k)|^2 = |\psi(k,0)|^2 \frac{1 + e^{-\gamma t_{\max}} - 2e^{-\gamma t_{\max}/2}\cos[(\omega - D_k)t_{\max}]}{(\omega - D_k)^2 + \gamma^2/4},$$

$$\xrightarrow[t\to\infty]{} \frac{|\psi(k,0)|^2}{\left(\omega - \frac{D_k}{\hbar}\right)^2 + \frac{\gamma^2}{4}}$$

Introduction

$$i\hbar\partial_t \left|\psi\right\rangle = (D + V + \mathcal{I} + \Gamma) \left|\psi\right\rangle + F$$

$\mathcal{I}$ is a nonlinear term in ψ due to interactions

$$\mathcal{I} \left|\psi\right\rangle = U|\psi|^2 \left|\psi\right\rangle$$

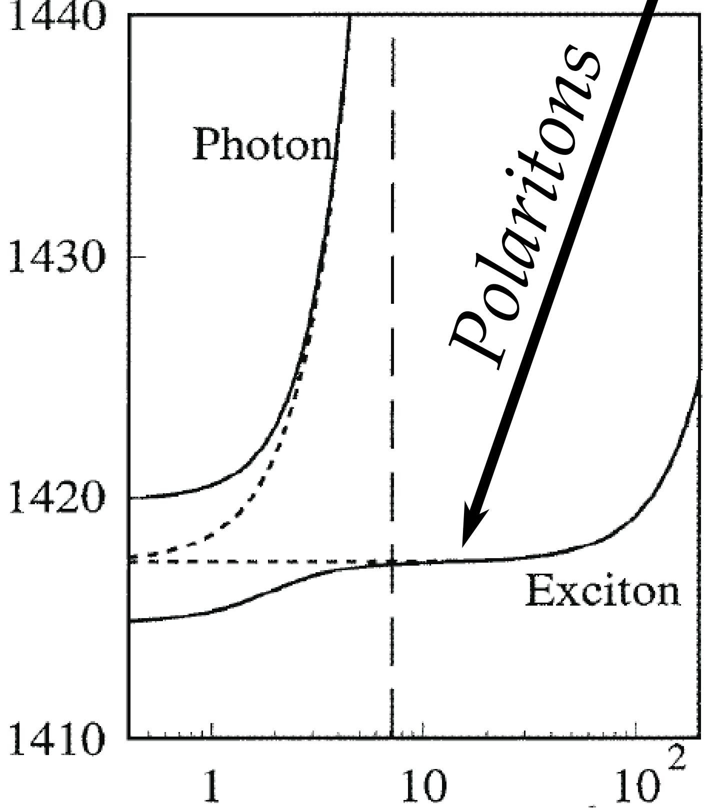
$$\left|\psi(\mathbf{r}, t)\right\rangle = \exp\left(-\frac{i}{\hbar}U|\psi(\mathbf{r}, t)|^2\right) \left|\psi(\mathbf{r}, 0)\right\rangle$$

In the context that interests us (see later), this term has been analyzed extensively, notably by *Ciuti, Carusotto and Wouters*. See also its original use to a polariton spin-wave (by *Kavokin, Shelykh, Rubo, Malpuech et al.*)

What we are aiming for:

$$i\hbar\partial_t \left|\psi\right\rangle = (D + V + \mathcal{I} + \Gamma) \left|\psi\right\rangle + F$$

Polaritons



is there in a realistic system, is useful to probe the propagation.

is important in a BEC, is interesting.

is large (short lifetime).

looks required to compensate the short lifetime.

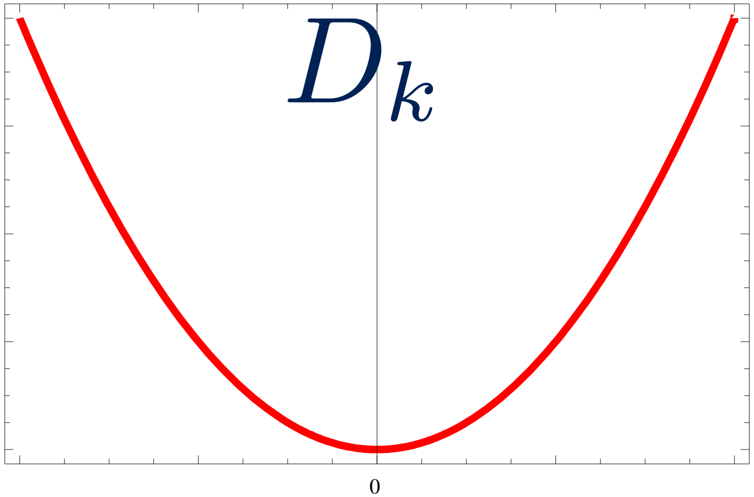
Introduction

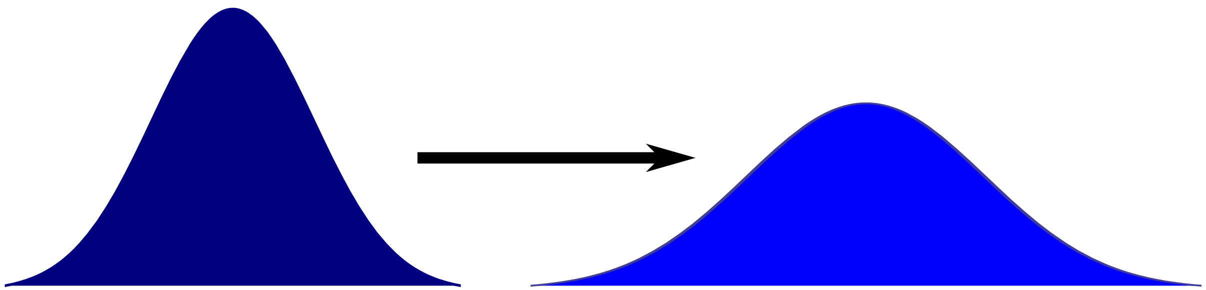
We appreciate the author's wish to limit the damage caused by a fall in rock climbing, but since one of us has survived a 30 m fall at a factor of 1.5 or thereabouts, the effects of sudden deceleration are perhaps less serious than the authors believe.

Propagation of the free spinless particle

Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK

PII: S0143-0807(97)80001-3





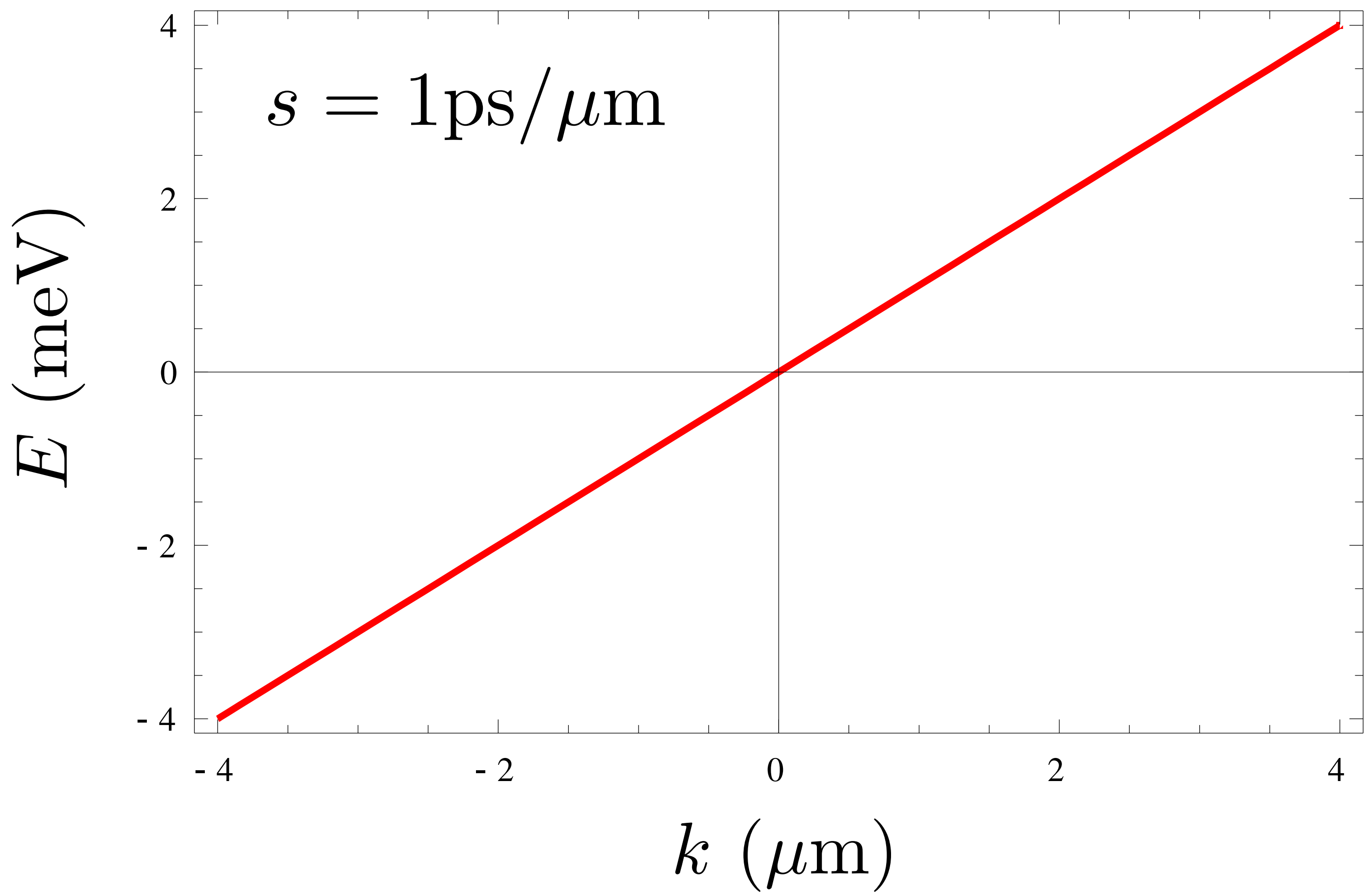
Analysis of the spreading Gaussian wavepacket

Abstract. The $\psi(x,t)$ wavefunction of a Gaussian wavepacket spreading in free space ($V(x) \equiv 0$) is expressed in a didactic form. The expression found is a product of pure real factors and pure phase factors. This makes it very easy to derive the expression for the probability density from the wavefunction. The physical meaning of each of the factors is analysed.

G. I. Mark. *Analysis of the spreading gaussian wavepacket.* Eur. Phys. J. B, **18**:247, 1997.

The linear dispersion

$$D_k = s\hbar k$$



The linear dispersion

Schrödinger equation

$$i\hbar\partial_t\psi(x) = \left(-is\hbar\partial_x + V(x) - i\frac{\gamma}{2}\right)\psi(x)$$

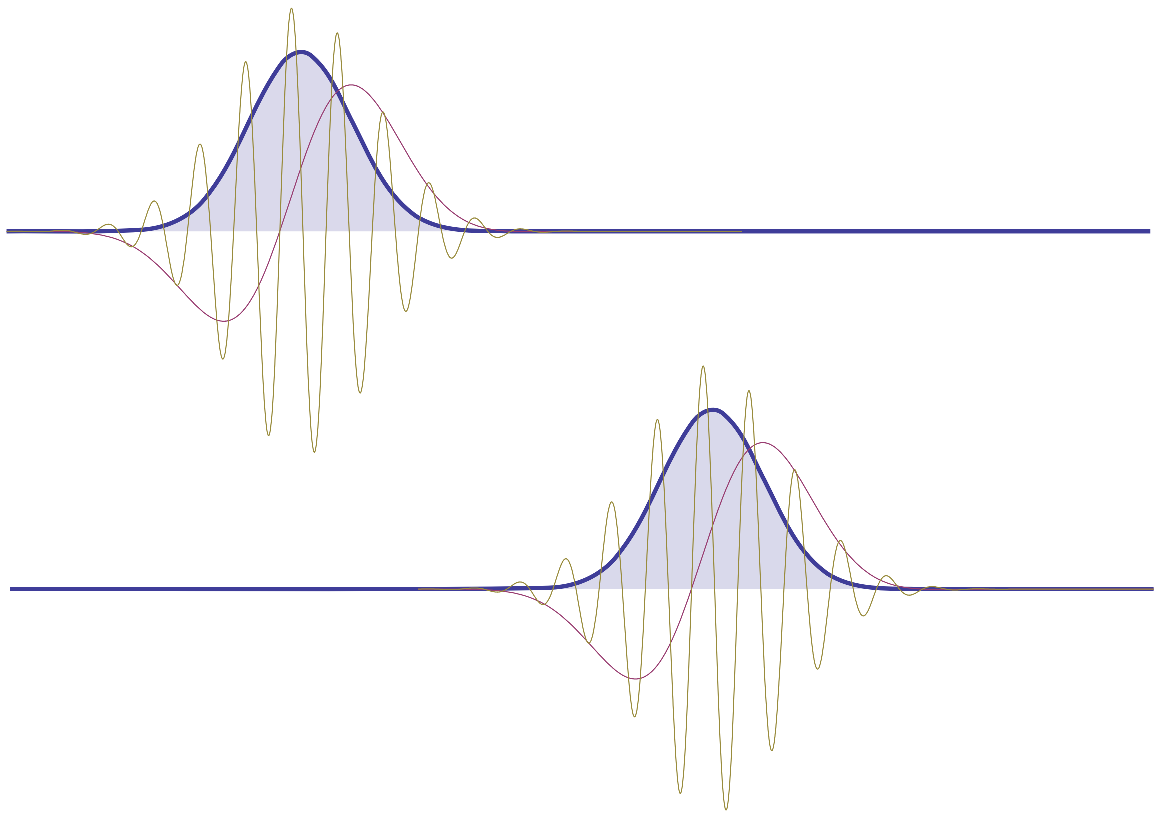
has solution

$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right)\exp\left(ik_0(x-st)\right)\exp(-\gamma t/2)$$
$$\exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right)-\mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$

The linear dispersion

$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right) \exp\left(ik_0(x-st)\right) \exp(-\gamma t/2) \exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right) - \mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$

(from previous slide)



t_1

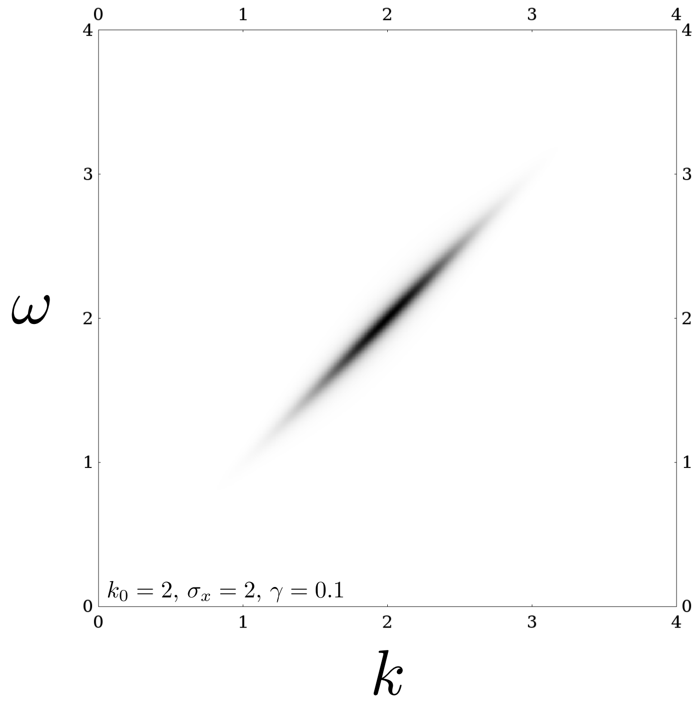
t_2

No "diffusion" of the wavepacket.

The linear dispersion

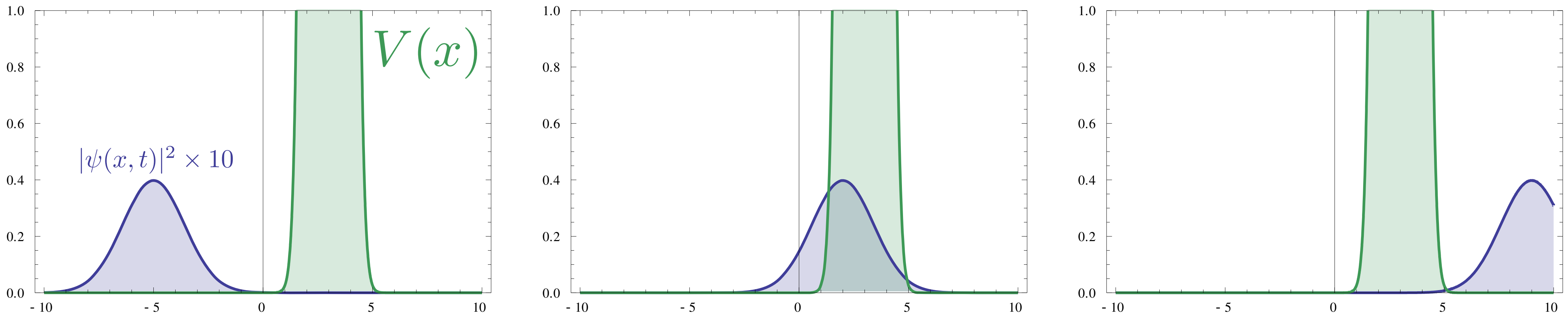
$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right) \exp\left(ik_0(x-st)\right) \exp(-\gamma t/2) \exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right) - \mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$

$$\psi(k,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\Big(-ik(st+x_0)\Big) \exp\Big(-\tfrac{1}{2}(k-k_0)^2\sigma_x^2\Big) \exp(-\gamma t/2) e^{ik_0x_0}$$



The linear dispersion

$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right) \exp\left(ik_0(x-st)\right) \exp(-\gamma t/2) \exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right) - \mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$



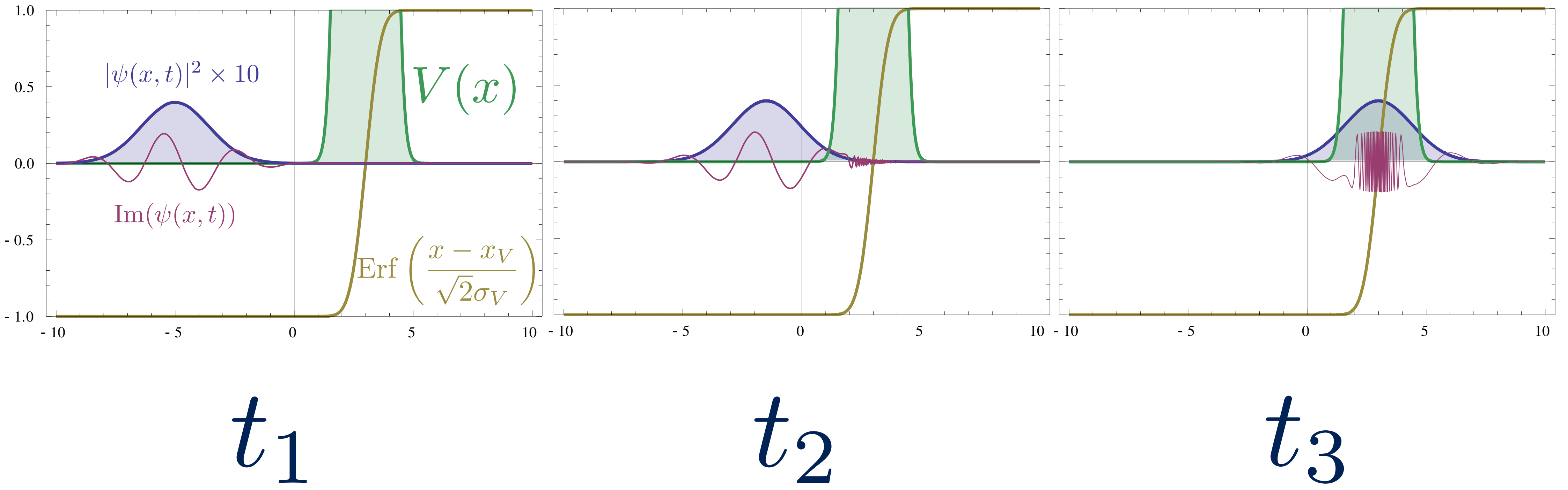
t_1 t_2 t_3

The wavepacket envelope tunnels through unaffected

The linear dispersion

$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right) \exp\left(ik_0(x-st)\right) \exp(-\gamma t/2) \exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right) - \mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$

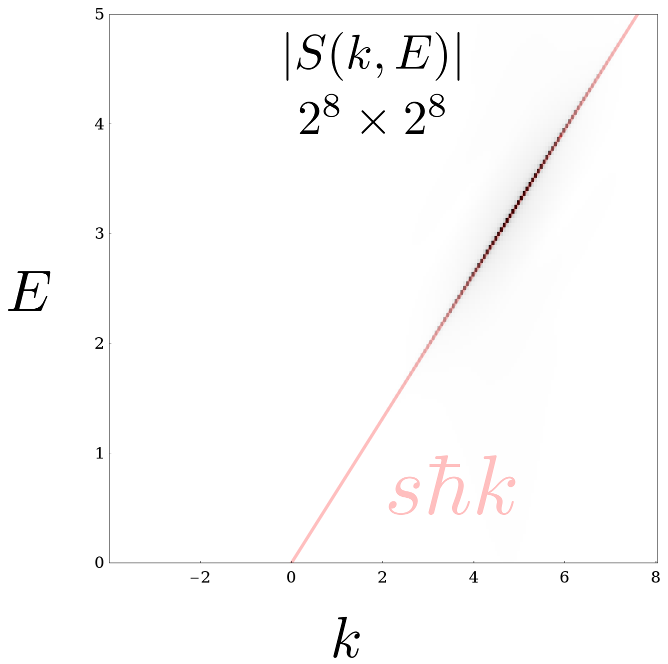
what happens underneath



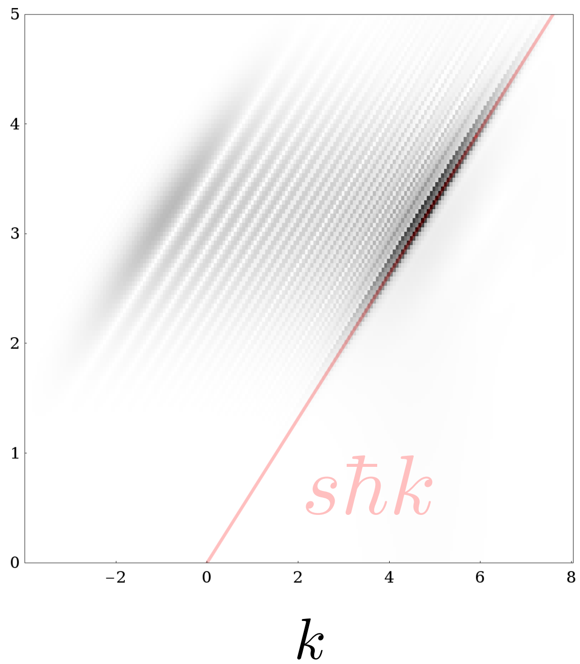
please see 1--tunnel.avi at this point

The linear dispersion

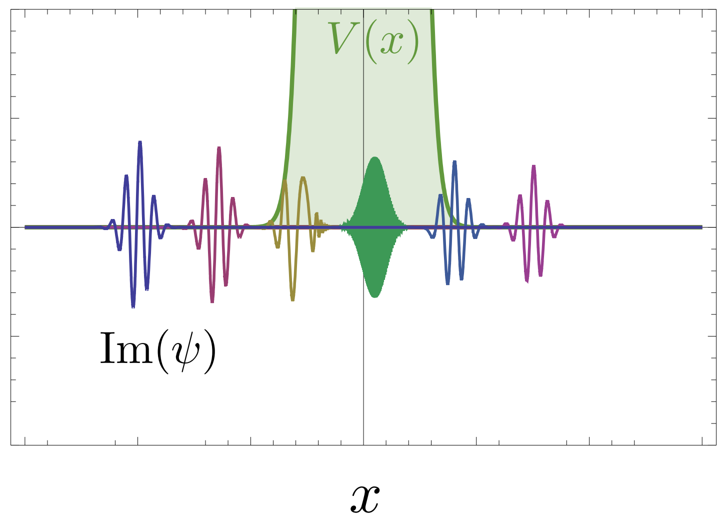
$$\psi(x,t) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-st-x_0)^2}{2\sigma_x^2}\right) \exp\left(ik_0(x-st)\right) \exp(-\gamma t/2) \exp\left(-\frac{iV_0}{2\hbar s}\left\{\mathrm{Erf}\left(\frac{x-x_V}{\sqrt{2}\sigma_V}\right) - \mathrm{Erf}\left(\frac{x-x_V-st}{\sqrt{2}\sigma_V}\right)\right\}\right)$$



$V = 0$



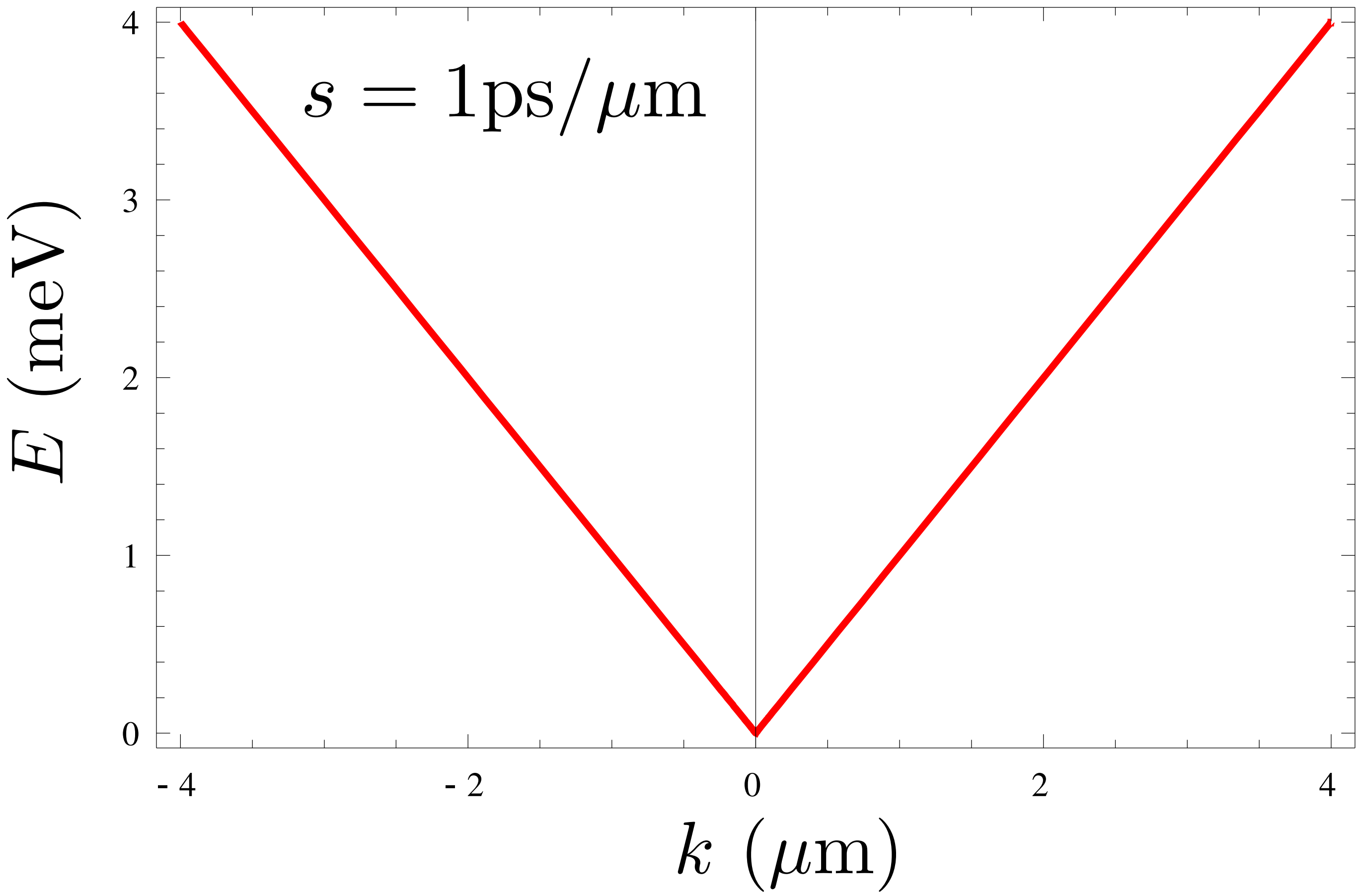
$V = 10^2$



The wavepacket is looking for somewhere to scatter-off the potential (but doesn't find any final state)

The conic dispersion *(free wave equation)*

$$D_k = s\hbar|k|$$



The conic dispersion

Schrödinger equation is not "local" anymore:

$$i\hbar\partial_t\psi(k) = s\hbar|k|\psi(k) + \frac{1}{\sqrt{2\pi}}\int e^{-ikx}V(x)\psi(x)\,dx$$

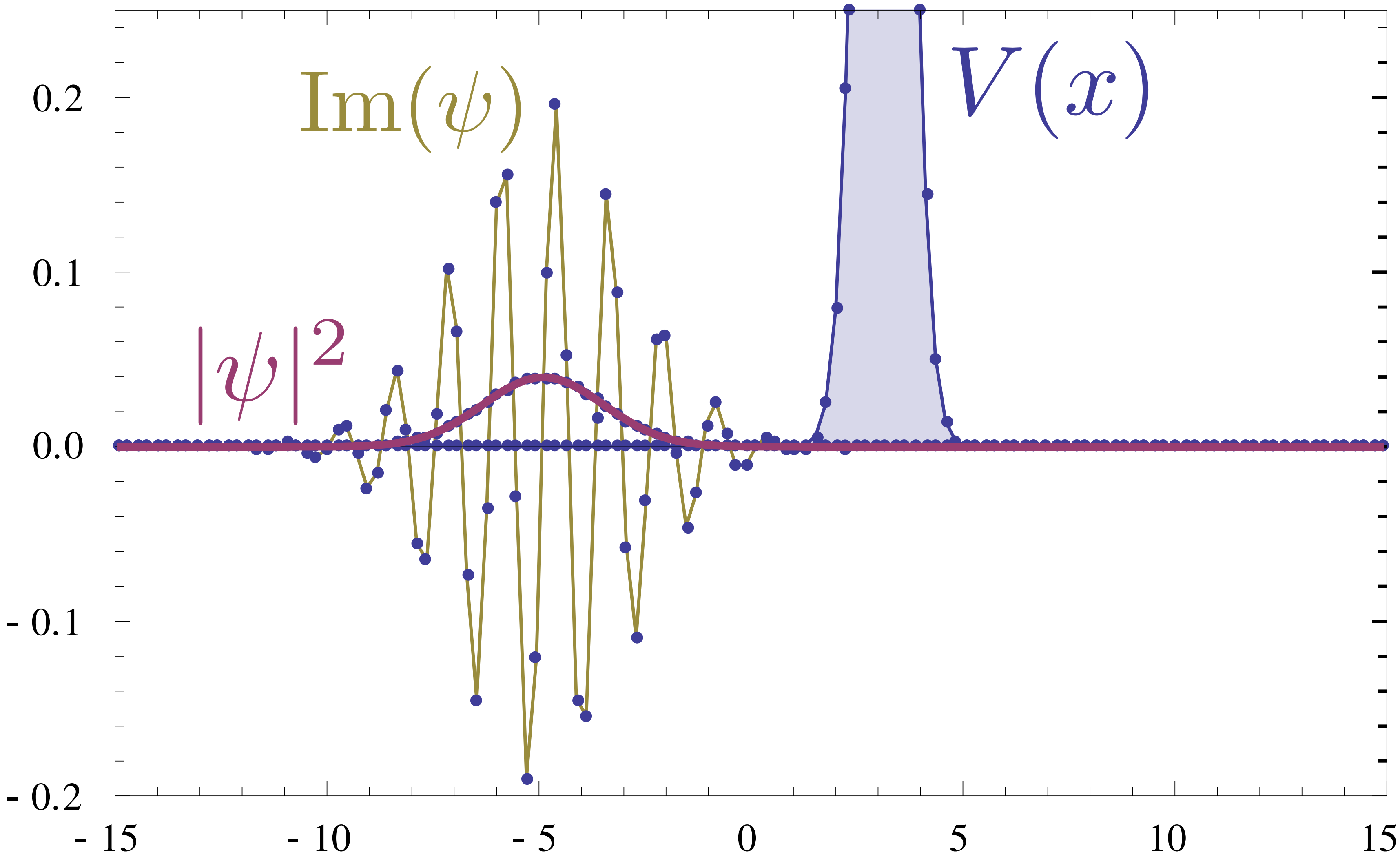
so we go "numerical"

(We used both a split-operator method and a more standard ADI integration method.

The analysis of the numerical-methods for the driven-dissipative nonlinear Schrödinger equation ("Gross-Pitaevskii") seems an unadressed issue.

We didn't prove our algorithm to be the most efficient.)

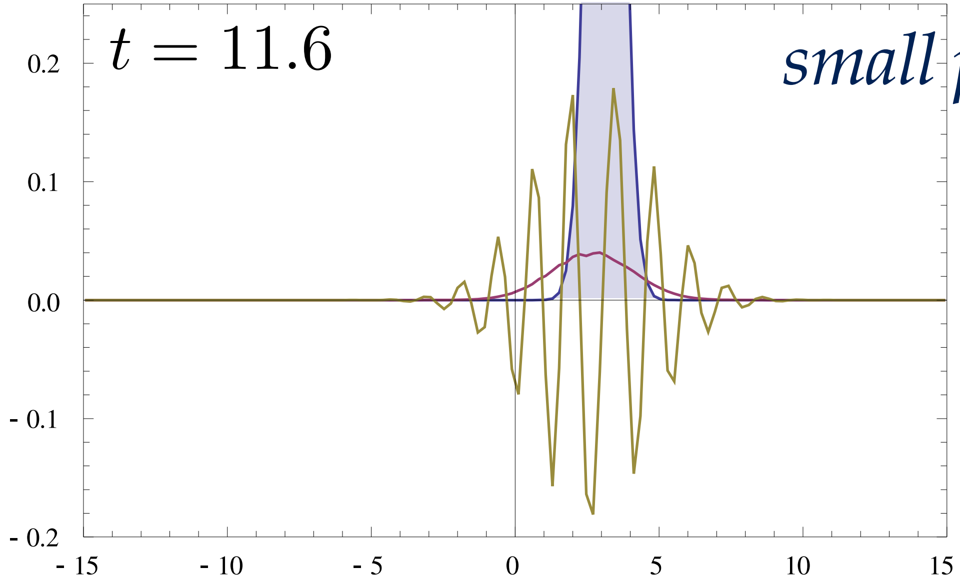
The conic dispersion

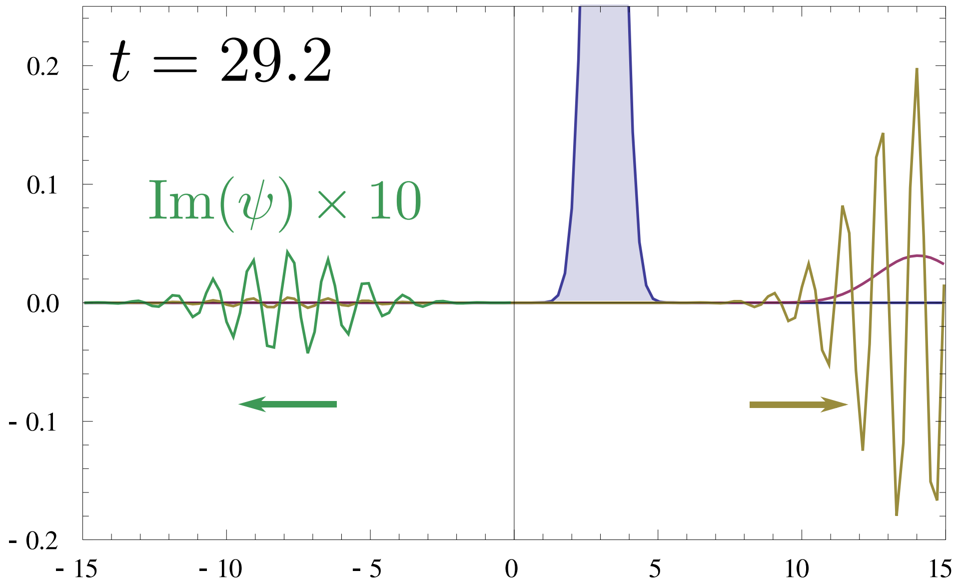


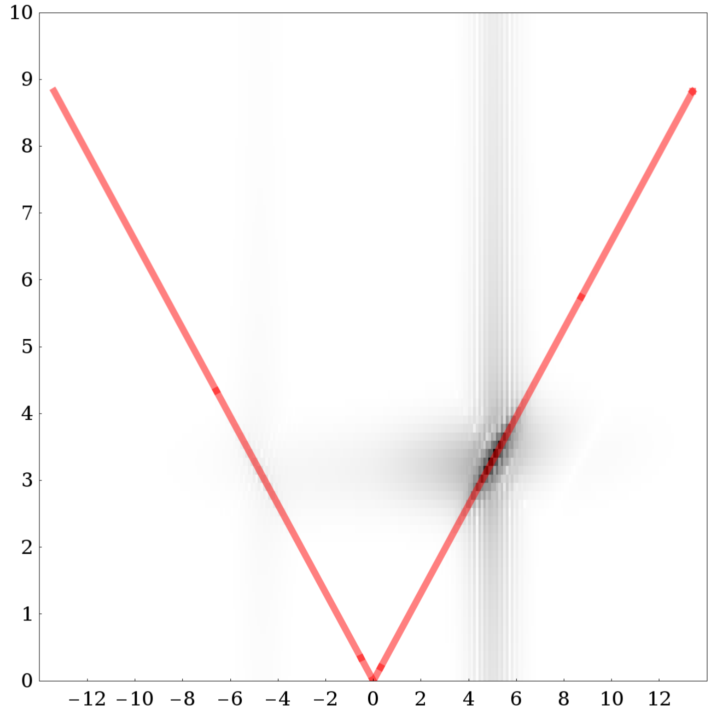
This particular run: $s_t = 2^9 \quad s_z = 2^8$

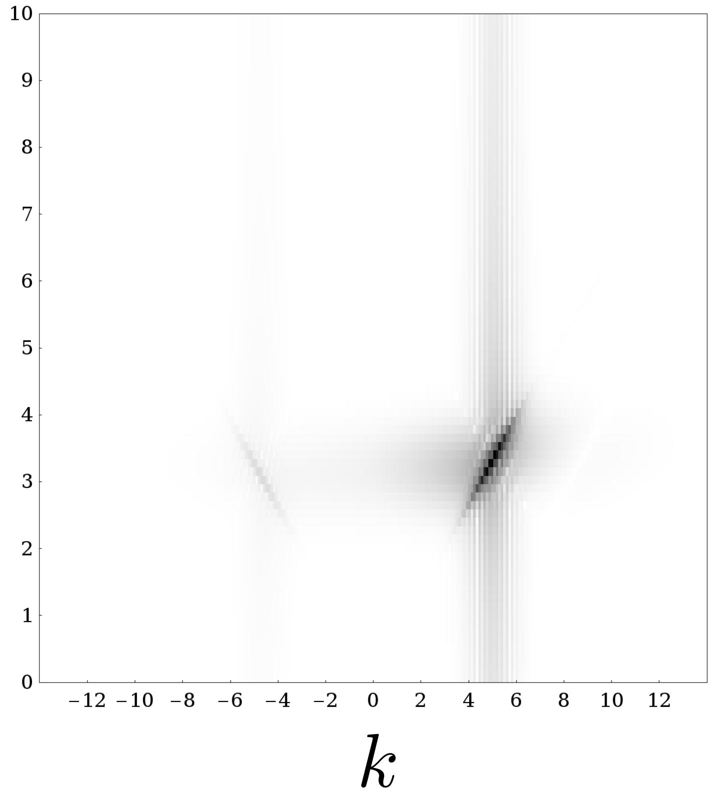
$-z_{\mathrm{min}} = z_{\mathrm{max}} = 30 \qquad t_{\mathrm{max}} = 50$

The conic dispersion



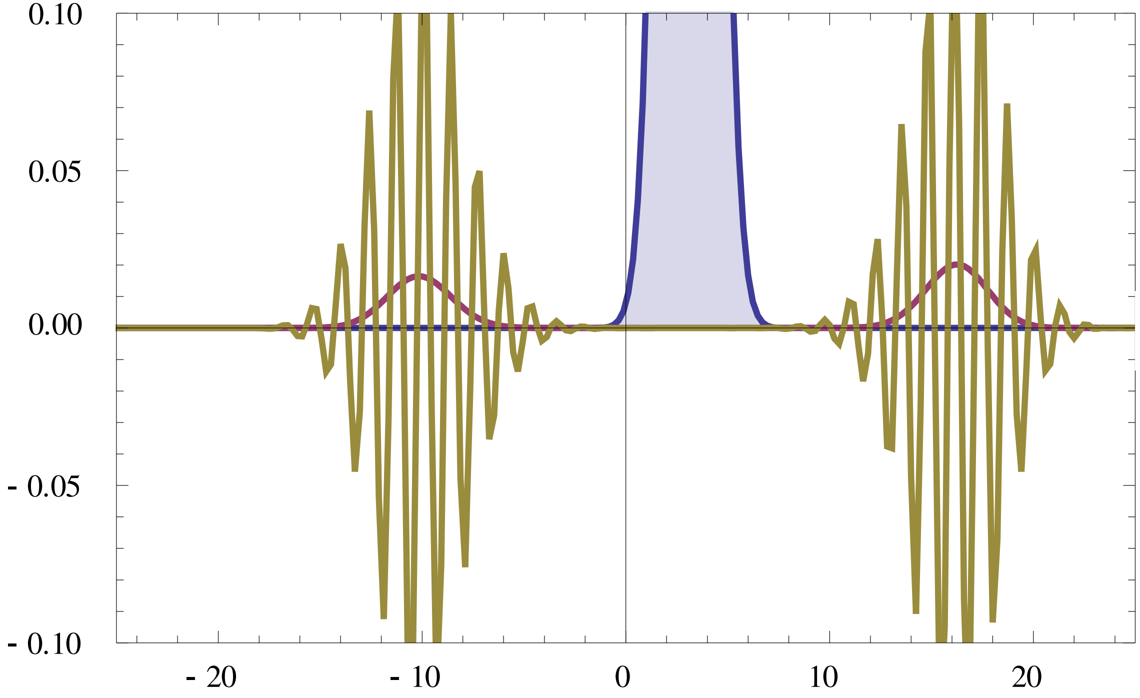






The conic dispersion

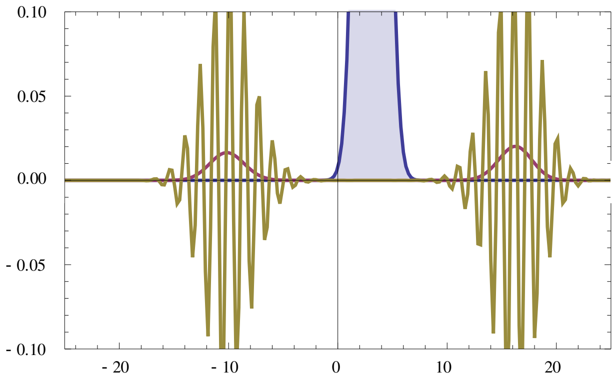
Large V

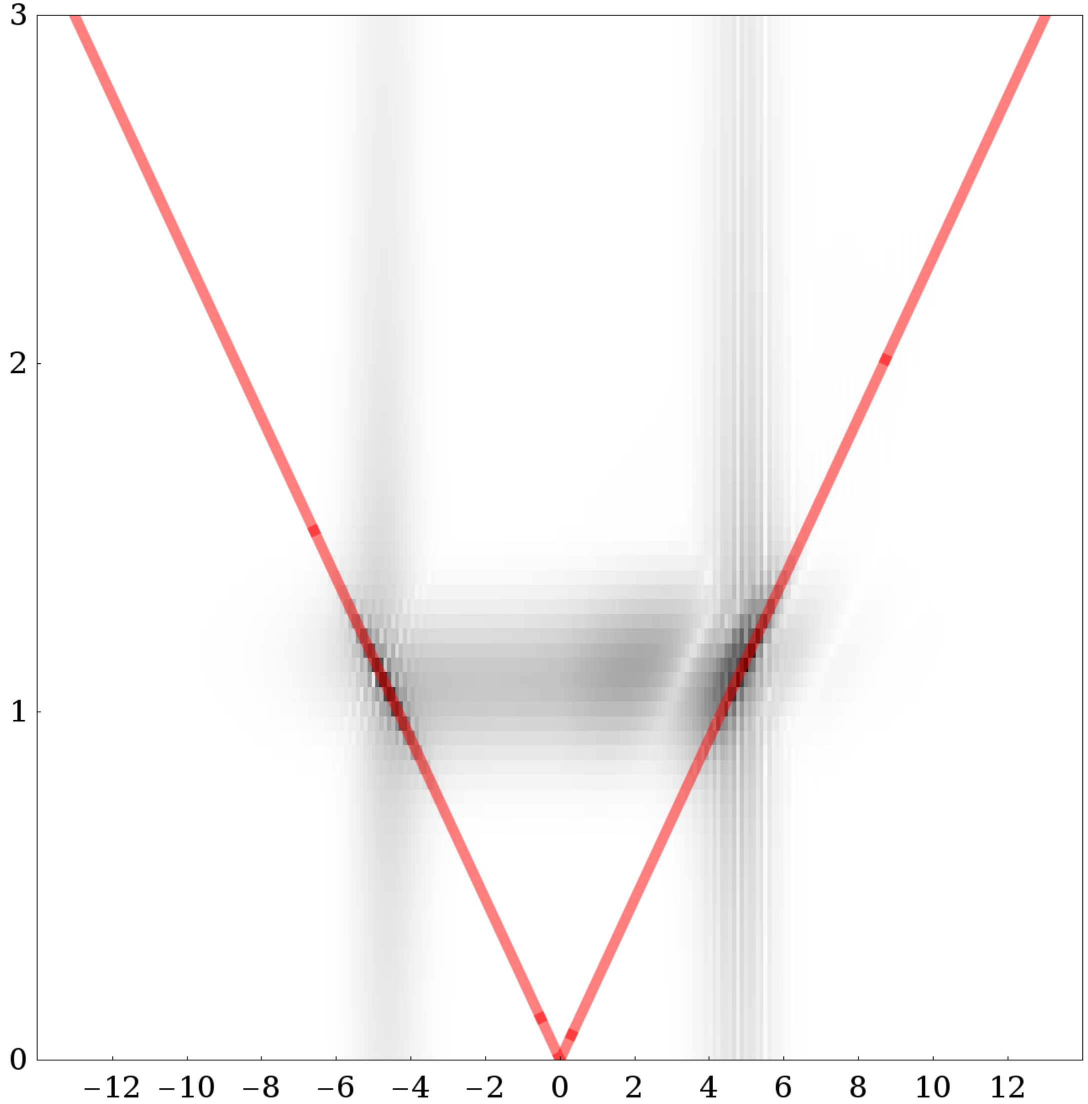


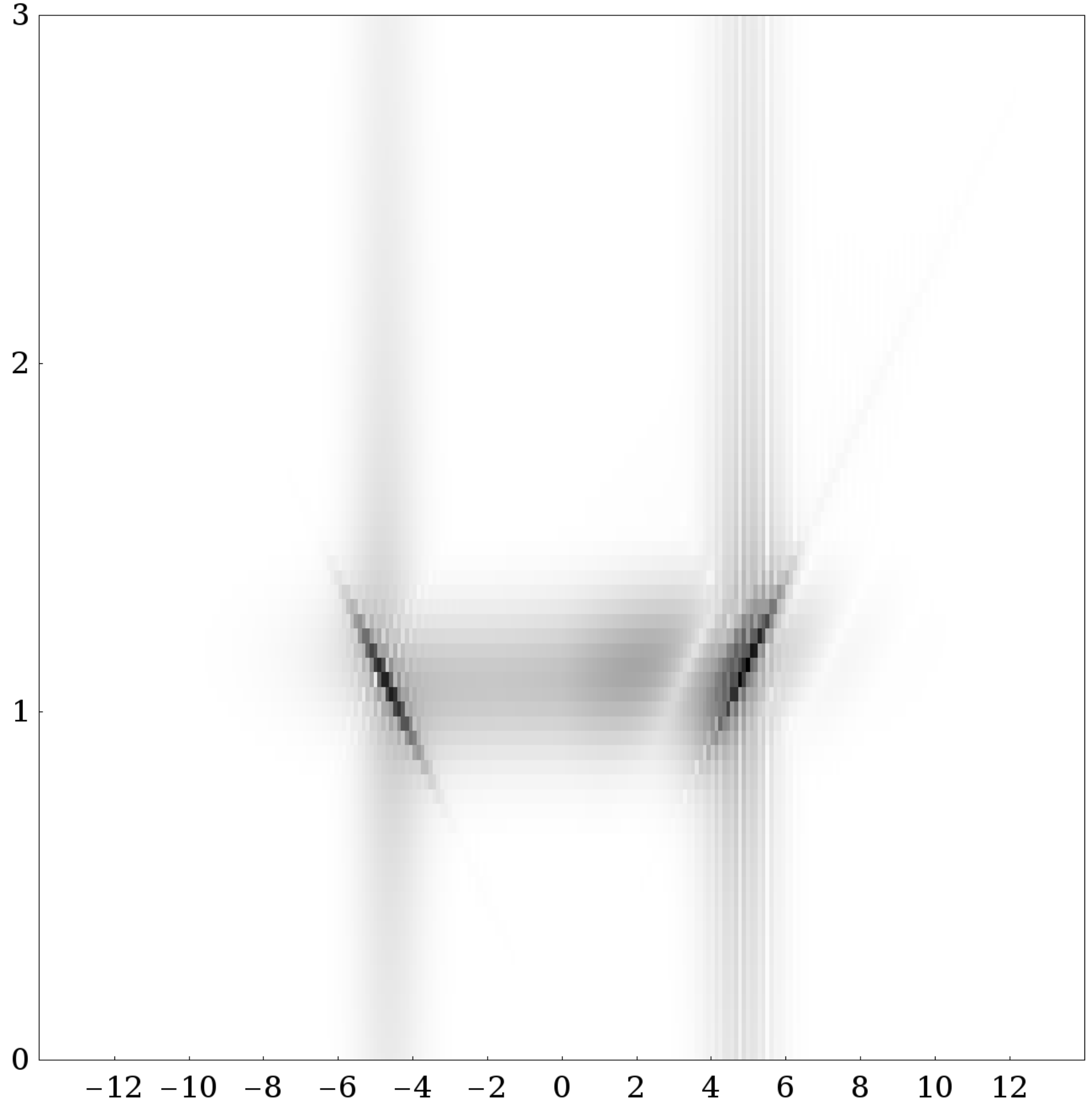
please see 2--conic.avi at this point

The conic dispersion

large potential







A preliminary note on shape preserving motion...

Is it a soliton?

Soliton like solutions of the Schrödinger equation for simple harmonic oscillator

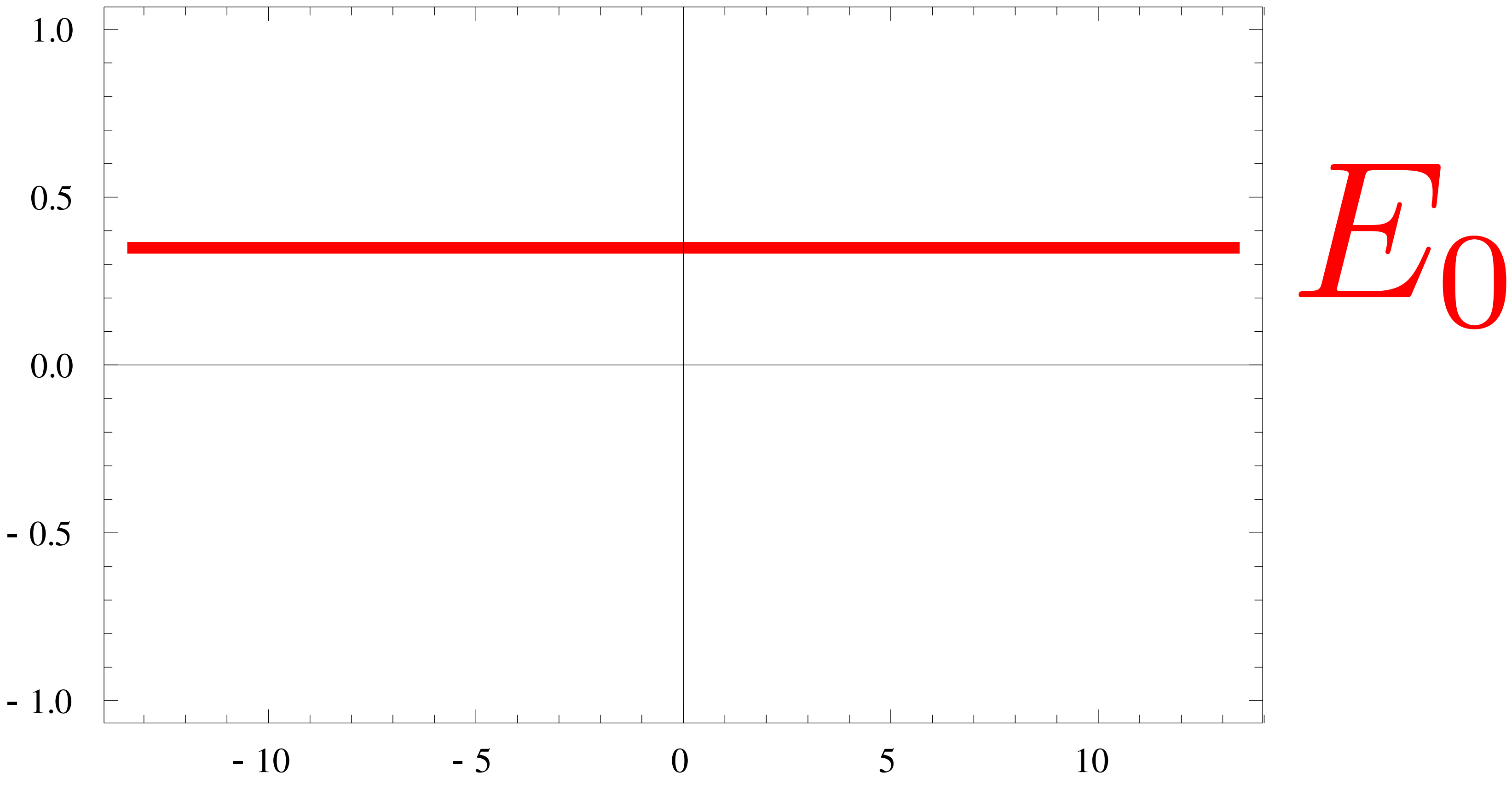
C. C. Yan
Instituto de Fisica, Universidade Federal do Rio de Janeiro, Ilha do Fundão, Rio de Janeiro, RJ, Brazil

(Received 30 November 1992; accepted 4 October 1993)

By solving exactly with a simple method the time dependent Schrödinger equation for simple harmonic oscillator, we demonstrate that the probability density associated with any displaced eigenstate has the following properties: (a) Its centroid oscillates according to the classical law. (b) Its wave packet form remains rigid. The same method is also applied to solve exactly the problem of simple harmonic oscillator with the force constant depending on time. In this case, probability densities with the same wave packet structures as that of the usual simple harmonic oscillator can be found to move in such a way that their centroids follow the classical law but their wave packets deform in time. Depending on how the force constant varies with time, the wave packets can spread, contract or pulsate.

Am. J. Phys. **62**, 147 (1994)

The flat dispersion



The flat dispersion

$$\psi(x,t) = \frac{e^{-\frac{(x-x_0)^2}{2\sigma_x^2} + ik_0 x - \frac{it\left(2E_0 + \frac{e^{-\frac{(x-x_V)^2}{2\sigma V^2}}\sqrt{2}\pi V_0}{\sigma V}\right)}{2\hbar}}}{\sqrt{2\pi}\sigma_x}$$

The wavepacket doesn't move

But its phase has a dynamics...

Galilean boost

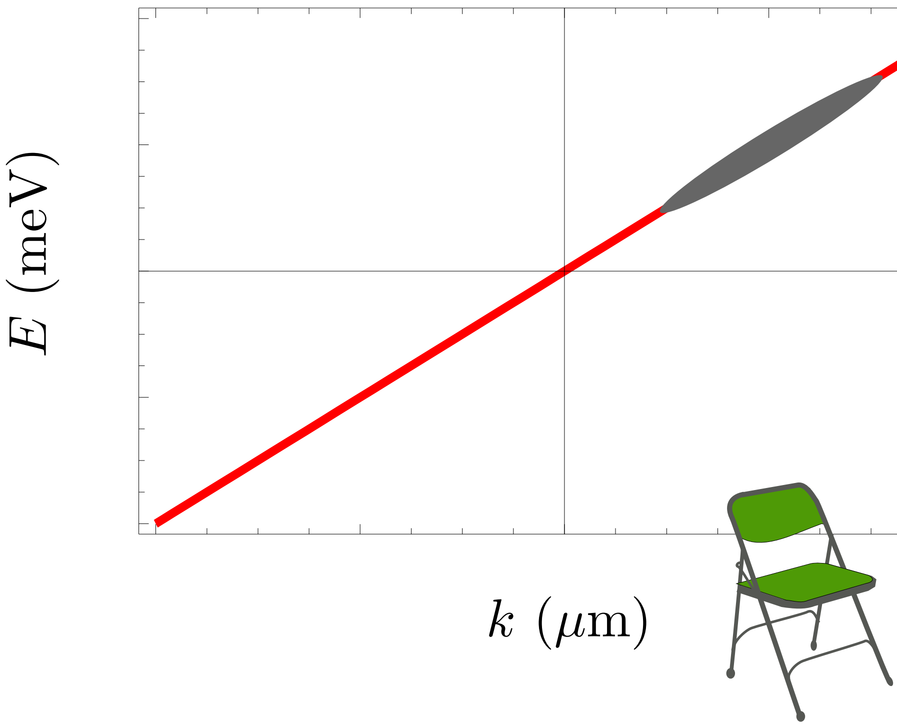
It is "known" that Schrödinger equation
[for the free (spinless) particle]
is not *Galilean invariant.*

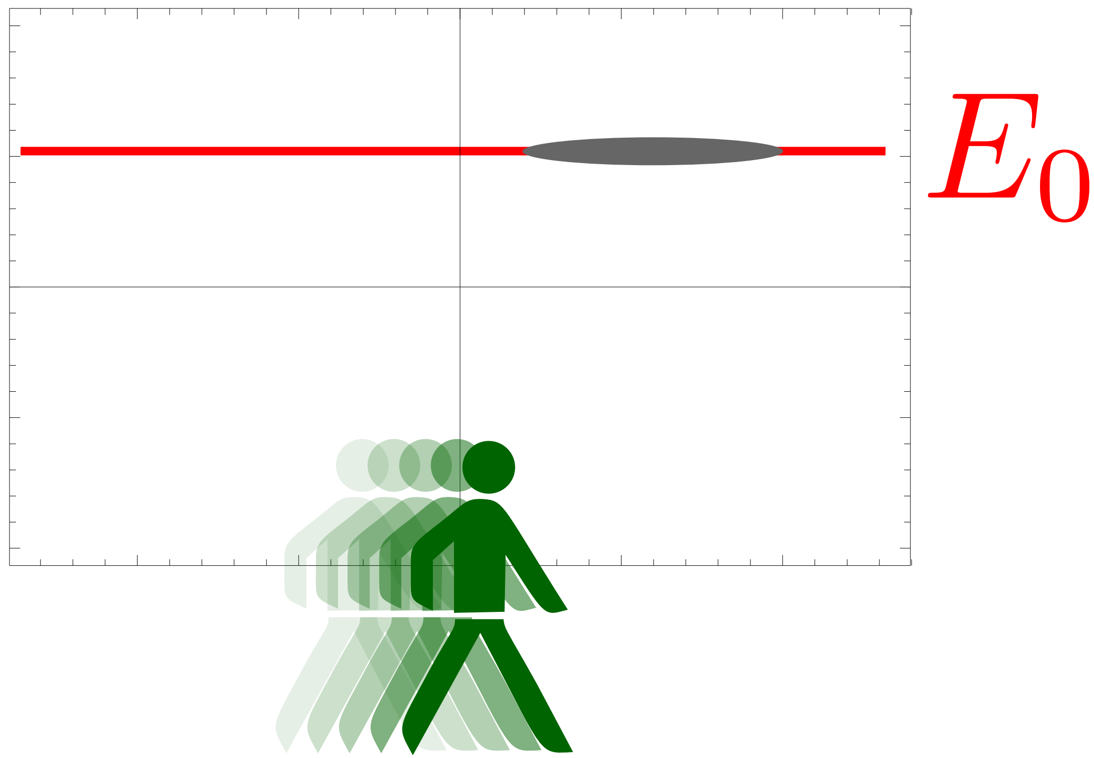
It is covariant, *but in a "clumsy" way:*

The "boost" is mass dependent.

Galilean boost

For the "free wave" wavepacket, the boost is k-dependent:



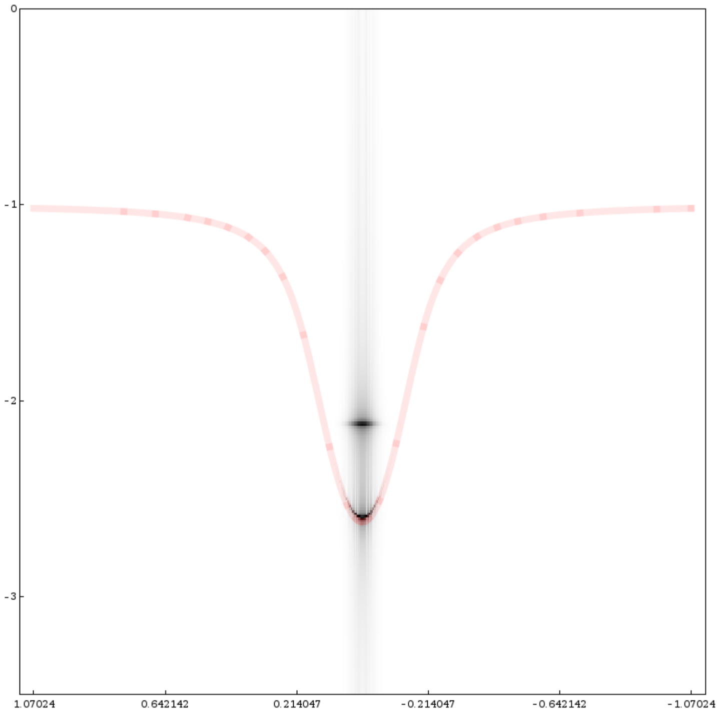


Pumping

$$i\hbar\partial_t |\psi\rangle = (D + V + \mathcal{I} + \Gamma)|\psi\rangle + F$$

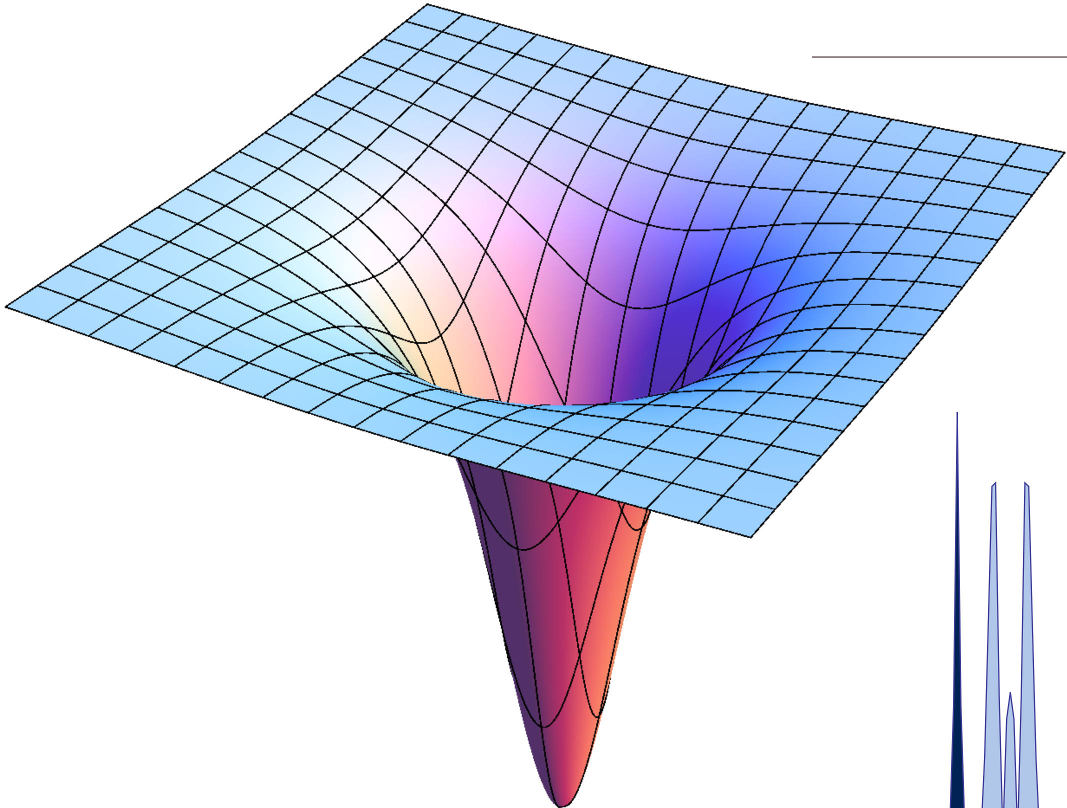
$$\psi(\mathbf{k},t) = \frac{F_{\mathrm{P}}(\mathbf{k})}{E_{\mathrm{LP}}(\mathbf{k}) - \hbar\omega_{\mathrm{P}} - i\frac{\gamma}{2}}\left(e^{-i\omega_{\mathrm{LP}}t}e^{-\gamma t/(2\hbar)} - e^{-i\omega_{\mathrm{P}}t}\right)$$

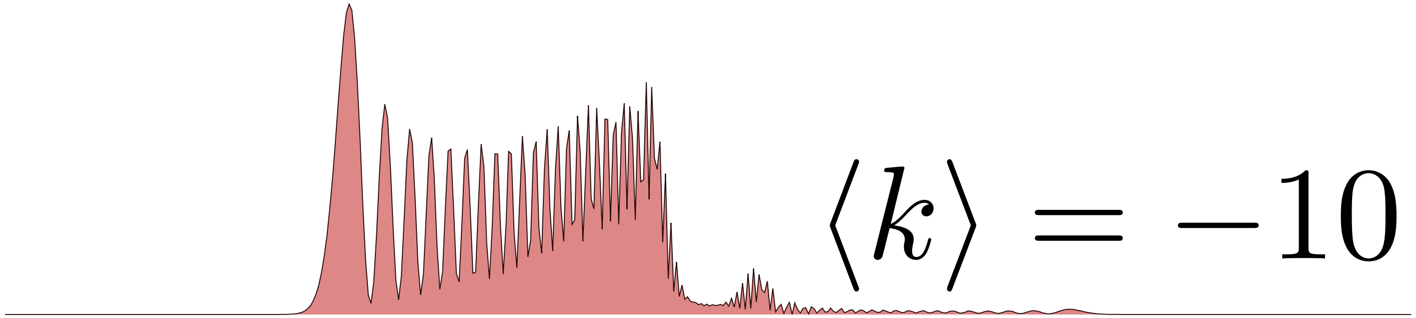
$$+\,\psi(\mathbf{k},0)e^{-i\omega_{\mathrm{LP}}t}e^{-\gamma t/(2\hbar)}$$



The coherent pumping drives a states that has the shape of a cross +

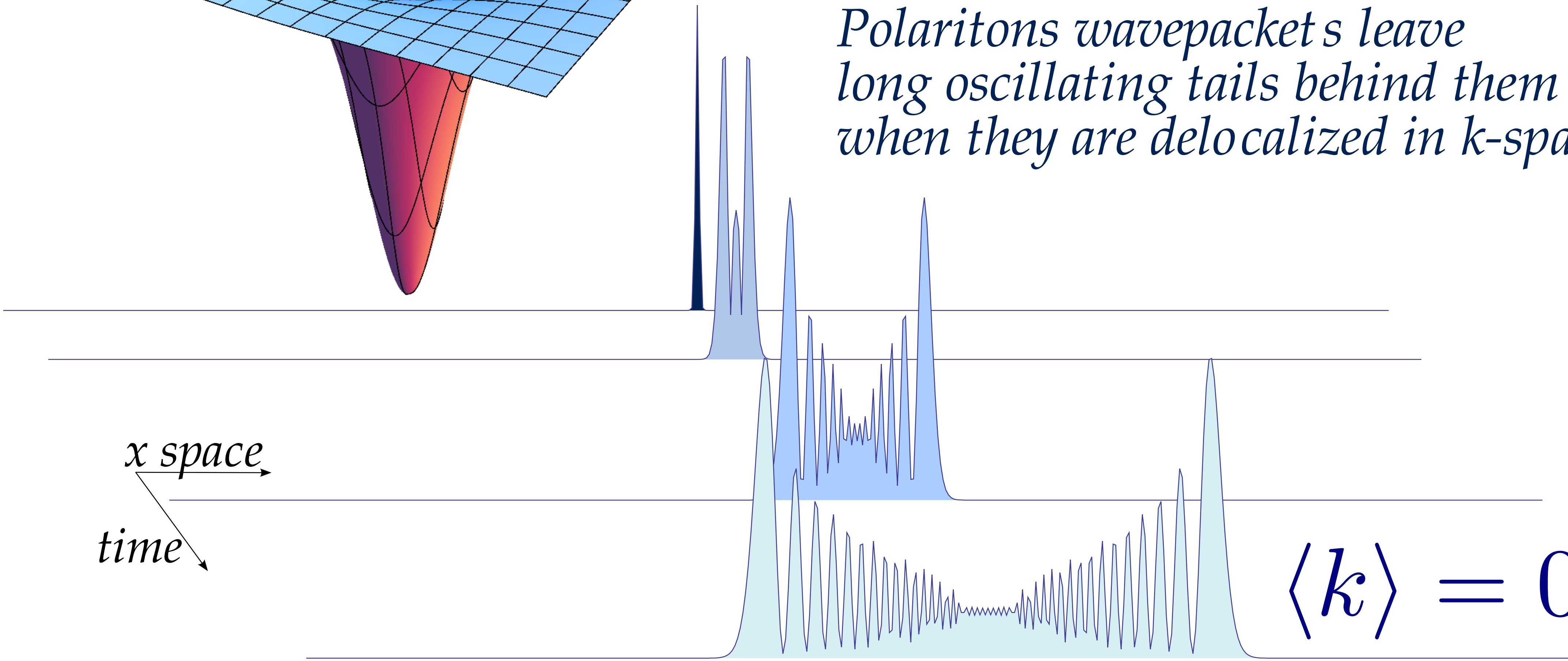
Polaritons



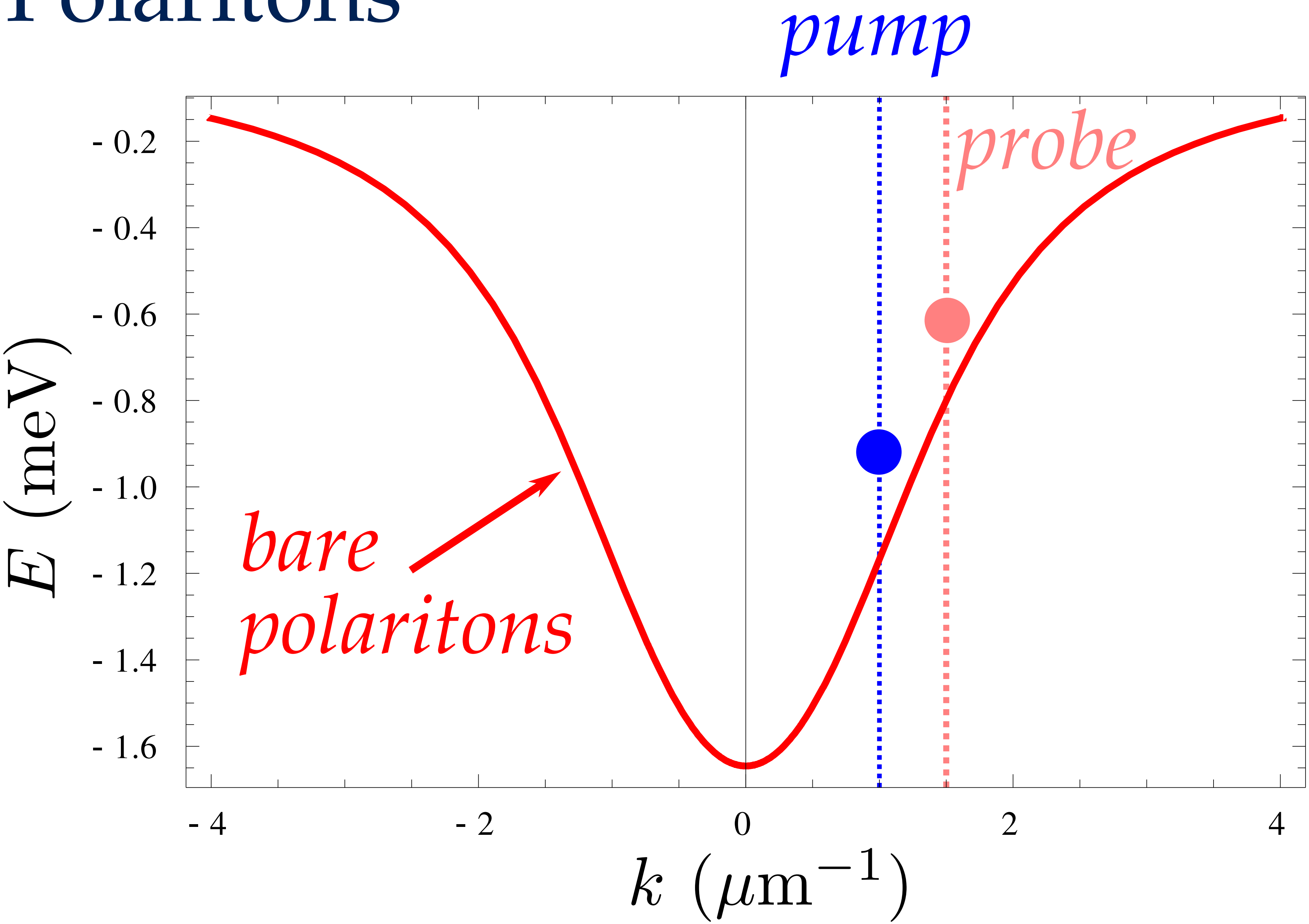


Their free-space propagation is already interesting

Polaritons wavepackets leave long oscillating tails behind them when they are delocalized in k-space

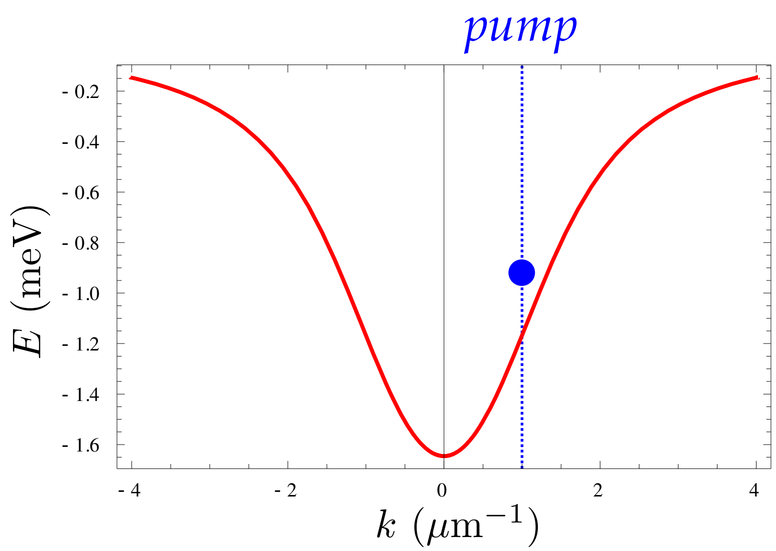


Polaritons



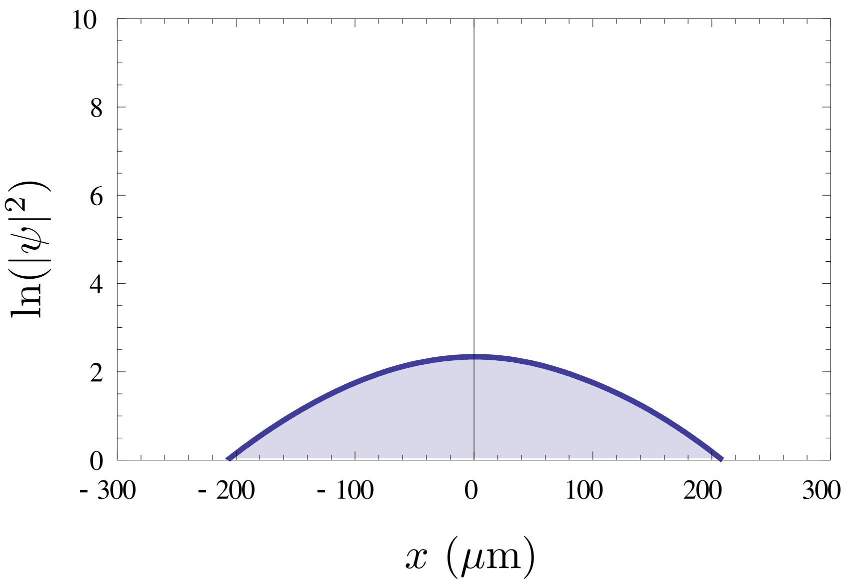
pump
probe
bare
polaritons
- 0.2
- 0.4
- 0.6
- 0.8
- 1.0
- 1.2
- 1.4
- 1.6
E (meV)
- 4
- 2
0
2
4
k (μm^{-1})

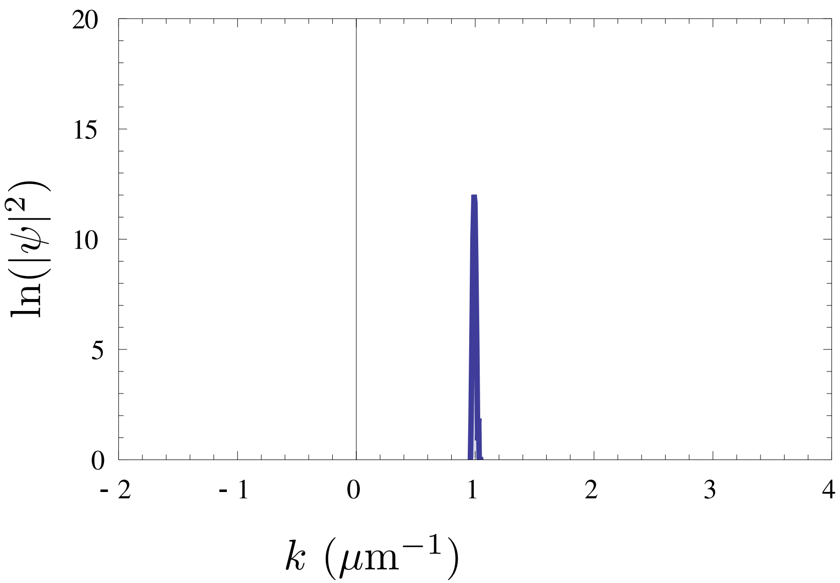
Polaritons



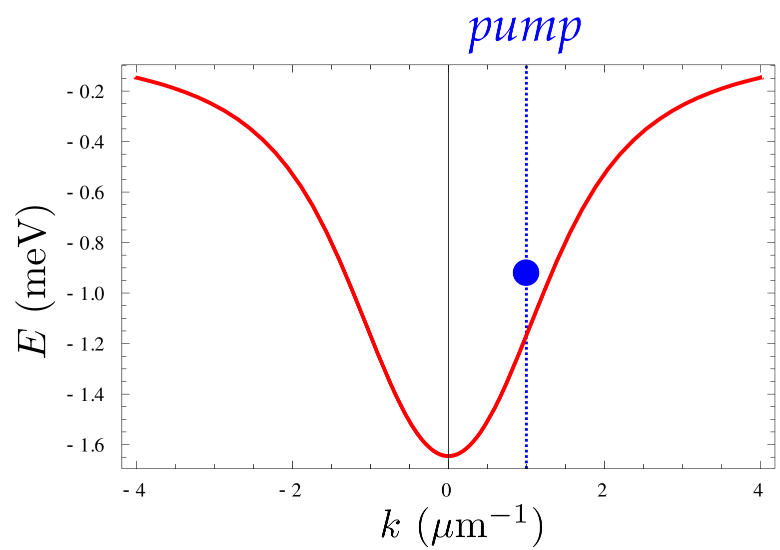
Our procedure:

1 - We reach the "steady state" with the pump only (from vacuum).



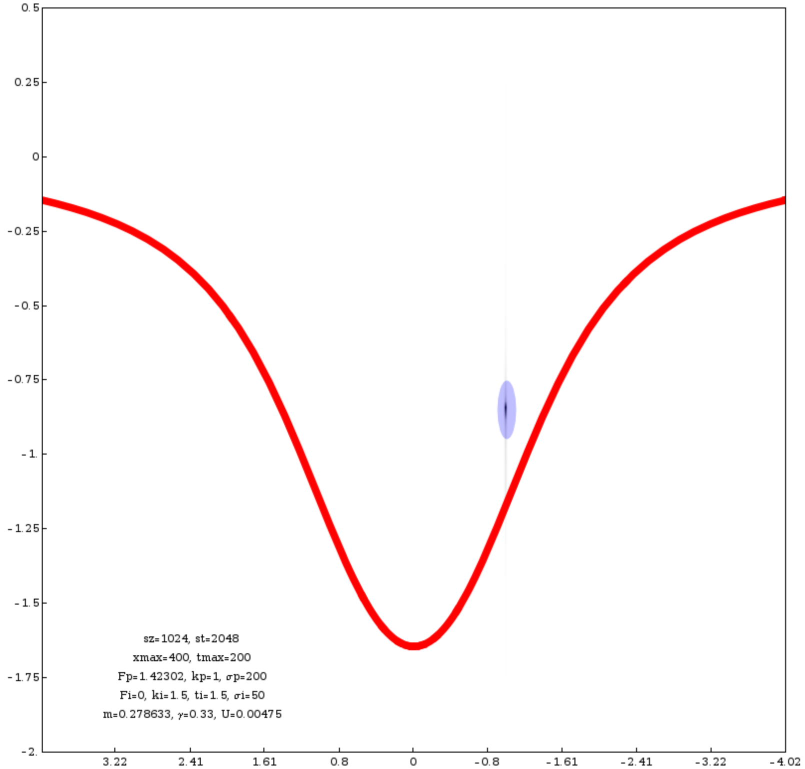


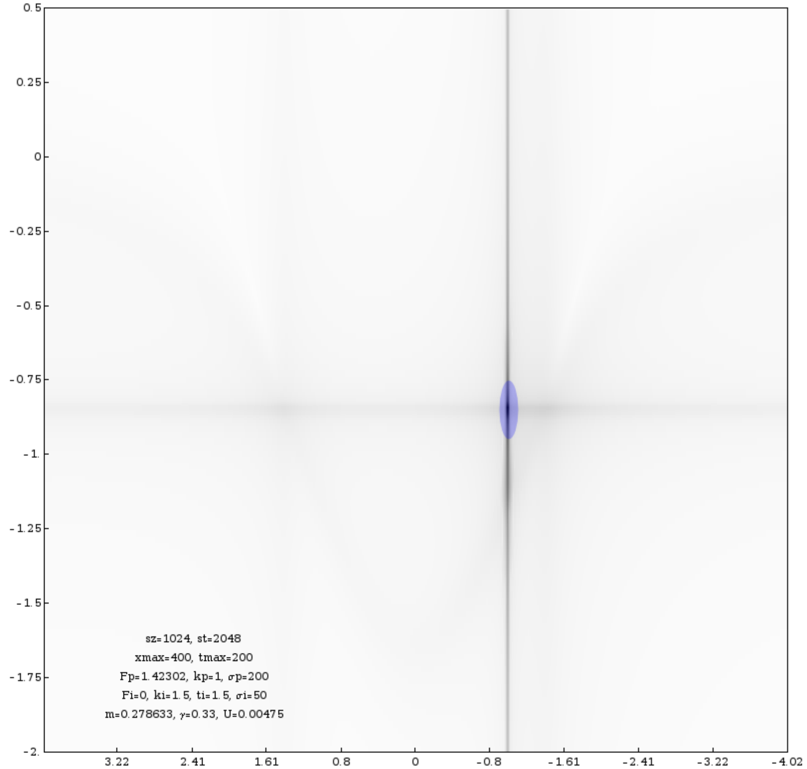
Polaritons



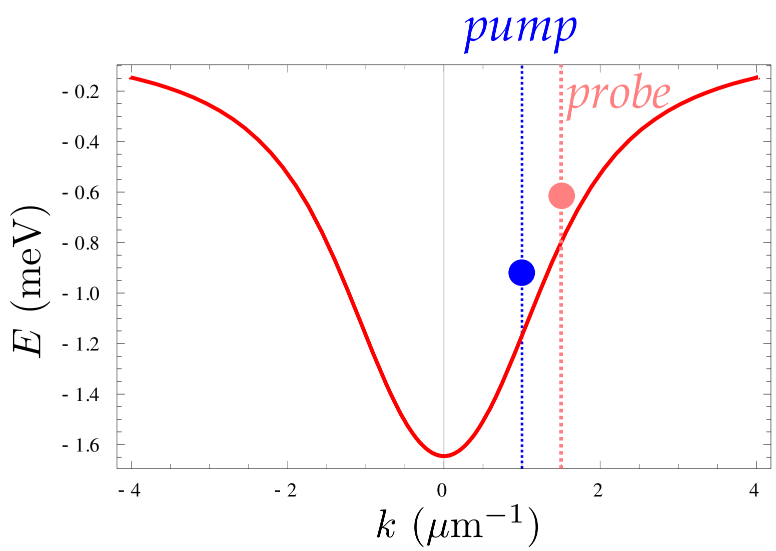
Our procedure:

In this particular case,
we don't go above OPO threshold, that is numerically unstable



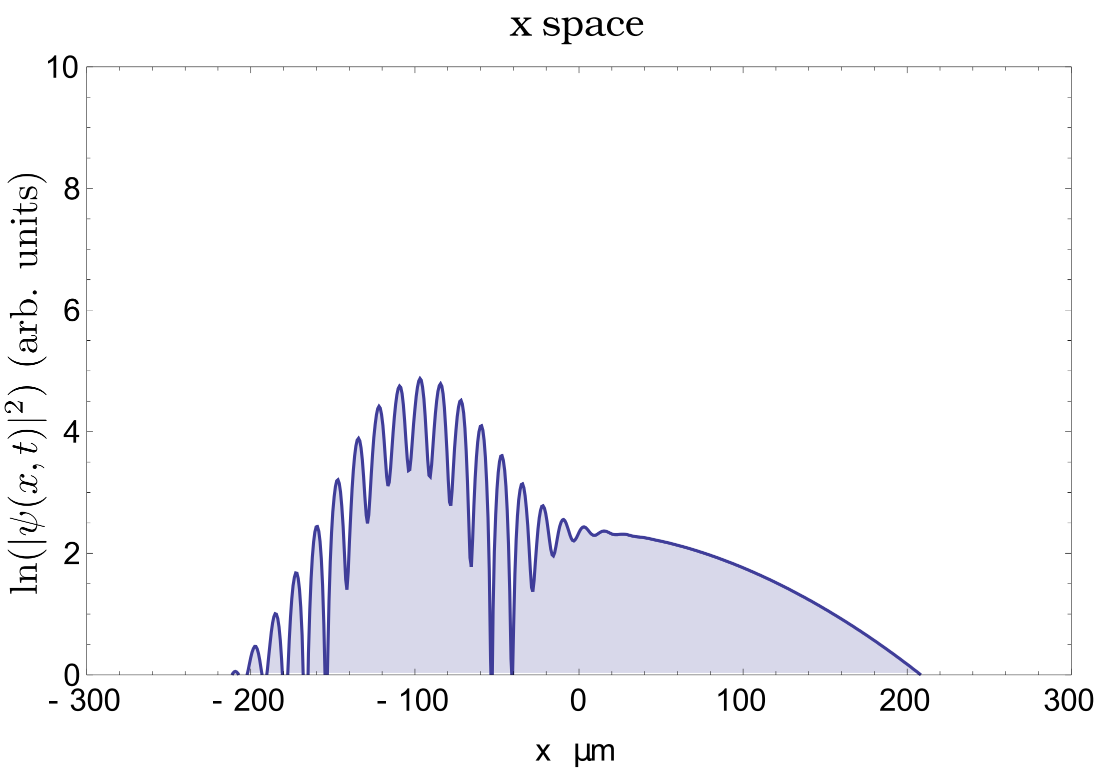


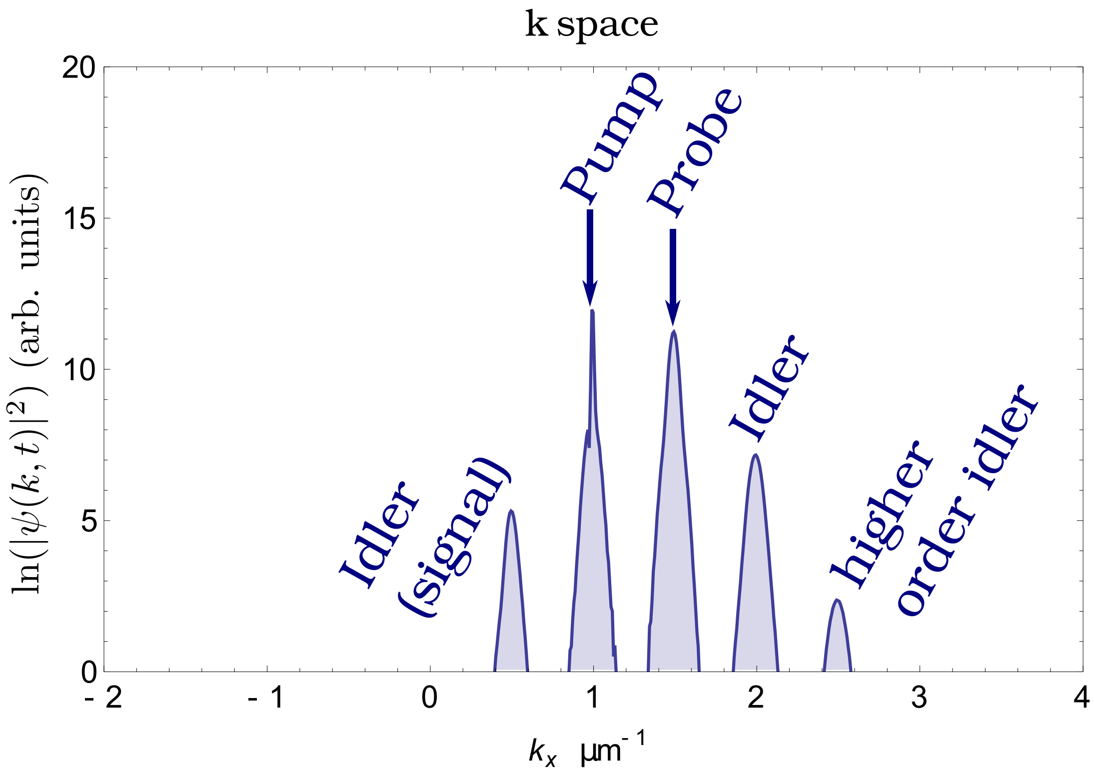
Polaritons



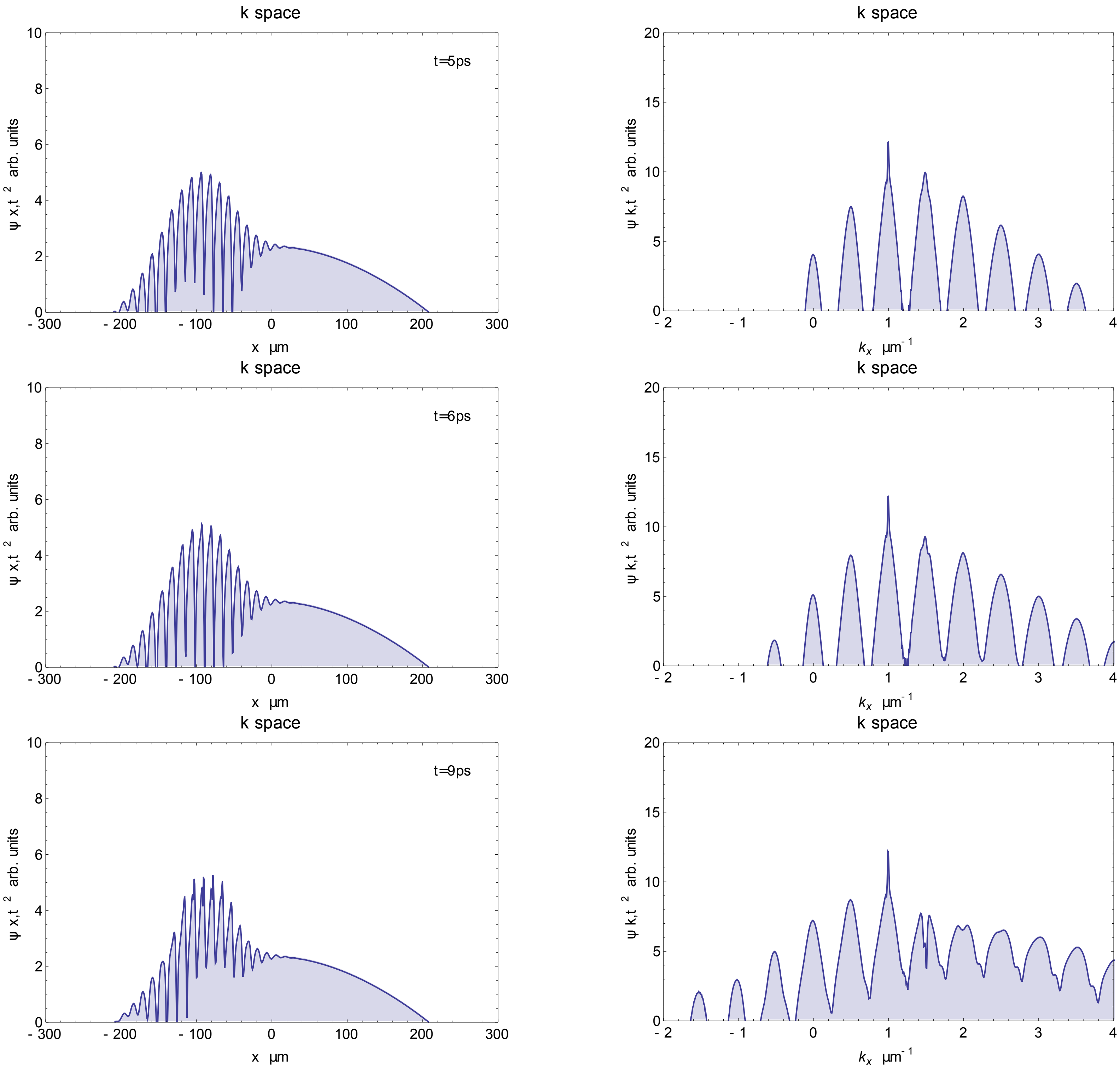
Our procedure:

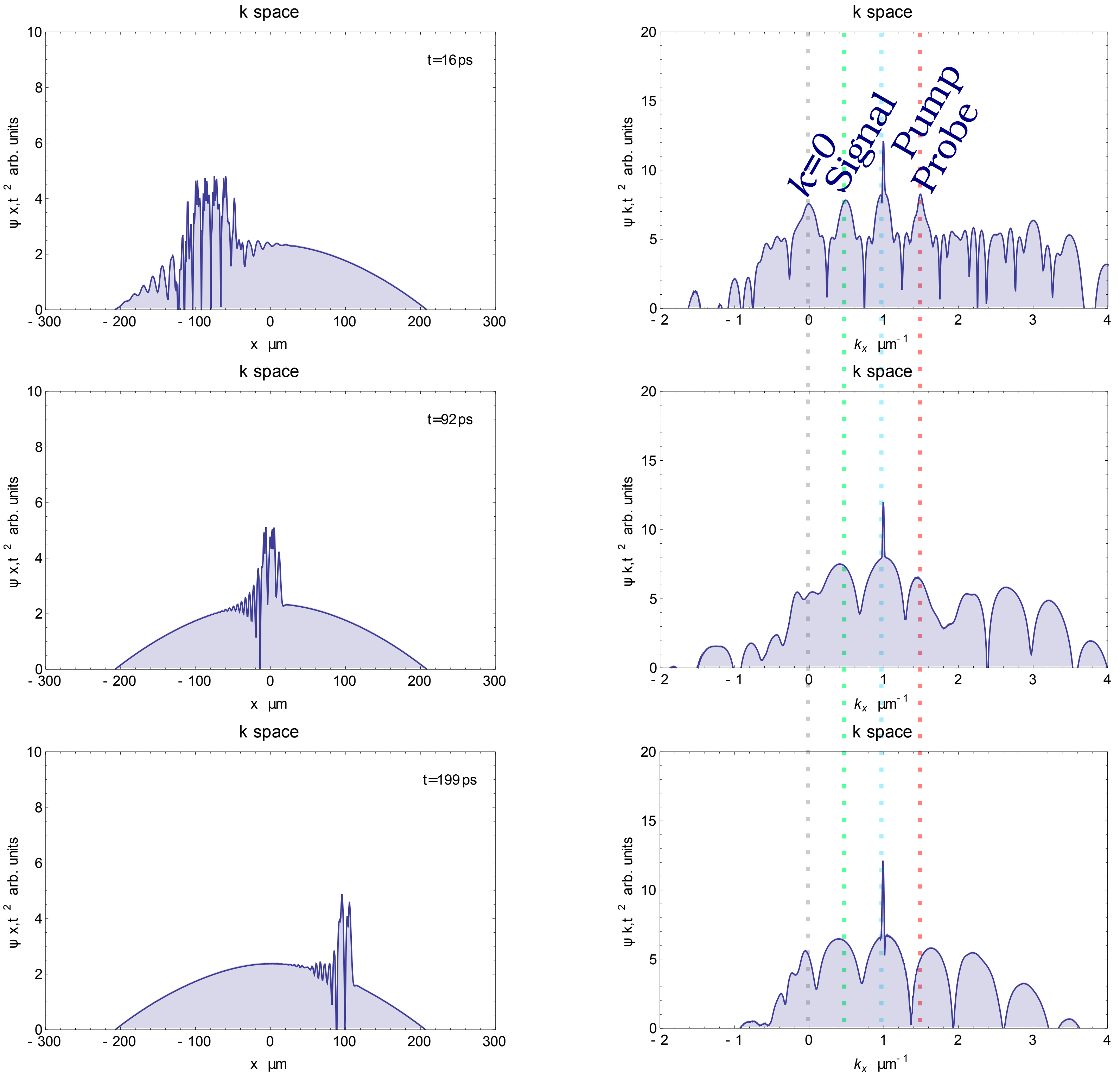
2 - We kick the "steady state" with a probe.





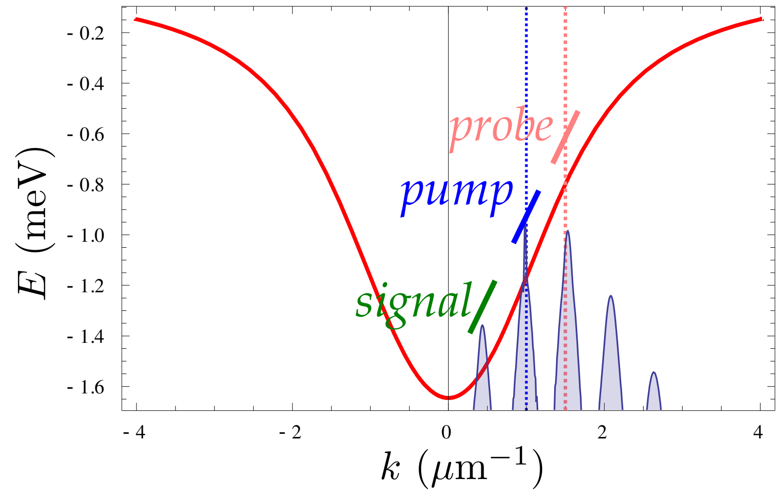
log scales


k space
ψ x,t 2 arb. units
t=5ps
x μm
k space
ψ k,t 2 arb. units
kx μm-1
k space
ψ x,t 2 arb. units
t=6ps
x μm
k space
ψ k,t 2 arb. units
kx μm-1
k space
ψ x,t 2 arb. units
t=9ps
x μm
k space
ψ k,t 2 arb. units
kx μm-1



please see
3--dyn-xk.avi
at this point

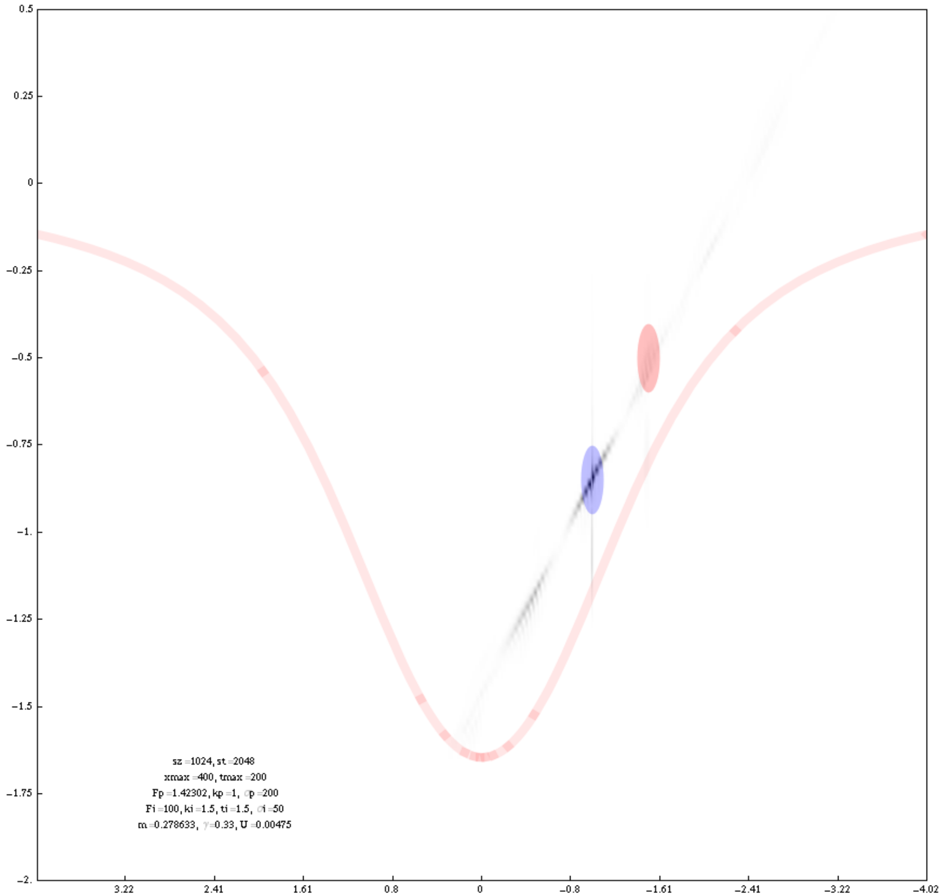
Polaritons



Our procedure:

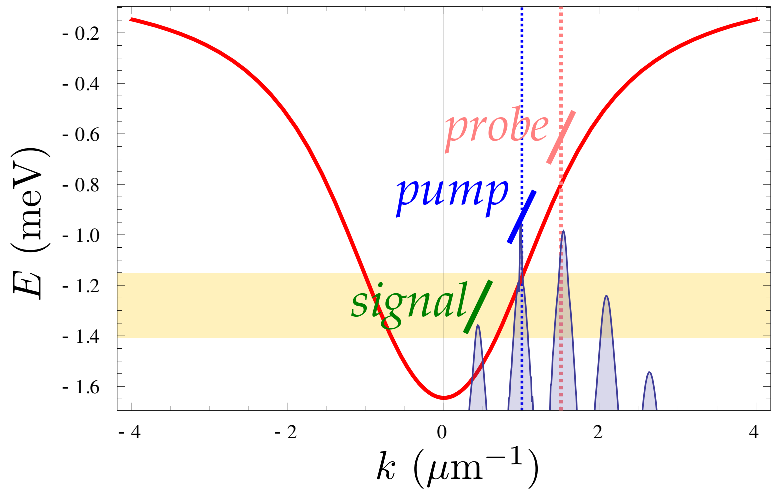
3 - We analyze the spectra.

(we have subtracted the "dull" & strong steady state spectrum)



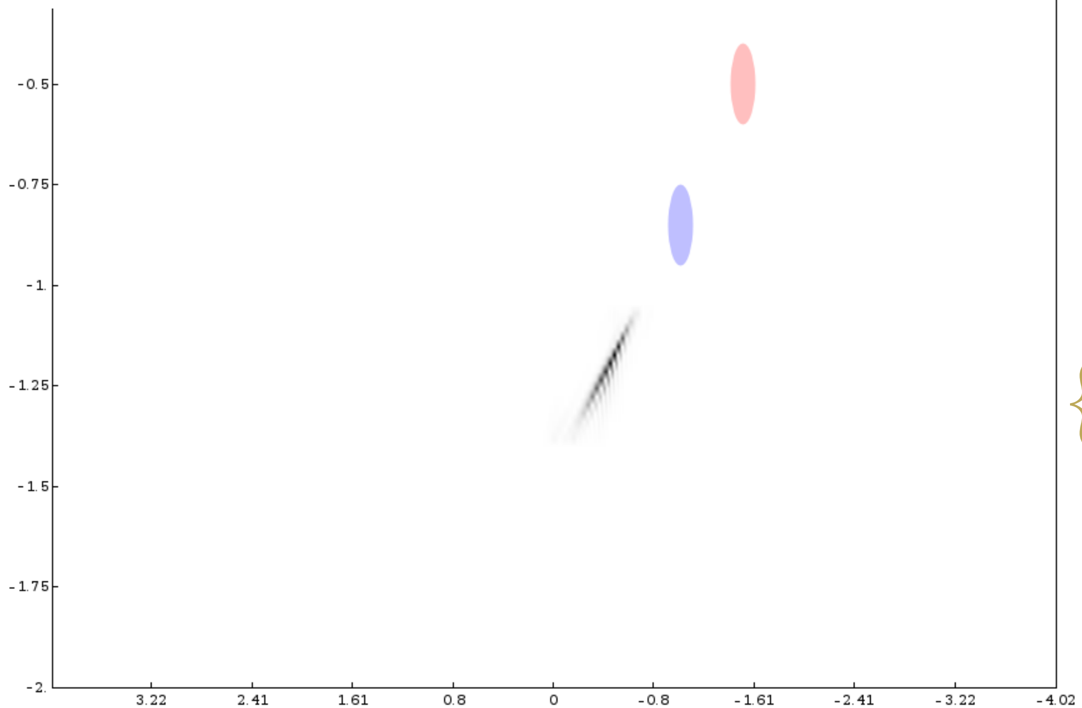
Linearization of phase-matched pair-scattered final states

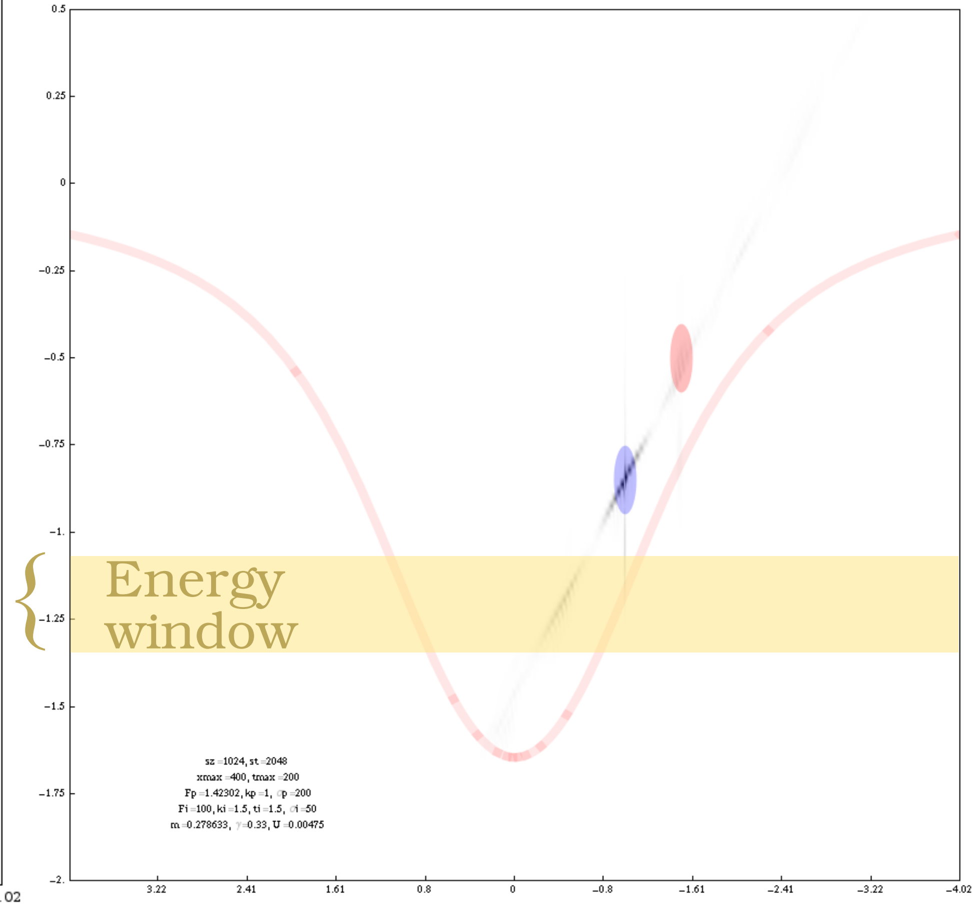
Polaritons



Our procedure:

4 - We filter out the signal.

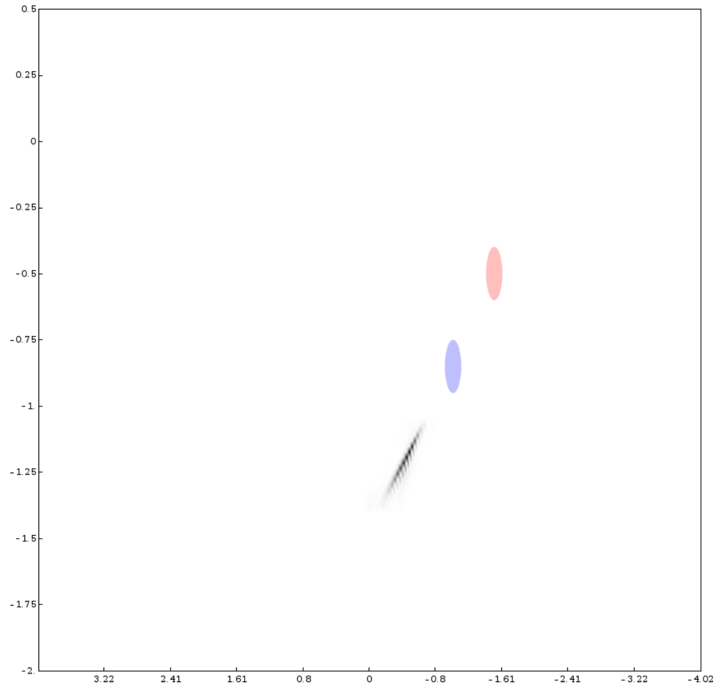


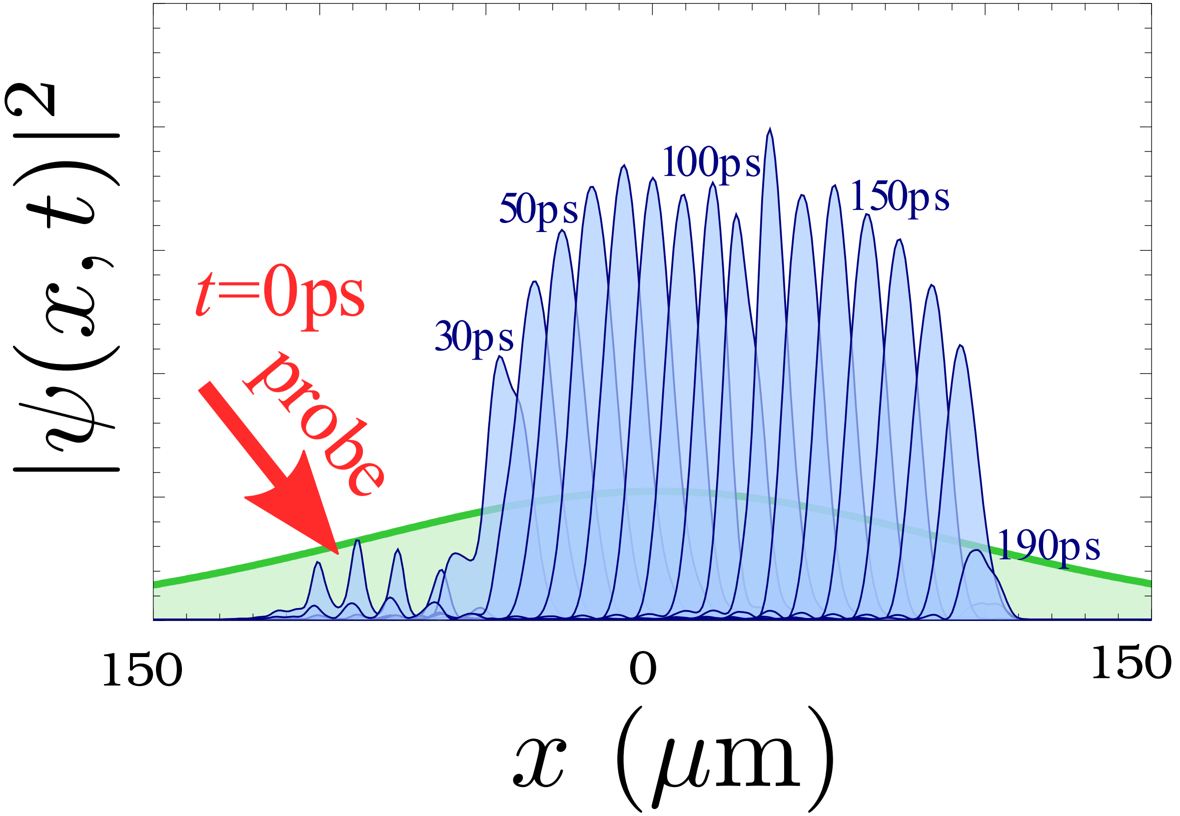


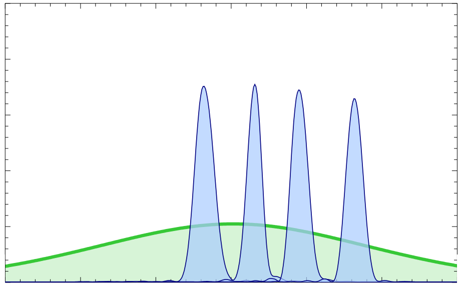
Polaritons

Our procedure:

5 - We go back in real-space







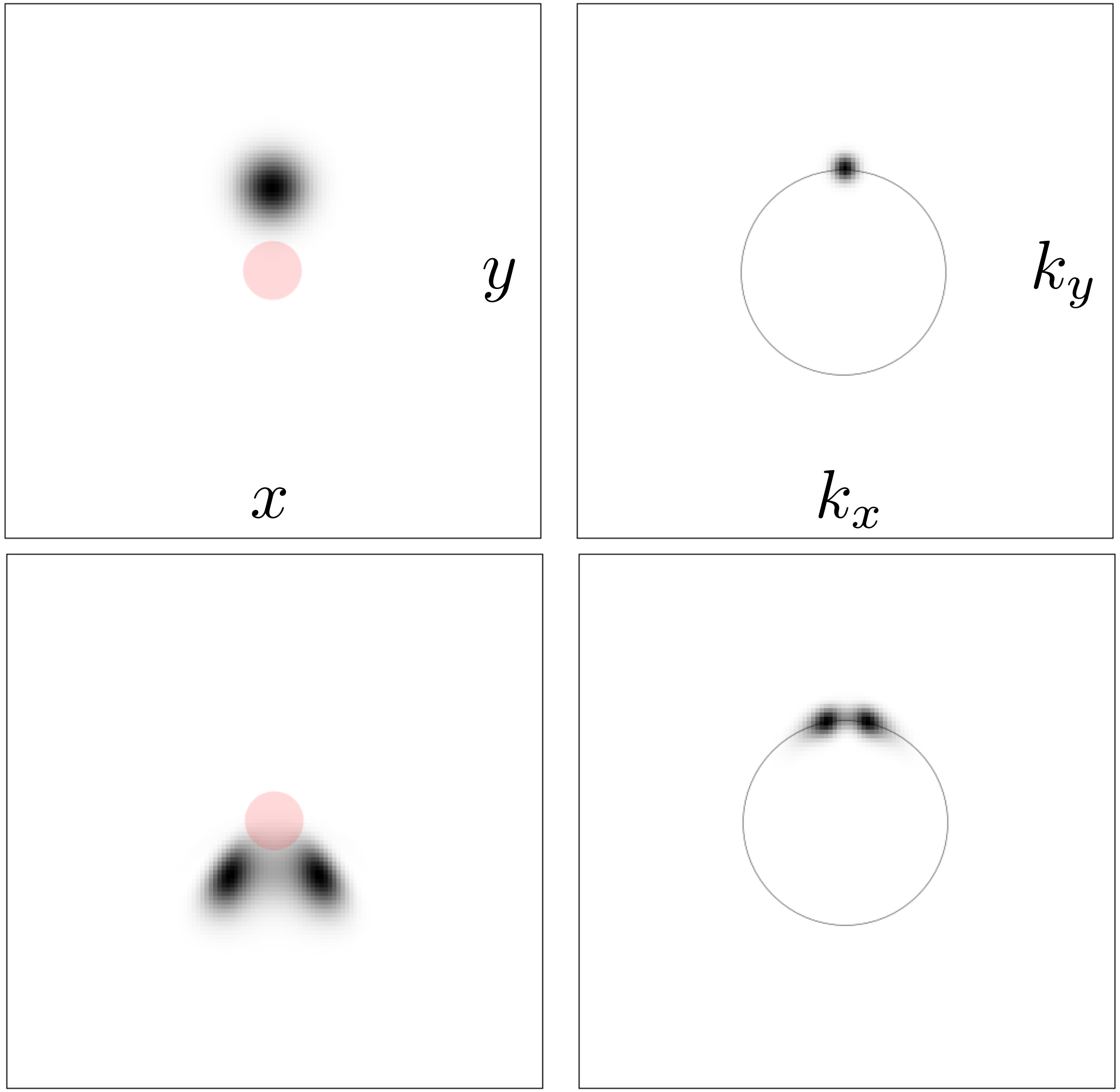
A polariton bullet propagates!

Collisions

(please see 5--fragmentation.avi at this point)

t_1

t_2

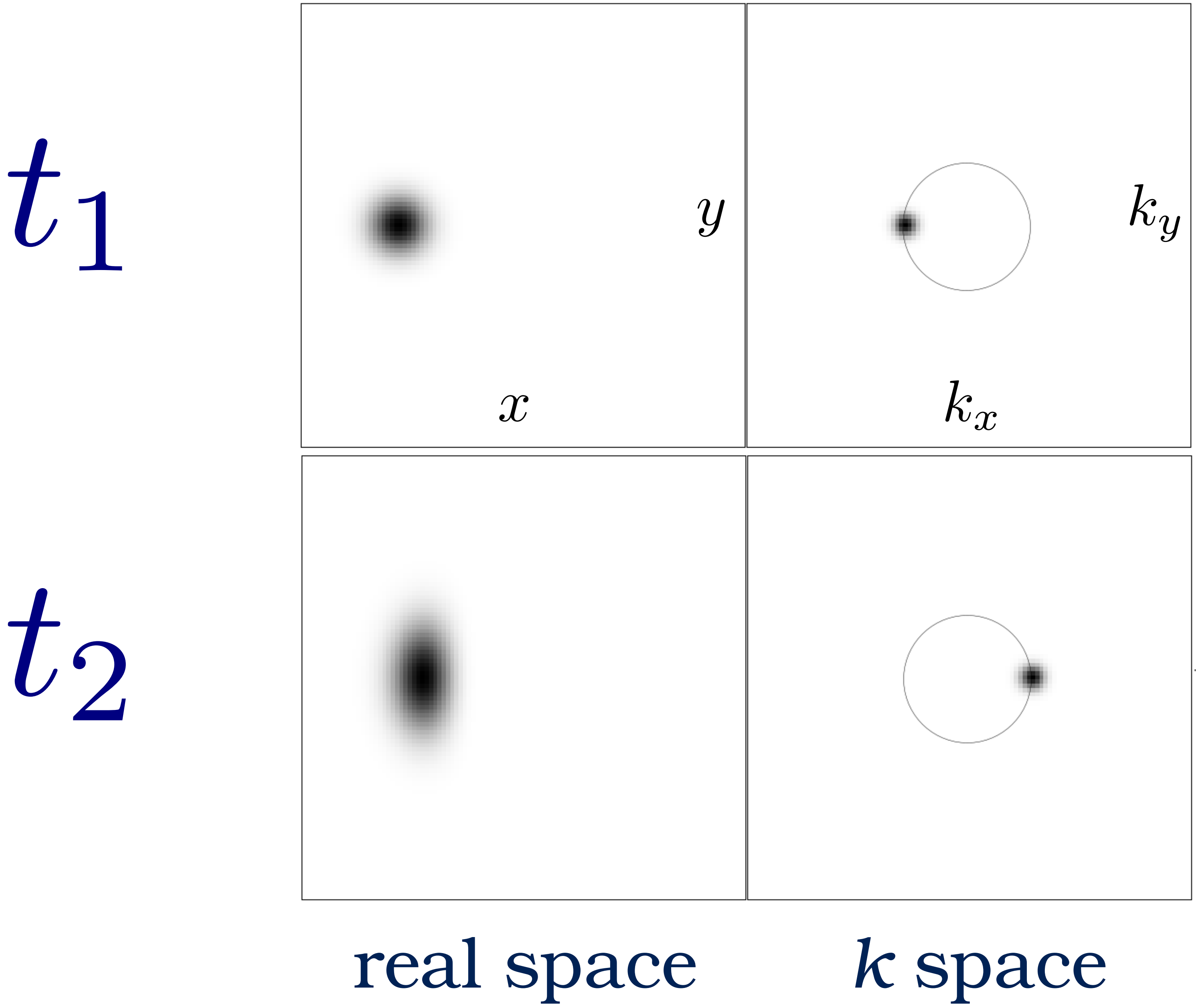


real space

k space

Collisions

(please see 6--bouncing.avi at this point)



Conclusions

- Microcavity polaritons open new possibilities for investigating quantum wavepacket propagation.
- Ultra-fast propagation of a quantum light-matter wavepacket reported by the Madrid group.
- Propagation of an interacting polariton BEC suggests Superfluidity, but of a non-conventional type. The same applies for its solitonic properties.
- Striking behaviours of fundamental physics with palpable technological applications.
- Still require extensive experimental & theoretical investigations.